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Stone–Geary meets CES: the properties of an extended linear expenditure system

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Abstract

The structure of demand plays a fundamental role in shaping economic equilibrium. In computable general equilibrium analysis, comparative statics allow us to simulate how policies or external adjustments impact the equilibrium state. The results of these simulations are significantly influenced by the nature of the adopted demand structure. Typically, homothetic demand structures are frequently used for simplicity in these equilibrium models, but they do not accurately capture demand behavior. In this text, we study the properties of a demand system that eliminates demand constraints often found in general equilibrium models. Specifically, we first explore the characteristics of a demand system that generalizes the standard Stone–Geary linear demand structure. Second, we illustrate its possibilities with an empirical application using available Spanish data.

Keywords: Linear expenditure systems, Constant elasticity of substitution, Calibrated demand functions, Computable general equilibrium

JEL Classification: C55, D11, D12

1 Introduction

The Stone–Geary utility function (Geary 1950; Stone 1954) is an extension of the Cobb–Douglas function that incorporates minimum (or subsistence) levels of consumption. It can be seen as a shifted version of the Cobb–Douglas function, where the shifting is defined by the minimum consumptions. A notable and well-known feature of the Stone–Geary function is that its demand function results in a linear expenditure system (LES; Deaton and Muellbauer 1980, chapter 3). For any given good, the total expenditure includes the expenditure for its minimum consumption plus a fixed proportion of the supernumerary income.

The Stone–Geary utility function is commonly employed in the field of multi-sectoral and computable general equilibrium (CGE) models to overcome the limitations of the more restrictive, yet easily applicable, Cobb–Douglas functions, particularly their homotheticity. For instance, Logfren et al. (2002) developed the well-known CGE model of the International Food Policy Research Institute using a LES demand system, whereas Washizu and Nakano (2010) embedded a LES demand system within an input–output quantity model to study the environmental impact

of changing lifestyles. More recently, Guerra et al. (2022) used a LES demand specification within a CGE model to evaluate the deadweight loss of taxation, while De Boer et al. (2023) compared LES with alternative econometric specifications of the demand system.

The advances in computing software have greatly facilitated the widespread use of CGE models. This diffusion has had two significant consequences worth mentioning. On the positive side, general equilibrium analysis has become a common instrument in the economist's toolkit, enabling the study of situations that would have been unthinkable a few decades ago. However, on the negative side, some contributions have fallen victim to the 'black box syndrome', meaning that the structure of the models becomes opaque, and the underlying theoretical properties are often forgotten or neglected, making it difficult to rationalize the results. The more complex a model is, the more likely it is to fall into the syndrome if we do not explicitly clarify the basic theoretical properties that justify its structure. For these reasons, it is necessary that the functional forms adopted within the context of a model, CGE or otherwise, are adequately justified in terms of their empirical representativeness and well understood in terms of their theoretical implications. The increasing use of LES-type specifications falls within this consideration; hence, the need to understand their technical properties in detail as a minimum guarantee that the implications of their use are well understood and that the interpretations derived from the results are in accordance with the theory used.

In the above context, the Stone–Geary function still exhibits some limitations that do not align well with empirical evidence. One such limitation is the unitary elasticity of substitution implicit in the base Cobb–Douglas function. Another limitation is that the proportion of supernumerary income in the demand function is fixed and is not responsive to changes in prices. These limitations have led to the adoption of more general utility functions of the Stone–Geary variety, as seen in the structural change literature (Herrendorf et al. 2013; Comin et al. 2021) or in CGE modeling (Bchir et al. 2002). These extensions of LES use a constant elasticity of substitution (CES, Arrow et al. 1961) shifted function instead of the typical Cobb–Douglas function. However, the technical background and the conceptual advantages of using this extension to remove some of the limitations of standard LES have not been explored in detail.

The purpose of this brief note is therefore twofold. First, we demonstrate how the use of CES shifted functions helps eliminate constraints affecting the LES system, providing a formal explanation for this. We also explicitly demonstrate various properties of this formulation, properties that are often ignored or insufficiently known in some cases. Second, we illustrate how the standard LES identification procedure in CGE modeling (Jussila et al. 2012) easily generalizes to the case of a shifted CES function. For this purpose, we rely on recent estimates of the income elasticity of demand in the Spanish economy (Garcia-Villar 2018).

The paper adopts the following structure. In Sect. 2, we recall the essential properties of the LES system. In Sect. 3, we demonstrate the properties of the CES shifted expenditure system and compare it with the properties of standard LES. Section 4 presents an example of a calibrated demand function under the CES shifted property. Section 5 concludes.

2 The Cobb–Douglas shifted linear expenditure system

For the n -good case the Stone–Geary utility function is defined by:

$$u(x_1, x_2, \dots, x_n) = \prod_{j=1}^n (x_j - z_j)^{\alpha_j}, \quad (1)$$

where $z_j \geq 0$ are minimum levels of consumption and α_j non-negative weights that add up to 1. The solution to the utility maximization problem under the budget constraint imposed by income level m and given minimum consumptions z_j yields demand functions:

$$x_j = z_j + \frac{\alpha_j}{p_j} \cdot \left(m - \sum_{i=1}^n p_i \cdot z_i \right). \quad (2)$$

These demand functions satisfy that the expenditure on good j includes a fixed part—given by the value at the current price of the minimal consumption of j —and a variable part that is a fixed proportion α_j of the supernumerary (i.e., leftover) income:

$$p_j \cdot x_j = p_j \cdot z_j + \alpha_j \cdot \left(m - \sum_{i=1}^n p_i \cdot z_i \right). \quad (3)$$

3 Extension to CES utility functions

The CES utility function takes the form:

$$u(x_1, x_2, \dots, x_n) = \left(\sum_{j=1}^n a_j \cdot x_j^\theta \right)^{\frac{1}{\theta}}. \quad (4)$$

In this expression $\theta = (\sigma - 1)/\sigma$ and $\sigma = 1/(1 - \theta)$, where $0 \leq \sigma < \infty$ is the elasticity of substitution, which takes non-negative values only, and thus $-\infty < \theta \leq 1$. If desired, the non-negative coefficients in (4) a_j can be defined to add up to 1. We now shift the CES function using the minimum consumption levels $z_j \geq 0$:

$$u(x_1, x_2, \dots, x_n) = \left(\sum_{j=1}^n a_j \cdot (x_j - z_j)^\theta \right)^{\frac{1}{\theta}}. \quad (5)$$

The shifted CES function inherits most of the properties of the standard CES function (monotonicity, convexity, differentiability but not homotheticity). The solution of the utility maximization problem

$$\begin{aligned} \text{Max } u(x_1, x_2, \dots, x_n) &= \left(\sum_{j=1}^n a_j \cdot (x_j - z_j)^\theta \right)^{\frac{1}{\theta}} \\ \text{st. } \sum_{j=1}^n p_j x_j &= m \text{ and } x_j \geq z_j. \end{aligned} \quad (6)$$

involves combining the standard procedure¹ under a simplifying change of variable. We obtain the demand functions for the CES shifted utility:

$$x_j = z_j + \frac{a_j^\sigma \cdot p_j^{-\sigma} \cdot \left(m - \sum_{i=1}^n p_i \cdot z_i\right)}{\sum_{i=1}^n a_i^\sigma \cdot p_i^{1-\sigma}}. \quad (7)$$

The expenditure system becomes now:

$$p_j \cdot x_j = p_j \cdot z_j + s_j(p) \cdot \left(m - \sum_{i=1}^n p_i \cdot z_i\right) \quad (8)$$

with

$$s_j(p) = \frac{a_j^\sigma \cdot p_j^{1-\sigma}}{\sum_{i=1}^n a_i^\sigma \cdot p_i^{1-\sigma}}. \quad (9)$$

This demand system satisfies the following properties:

Property 1 *The terms $s_j(p)$ are non-negative proportions. Their sum over j is clearly 1. But unlike the fixed proportions in the standard LES system (3), the proportions $s_j(p)$ in (9) are price and substitution elasticity dependent.*

Property 2 *When $\sigma = 1$, which represents the LES Cobb–Douglas case, the proportions become constant. Indeed:*

$$s_j(p) = \frac{a_j^\sigma \cdot p_j^{1-\sigma}}{\sum_{i=1}^n a_i^\sigma p_i^{1-\sigma}} = \frac{a_j}{\sum_{i=1}^n a_i} = \alpha_j.$$

Property 3 *The own derivatives have opposite signs depending on the substitution elasticity:*

$$\frac{\partial s_j(p)}{\partial p_i} > 0 \text{ for } 0 < \sigma < 1 \text{ and } \frac{\partial s_j(p)}{\partial p_i} < 0 \text{ for } \sigma > 1.$$

Take the derivative from expression (9) to obtain:

$$\frac{\partial s_j(p)}{\partial p_j} = \frac{(1 - \sigma) \cdot a_j^\sigma \cdot \sum_{i \neq j} a_i^\sigma \cdot p_i^{1-\sigma}}{\left(\sum_{i=1}^n a_i^\sigma \cdot p_i^{1-\sigma}\right)^2}.$$

The sign of the derivative depends only on whether $\sigma < 1$ or $\sigma > 1$. For complementary goods ($0 < \sigma < 1$) any increase in the price of good j will require a larger fraction of the

¹ See Varian (1992), Chapter 7, and Jehle and Reny (2011), Chapter 1.

leftover income to be devoted to the good getting more expensive. The reason is that for complements consumptions tend to move in the same direction and the good getting relatively more expensive will be the most affected in terms of expenditure. The consumer needs to devote a greater proportion of the leftover income to purchase the good in question. We can see this more clearly in the limit case of perfect complements. In this extreme case, consumption proportions are constant and, even if the allotted income falls, the share of leftover income needed for the good whose price increase becomes larger. The opposite occurs when goods tend to be substitutes ($\sigma > 1$).

Property 4 *The cross derivatives have opposite signs depending on the substitution elasticity:*

$$\frac{\partial s_j(p)}{\partial p_i} < 0 \text{ for } 0 < \sigma < 1 \text{ and } \frac{\partial s_j(p)}{\partial p_i} > 0 \text{ for } \sigma > 1.$$

The same intuition as in Property 3 helps to explain why. If good i becomes more expensive, and goods are complements, demand for j will fall too but the share $s_j(p)$ becomes smaller for good j since it is getting relatively cheaper than good i .

Property 5 *Similar to the standard LES, the CES utility function with minimal consumptions is no longer homothetic. We calculate the marginal rate of substitution $MRS_{i,j}$ for the utility in (5) and along a ray $x_j = \beta \cdot x_i$ with $\beta > 0$ we obtain:*

$$MRS_{i,j} = - \frac{a_i \cdot (x_i - z_i)^{\theta-1}}{a_j \cdot (\beta \cdot x_i - z_j)^{\theta-1}},$$

whose value depends on the value of x_i and thus the marginal rate of substitution is not constant. One of the criticisms to the use (or abuse) of homothetic utilities in applied work is that the real world does not seem to be homothetic. The shifted CES utility function with minimal consumption levels does not suffer from this problem.

We can visually illustrate the absence of homothecy in two ways. In Fig. 1, for example, we show how the marginal rate of substitution changes along a ray that passes through the coordinate origin. In Fig. 2, in contrast, we set a value for the marginal rate of substitution and verify that there is no ray passing through the origin that gives rise to that set value.

4 A calibrated demand function

When econometric estimates are not available, a circumstance we often face, an alternative to implement demand systems in numerical general equilibrium models (Dervis et al 1982; Shoven and Whalley 1984) is to use the simpler method of calibration. This method uses all available information to derive demand functions that are consistent with the data and the restrictions on parameters implied by utility maximization (Dawkins et al 2001). The goal of calibration is to have an empirical specification of expression (5) that has the property that when used to maximize utility, the solution endogenously yields the observed empirical data registered in some database.

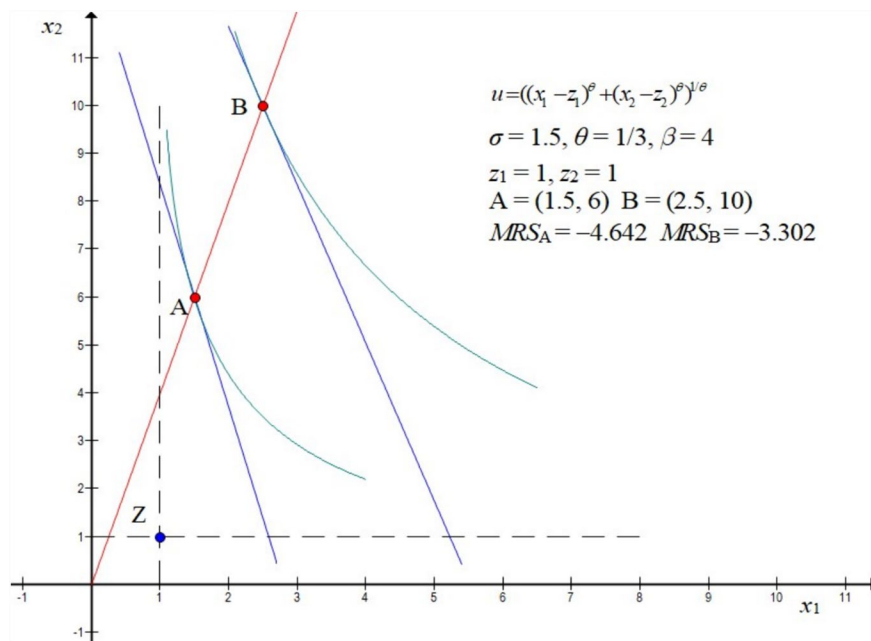


Fig. 1 MRS not constant along a ray through the cartesian origin. Source: Our calculations using Graph (<https://www.padowan.dk>)

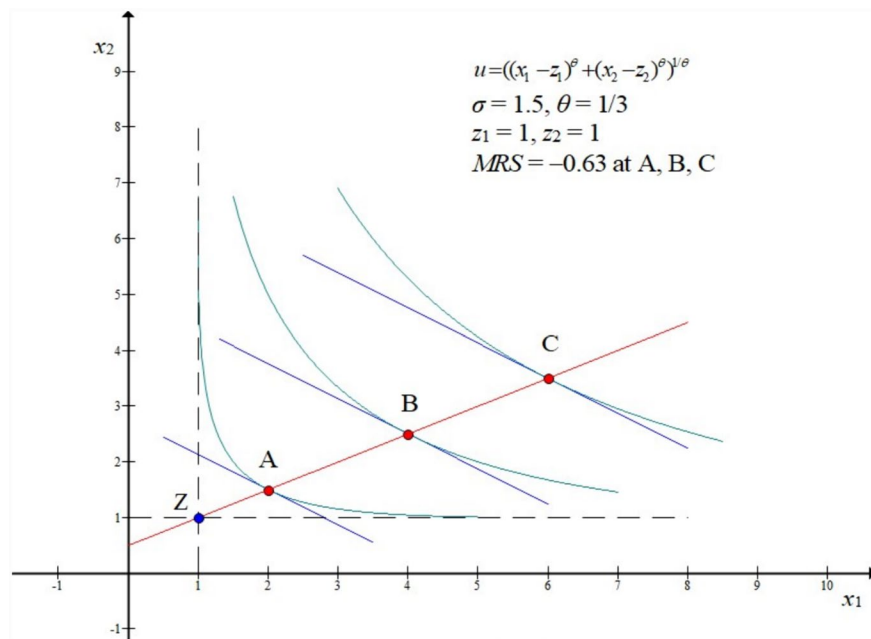


Fig. 2 MRS constant but not along a ray through the cartesian origin. Source: Our calculations using Graph (<https://www.padowan.dk>)

A look at (5) shows that we need values for the n share coefficients a_j , the elasticity of substitution σ (which gives us θ) and the n minimum consumption levels z_j . In total, $2n + 1$ parameters. If the value of s is known, or assumed, this leaves $2n$ parameters

to be determined, or "calibrated". If we use input–output or social accounting matrix data, however, only n consumption observations are available.

We need some calibration tricks. The first one is to assume that all prices reflected in the database are unitary $p_j = 1$. This entails a redefinition of the units in such a way that one physical unit has the worth of one currency unit. With this redefinition, all observed data in the database are now both value as well as physical units. This allows us to simplify expression (9) to:

$$s_j(1) = \frac{a_j^\sigma}{\sum_{i=1}^n a_i^\sigma}. \quad (10)$$

The second trick involves, when they are available, the use of income elasticities of demand η_j . From expression (7), we can easily check that

$$s_j(1) = \eta_j \cdot \alpha_j(1), \quad (11)$$

where $\alpha_j(1)$ is the share of expenditure on good j over total income at the initial unitary prices. Substituting (11) into (10) we obtain a system of linear equations that can be solved conditional on the value of σ . The system in (10) has multiple solutions since if a_j is a solution so is $\mu \cdot a_j$ for any scalar μ . Remember, however, that from the definition of the CES function the sum of these coefficients can be directly normalized to be 1. Thus, the system in (10) is determined.

The final trick needs to deal with the determination of the subsistence levels z_j . This step requires the use of the Frisch parameter ϕ (Frisch 1959) which measures the flexibility in the distribution of income between the fixed and the variable parts of consumptions. At the initial unitary prices this gives:

$$-\phi = \frac{m}{m - \sum_{j=1}^n 1 \cdot z_j}. \quad (12)$$

Substituting into expression (8) we can solve for the subsistence levels:

$$z_j = x_j - s_j(1) \cdot \frac{m}{-\phi}. \quad (13)$$

As an illustration of the calibration procedure, we borrow income elasticities calculated for the two-digit 12 ECOICOP sectors for the Spanish economy from Garcia-Villar (2018). The fact that these estimated income elasticities are not unitary challenges the typical use of homothetic utility functions in numerical general equilibrium since, most commonly, the typical selection of preferences give rise to unitary income elasticities.² This justifies departing from homothetic functions and endorses the use of LES or the CES extended Stone–Geary utility functions. We use the reported value of the Frisch parameter ($\phi = -2$) from Deaton and Muellbauer (1980) along with two sensible small deviations around it. The central elasticity of substitution value corresponds to the

² The econometrics literature provides ample evidence for non-unitary income elasticities. See Lecocq & Robin (2006), Christensen (2014) and García-Enríquez & Echevarría (2016).

Table 1 Data on estimated elasticities and reported expenditures for 2017

Goods	Income elasticities	Expenditure data
1.—Food and non-alcoholic beverages	0.546	76.042
2.—Alcoholic beverages and tobacco	0.563	9.927
3.—Clothing	1.094	28.043
4.—Housing	0.413	162.431
5.—Household articles	1.120	24.762
6.—Health	1.085	18.149
7.—Transportation	1.383	67.890
8.—Communication services	0.421	17.209
9.—Recreational services	1.148	30.770
10.—Education	1.282	7.668
11.—Hotels and restaurants	1.261	55.588
12.—Other services	0.880	41.864

Source: National Institute of Statistics for expenditure, García-Villar (2018) for the income elasticities. Expenditure in millions of current Euros

Table 2 Utility coefficients for alternative substitution elasticity values and $\phi = -2$

Goods	Coefficients	$\sigma = 0.75$	$\sigma = 1$	$\sigma = 1.25$
1.—Food and non-alcoholic beverages	a_1	0.087	0.093	0.095
2.—Alcoholic beverages and tobacco	a_2	0.006	0.013	0.019
3.—Clothing	a_3	0.058	0.069	0.075
4.—Housing	a_4	0.165	0.151	0.140
5.—Household articles	a_5	0.051	0.062	0.069
6.—Health	a_6	0.032	0.044	0.052
7.—Transportation	a_7	0.259	0.211	0.183
8.—Communication services	a_8	0.009	0.016	0.024
9.—Recreational services	a_9	0.070	0.079	0.084
10.—Education	a_{10}	0.013	0.022	0.030
11.—Hotels and restaurants	a_{11}	0.175	0.157	0.145
12.—Other services	a_{12}	0.074	0.083	0.086

Source: Our model computations

widely used Cobb–Douglas case ($\sigma = 1$) but, again, we introduce deviations from this unitary elasticity value to appraise the sensitivity of the results. Finally, we use expenditure data for the same classification of goods taken from the ECOICOP data published by the National Institute of Statistics for 2017.³ Table 1 shows the calibration of the utility function coefficients.

Table 1 shows the data on income elasticities and expenditure for 2017, whereas Tables 2 and 3 show the calibrated parameters for the shifted utility function. To validate our results, we compare our findings with the data published by the National Institute of Statistics of Spain for the first income quintile in the base year 2017. Since the published data are presented as percentages, we calculate corresponding percentages derived from the calibrated minima for the central case of the Frisch parameter $\phi = -2$. Table 4 presents the comparison between model results and actual data for the Spanish economy.

³ <https://www.ine.es/jaxiT3/Tabla.htm?t=24765&L=0>

Table 3 Minimum consumptions for alternative values of the Frisch parameter φ

Goods	Minima	$\varphi = -1.75$	$\varphi = -2$	$\varphi = -2.25$
1.—Food and non-alcoholic beverages	z_1	47.268	50.864	53.662
2.—Alcoholic beverages and tobacco	z_2	6.054	6.538	6.914
3.—Clothing	z_3	6.781	9.439	11.506
4.—Housing	z_4	115.939	121.750	126.270
5.—Household articles	z_5	5.542	7.944	9.813
6.—Health	z_6	4.502	6.208	7.535
7.—Transportation	z_7	2.819	10.953	17.279
8.—Communication services	z_8	12.188	12.816	13.304
9.—Recreational services	z_9	6.289	9.349	11.729
10.—Education	z_{10}	0.855	1.707	2.369
11.—Hotels and restaurants	z_{11}	7.008	13.081	17.804
12.—Other services	z_{12}	16.332	19.524	22.006

Source: Our model computations

Table 4 Calculated minima for the central calibration ($\varphi = -2$) versus actual data for the first income quintile in 2017 in expenditure proportions

Goods	Minima	$\varphi = -2$	Model Proportions	Actual proportions
1.—Food and non-alcoh. beverages	z_1	50.864	18.83%	20.30%
2.—Alcoholic beverages and tobacco	z_2	6.538	2.42%	2.48%
3.—Clothing	z_3	9.439	3.49%	3.76%
4.—Housing	z_4	121.750	45.06%	39.59%
5.—Household articles	z_5	7.944	2.94%	3.20%
6.—Health	z_6	6.208	2.30%	2.25%
7.—Transportation	z_7	10.953	4.05%	7.16%
8.—Communication services	z_8	12.816	4.74%	4.62%
9.—Recreational services	z_9	9.349	3.46%	3.04%
10.—Education	z_{10}	1.707	0.63%	0.60%
11.—Hotels and restaurants	z_{11}	13.081	4.84%	5.91%
12.—Other services	z_{12}	19.524	7.23%	7.10%

Source: Data from <https://www.ine.es/jaxiT3/Tabla.htm?t=24900&L=0> and our computations

The striking similarity observed in the numerical results of the last two columns of the table lends support to the calibration exercise we have carried out.

5 Concluding remarks

The Stone–Geary linear expenditure system correctly captures some rigidity properties of consumption demand by recognizing the existence of minimum consumption thresholds for specific goods. In a conventional linear expenditure system (LES), any consumption surpassing these minimum thresholds is allocated using fixed share coefficients. However, when we consider broader substitution possibilities, as seen in the CES displaced utility functions we examine here, the shares of excess consumption become sensitive to prices and to the elasticity of substitution, thereby enhancing households' ability to react to fluctuations in prices that affect their real income. This type of response captures a more realistic empirical characteristic and may help in enriching the modeling

of the demand side in numerical general equilibrium models. It may also enable more reliable and realistic assessments of the welfare implications of various policies. Moreover, the CES displaced functions, being non-homothetic, adequately reflect the empirical nature of income elasticities. This concurs with numerous econometric studies that indicate that income elasticities do not exhibit unitary values. These conceptual properties of the demand structure must be adequately understood to be able to discern the reasons for the results that a model, CGE or of any other type, generates, even if it is only in a first approximation, so as not to fall into the opacity of black box type results.

Notice also that the focus on utility and demand theory presented here can readily be reframed within the context of production theory. In this case, the technology would require a minimum level of primary factors—labor and capital—in order to operate. As a result, the resulting cost function would distinguish between variable and fixed costs. This contrasts with the customary approach in computable general equilibrium analysis, where the inclusion of fixed costs often relies on ad hoc rules with little theoretical backing. The challenges of this extension for CGE modeling are more empirical than formal. Conceptually, the extension is straightforward, as it only requires a redefinition of the cost minimization problem under a displaced production function of the CES type explored here. However, on the empirical side, a concept similar to that of the Frisch parameter should be defined within production theory to enable the calibration of the minimum use of factors.

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Availability of data and materials

The data used are openly available in the listed references and publications.

Declarations

Competing interest

There are no competing interests to disclose.

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