

Synthesis of In-Line Fully Canonical Filters by Solving a Matrix Completion Problem Under AW Ladder Constraints

Joel Mesas^{ID}, *Student Member, IEEE*, Jordi Verdú^{ID}, *Senior Member, IEEE*,
and Pedro de Paco^{ID}, *Senior Member, IEEE*

Abstract—This work extends a matrix-based numerical methodology to cover fully canonical generalized Chebyshev (GC) transfer functions by reconfiguring canonical filter topologies into dangling inline structures, with a direct impact on the consideration of ladder circuit filters. The transformation matrix that maps a canonical matrix to the dangling inline is defined in such a way that, within a mathematical matrix completion framework, it can nullify the source-load coupling, modify the source and load reactances to accommodate the phase at the first and last extracted-pole sections of the network, and capture the response information in a different size matrix.

Index Terms—Acoustic wave (AW), coupling matrix (CM), filter synthesis, matrix completion, topology reconfiguration.

I. INTRODUCTION

AMONG its many advantages, coupling matrix (CM) theory [1] allows the systematic synthesis of filter topologies by leveraging canonical configurations—such as transversal or folded arrays—and transforming them through matrix algebra like similarity transformations, partitions, or splitting networks into cascaded subnetworks [2]. Although analytical frameworks yield exact solutions for specific cases, they are inherently constrained by standard topologies and fixed methodologies. As filter design demands evolve toward more complex or irregular structures, numerical synthesis methods become essential to address nonstandard configurations and broaden the design space.

Recent advances, such as [3], [4], [5], [6], and [7], have introduced robust frameworks based on analytical derivations leveraging Householder transformations, inverse nonlinear eigenvalue formulations, and topology projection to construct CMs that enable the synthesis of highly complex filter topologies, where traditional methods lack systematic analytical solutions.

Notably, the work presented by Zeng et al. [7], where the CM reconfiguration problem is formulated as a matrix completion problem to find the transformation matrix, whether orthogonal or congruent, becomes a very efficient strategy for reconfiguring topologies due to the use of advance matrix operations for all gradient calculations instead of element-wise computation within an optimization procedure. However,

this excellent approach presents a limitation when addressing dangling inline structures that accommodate fully canonical transfer functions as it does not allow for the nullification of the direct source-load coupling (M_{SL}) and it is incapable of generating the source and load frequency-invariant reactances (FIRs) to adjust input and output phases [8].

It is known that a set of similarity transformations, commonly referred to as a Given's rotations, is defined as

$$\mathbf{Q} = \prod_k \mathbf{R}(i_k, j_k, \theta_k)^T = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \mathbf{Q}' & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \quad (1)$$

where only internal nodes can be selected as pivots to preserve the filter response, and thus, the outermost columns and rows are zero except for the entries corresponding to the source and load nodes. Then, a canonical topology defined by $N + 2$ size matrices [9] $\mathbf{M}_0$, $\mathbf{C}_0$, and $\mathbf{R}_0$ is transformed into a target topology by applying a transformation matrix $\mathbf{Q}$

$$\mathbf{M}_T = \mathbf{Q}^T \mathbf{M}_0 \mathbf{Q}, \mathbf{C}_T = \mathbf{Q}^T \mathbf{C}_0 \mathbf{Q}, \mathbf{R}_T = \mathbf{Q}^T \mathbf{R}_0 \mathbf{Q}. \quad (2)$$

Note that the conductance matrix $\mathbf{R}_T$ remains equal to $\mathbf{R}_0$ due to the structure of $\mathbf{Q}$ in (1), which does not alter the entries associated with the normalized impedance ports. Nonetheless, transforming canonical topologies into fully canonical inline dangling structures—characterized by higher matrix size ($2N + 2$), the absence of M_{SL} , and the need to control input (B_S) and output (B_L) FIRs—is mathematically unfeasible, as the corner elements of the canonical CM cannot be modified under such transformations, as demonstrated in the following equation:

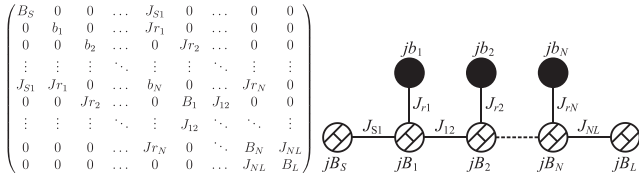
$$\mathbf{M}_T = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \mathbf{Q}' & \vdots \\ 0 & \cdots & 1 \end{pmatrix}^T \begin{pmatrix} 0 & \cdots & \mathbf{M}_{SL} \\ \vdots & \ddots & \vdots \\ \mathbf{M}_{SL} & \cdots & 0 \end{pmatrix} \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \mathbf{Q}' & \vdots \\ 0 & \cdots & 1 \end{pmatrix}. \quad (3)$$

Furthermore, the dangling inline structure is a fundamental topology able to model acoustic wave (AW) ladder networks, which have a huge industrial impact [10]. Although such networks have been extensively studied, existing methods rely on deterministic solutions based on sequential network extractions [11], [12], [13], direct matrix synthesis approach [14], or predefined rotation sequences [15], [16]. In the latter case, source and load nodes remain unaltered, as the synthesis process employs transformations like (1) to reach the dangling inline configuration. In contrast, the proposed method introduces a numerical strategy for determining the unknown coupling coefficients of arbitrary target topologies, allowing M_{SL} cancellation and modification of the source and load nonresonating nodes (NRNs).

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The authors are with the Department of Telecommunications and Systems Engineering, Universitat Autònoma de Barcelona, 08193 Barcelona, Spain (e-mail: joel.mesas@uab.cat).

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Fig. 1. Generic N th-order ladder low-pass CM and nodal representation.

II. CM COMPLETION TO FULLY CANONICAL NETWORKS

Following [7], we propose a solution where matrix $\mathbf{Q}$ has been redefined to incorporate topologies featuring extracted-pole sections, including mixed configurations where the extracted pole appears at the beginning or end of the network, addressing phase-related constraints. Additionally, this structural revision extends the framework's capabilities to accommodate fully canonical responses addressing the ladder-type structures. A general dangling inline topology as depicted in Fig. 1 is built by cascading extracted pole sections where the number of resonators equals the number of transmission zeros. The canonical CM representation in its transversal form for fully canonical transfer functions takes the form of (4). This matrix is redimensioned by the insertion of N independent NRN without being connected to other nodes, corresponding to the number of transmission zeros (also $\mathbf{C}_0$ and $\mathbf{R}_0$ must be rearranged). Notably, this representation is characterized by a nonzero coupling value between source and load nodes

$$\mathbf{M}_0 = \begin{pmatrix} 0 & M_{S1} & \dots & 0 & \dots & M_{SN} & M_{SL} \\ M_{1S} & B_1 & \dots & 0 & \dots & 0 & M_{1L} \\ M_{2S} & 0 & \ddots & 0 & \dots & 0 & M_{2L} \\ 0 & 0 & 0 & \pm \mathbf{B}_{NRN} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & 0 & \ddots & \vdots & \vdots \\ M_{NS} & 0 & \dots & 0 & \dots & B_N & M_{NL} \\ M_{LS} & M_{L1} & \dots & 0 & \dots & M_{LN} & 0 \end{pmatrix}. \quad (4)$$

To cancel M_{SL} , the transformation matrix must be redefined so that certain entries in the outermost columns can take nonzero values, as illustrated in the following equation:

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & & & \vdots \\ \vdots & & & 0 \\ q_{N+2,1} & \mathbf{Q}' & & q_{N+2,2N+2} \\ \vdots & & & \vdots \\ q_{2N+1,1} & & & q_{2N+1,2N+2} \\ 0 & \dots & 0 & \dots & 1 \end{pmatrix}. \quad (5)$$

To ensure that the network's admittance information is preserved and the response remains intact, the conductance matrix $\mathbf{R}$ must be included in the matrix completion algorithm. Accordingly, the cost function is defined similar to that in [7], with an additional term introduced to account for the new definition of $\mathbf{Q}$. This modification guarantees that, when applying (2), the conductance matrix remains unchanged

$$\begin{aligned} \mathbf{F}(\mathbf{Q}) = & \frac{1}{2} \langle \mathbf{Q}^T \mathbf{M}_0 \mathbf{Q} - \mathbf{P}_M (\mathbf{Q}^T \mathbf{M}_0 \mathbf{Q}) \rangle \\ & + \frac{1}{2} \langle \mathbf{Q}^T \mathbf{C}_0 \mathbf{Q} - \mathbf{P}_C (\mathbf{Q}^T \mathbf{C}_0 \mathbf{Q}) \rangle + \frac{1}{2} \langle \mathbf{Q} - \mathbf{P}_Q(\mathbf{Q}) \rangle \\ & + \frac{1}{2} \langle \mathbf{Q}^T \mathbf{R}_0 \mathbf{Q} - \mathbf{P}_R (\mathbf{Q}^T \mathbf{R}_0 \mathbf{Q}) \rangle \end{aligned} \quad (6)$$

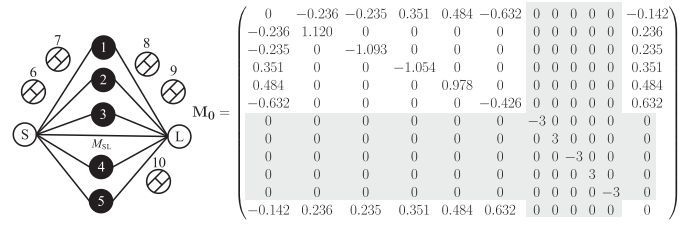


Fig. 2. Expanded canonical CM according to sign criteria in (11).

where the projection matrices are defined the same way by the Hadamard product ($\odot$) and introducing the new term

$$\mathbf{P}_R(\mathbf{R}) = (\mathbf{T}_R \odot \mathbf{R}) + \mathbf{X}_R \Big|_{\mathbf{R}=\mathbf{Q}^T \mathbf{R}_0 \mathbf{Q}} \quad (7)$$

$$\mathbf{X}_R = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \mathbf{0}^{2N \times 2N} & \vdots \\ 0 & \dots & 1 \end{pmatrix} \quad \mathbf{T}_R = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & \mathbf{0}^{2N \times 2N} & \vdots \\ 0 & \dots & 0 \end{pmatrix}. \quad (8)$$

In this way, defining the constrained matrix $\mathbf{X}_R$ and $\mathbf{T}_R$ as (8), the admittance information is maintained after the transformation. Then, the gradient of $\mathbf{F}(\mathbf{Q})$ is derived analytically via Fréchet derivative and Rietz representation [17] and expressed as a ordinary differential equation (ODE)

$$d\mathbf{Q}/dt = -\nabla \mathbf{F}(\mathbf{Q}) = -(\mathbf{g}_1(\mathbf{Q}) + \mathbf{g}_2(\mathbf{Q}) + \mathbf{g}_3(\mathbf{Q}) + \mathbf{g}_4(\mathbf{Q})) \quad (9)$$

where each g_i denotes the analytical derivative of the corresponding term in (6)

$$\begin{aligned} \mathbf{g}_1(\mathbf{Q}) &= 2\mathbf{M}_0 \mathbf{Q} (\mathbf{Q}^T \mathbf{M}_0 \mathbf{Q} - \mathbf{P}_M (\mathbf{Q}^T \mathbf{M}_0 \mathbf{Q})) \\ \mathbf{g}_2(\mathbf{Q}) &= 2\mathbf{C}_0 \mathbf{Q} (\mathbf{Q}^T \mathbf{C}_0 \mathbf{Q} - \mathbf{P}_C (\mathbf{Q}^T \mathbf{C}_0 \mathbf{Q})) \\ \mathbf{g}_3(\mathbf{Q}) &= (\mathbf{Q} - \mathbf{P}_Q(\mathbf{Q})) \odot (\mathbf{E} - \mathbf{T}_Q) \\ \mathbf{g}_4(\mathbf{Q}) &= 2\mathbf{R}_0 \mathbf{Q} (\mathbf{Q}^T \mathbf{R}_0 \mathbf{Q} - \mathbf{P}_R (\mathbf{Q}^T \mathbf{R}_0 \mathbf{Q})). \end{aligned} \quad (10)$$

Although the methodology is presented in the context of a fully canonical inline structure, such modifications are equally required for nonfully canonical solutions that start or end with an extracted-pole section, due to the need for precise control of B_S and B_L . Moreover, when particularizing this mathematical framework for AW ladder networks, specific considerations must be addressed.

To remain consistent with the AW reactance model described in [8], the susceptances B_i in the target matrix (Fig. 1)—associated with transmission zeros above the pass-band, corresponding to series AW resonators—must be negative. Likewise, the susceptance b_i of the resonant node introducing the transmission zero via the extracted-pole section must also be negative, as given by $\Omega_i = -b_i$. Conversely, when the transmission zero lies below the passband, both B_i and b_i must be positive.

This is achieved by inserting an arbitrary susceptance into the diagonal of the canonical matrix $\mathbf{M}_0$ being an alternating sign to be compatible with AW ladder structure. Thus, in a general transversal CM like (4), the entry corresponding to an upper stopband TZ must be assigned a negative susceptance, and analogously, a positive value for a lower stopband TZ

$$\mathbf{M}_0(i, j) \Big|_{j=i} = \begin{cases} -\mathbf{B}_{NRN}, & \text{if } \text{TZ}_k > +1, \mathbf{X}_M(i, j) = -\Omega_k \\ +\mathbf{B}_{NRN}, & \text{if } \text{TZ}_k < -1, \mathbf{X}_M(i, j) = \Omega_k. \end{cases} \quad (11)$$

Self-coupling positions (FIRs) of the prescribed frequency independent matrix ($\mathbf{X}_M$) must be set to $\pm\Omega_i$, to control

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