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Exploring Instruments for Assessing Initial Mathematical Knowledge in Primary Teacher Education

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Knowing the mathematical background of students entering primary teacher education is key to guiding admission or diagnoses to inform teacher education programs. To understand how this initial mathematical knowledge that students have is organised, we retrieved the idea of Fundamental Mathematical Knowledge which had been defined as the mathematical knowledge necessary for these students to be able to successfully follow subjects about mathematics and teaching during their teacher education. In this study, we examined two questionnaires that had been designed in Catalonia and Chile to gauge the disparity between students' mathematical knowledge at the beginning of their teacher education and a specification of Fundamental Mathematical Knowledge defined around curricular content blocks. The scores obtained in both questionnaires by two groups of students—one in Barcelona and another in Santiago de Chile—constituted our data. The quantitative analysis of the data led us to psychometrically validate them as instruments for the measurement of students' initial mathematical knowledge and to the realisation that the structure of this knowledge was unidimensional and behaved as a single latent construct. We concluded that teacher education programs should be aimed at progressing towards the development of a deeper understanding of elementary mathematics and its conceptual connections.

Keywords · fundamental mathematical knowledge, mathematics teacher education, student teachers, assessment instruments validation

Introduction

A major concern in mathematics education is whether teachers have sufficient content knowledge to teach mathematics effectively (Hiebert et al., 2019). This concern is also shared by teacher education programs as content knowledge is considered a crucial part of their curriculum (Feuer et al., 2013). Different theoretical frameworks have been developed to characterise mathematics teachers' knowledge, addressing various aspects of their education and professional practice (Carrillo et al., 2018; Rowland et al., 2009). In particular, there has been a focus on the mathematical knowledge required to teach primary school mathematics (Ball et al., 2001; Ball et al., 2012; Hiebert et al., 2002; Hill et al., 2007). Additionally, there are studies that explored the development of mathematical knowledge during teacher education (Ball et al., 2001; Ball et al., 2008; Cooney & Wiegel, 2003; Hiebert et al., 2002). Also, there are studies that established the characteristics of mathematics knowledge for teaching and identified subject areas to inform the curriculum of teacher education programmes (Association of

Mathematics Teacher Educators, 2017; Ball et al., 2008). There is, however, scant research exploring the mathematical knowledge of the students beginning their education as teachers.

The lack of a thorough theoretical debate on desirable prior knowledge for students entering teacher education programmes, combined with varied admission methods, poses significant challenges for teacher education institutions. Miller-Levy et al. (2014) pointed out that teacher education programs admission requirements should assess, among other aspects, candidates' prior knowledge of the content of the different subjects they will later have to teach. Teacher education students have successfully passed the previous educational stages established by each system and the applicable university entrance exams. Several studies, however, show that candidates admitted to the programs continued to display gaps in their mathematical knowledge (Ball, 1990; Beswick & Goos, 2012; Ingram & Linsell, 2013; Ryan & McCrae, 2005/06; Senk et al.; 2012; Tatto et al., 2008). Even so, a review of the published literature revealed that little attention has been given to the mathematical knowledge possessed by students beginning teacher education (Gorgorió et al., 2021; Linsell & Anakin, 2012, 2013).

At a theoretical level, there is no consensus on the specific mathematical content or competences desired of incoming students. Nor has it been defined what the instruments should be and how they could be used to characterise students' knowledge when they join teacher education programs (Gorgorió, Albarracín, & Villarreal, 2017). At the social level, too often, training programs do not have validated instruments to assess the mathematical knowledge, either for selective or diagnostic purposes, of incoming students. If we are to underpin policies affecting admissions processes, create instruments to assess applicants, or obtain diagnostic data to inform and guide training programs, it is important to have a theoretical framework to understand this knowledge and valid tools to assess it.

In this study, we retrieved the idea of Fundamental Mathematical Knowledge as that mathematical knowledge that students entering teacher education need to successfully pursue the mathematics and mathematics teaching syllabi included in their courses (Castro et al., 2014; Gorgorió & Albarracín, 2020; Gorgorió, et al., 2021). Following on from the work presented in Albarracín et al. (2021), we examined two questionnaires that had been designed in two different national contexts to study the disparity between students' mathematical knowledge at the beginning of their teacher education and a specification of Fundamental Mathematical Knowledge (Gorgorió & Albarracín, 2020). The scores obtained by administering the questionnaires to two groups of students, one in Barcelona and the other in Santiago de Chile, constituted the data for the study. A quantitative analysis of the data made it possible to achieve the study goals, namely: to psychometrically validate both questionnaires as instruments to measure the mathematical knowledge of students starting a teacher education program and to analyse whether the structure of the mathematical knowledge exhibited by these students matched the thematic grouping established theoretically when defining fundamental mathematical knowledge. In other words, in order to ascertain the mathematical knowledge structure of students entering a primary school teacher training programme, it is necessary to employ psychometrically validated instruments. The result of this validation will allow us, in addition to obtaining a valid instrument, to give an account of how this knowledge is organised, either by blocks mathematical content or as an interrelated whole.

Context of the Study

This study focused on two different educational contexts: Catalonia in Spain and Chile, and is the result of a collaboration between two teams, one from each country. In Spain, most studies assessing students' mathematical knowledge at the start of teacher training used instruments aimed at schoolchildren (Arce et al., 2017; Nortes & Nortes, 2013; 2018). For this reason, in Catalonia, the group led by Gorgorió developed a specific instrument to assess student teachers' mastery of the basic mathematical knowledge they would need to successfully complete their teacher education (Gorgorió & Albarracín, 2020). The results of Gorgorió's group suggested the need to consolidate tools to ensure that students' mastery of basic mathematical knowledge is sufficiently solid. The group's efforts led to the requirement that in Catalonia, from the 2017-2018 academic year, candidates for primary teacher training programmes have to pass a personal aptitude test that assesses their communicative and mathematical competence[1].

In Chile, the *Sistema de Desarrollo Profesional Docente* or Teacher Professional Development System (Law 20.903, April 1, 2016) mandates the use of two diagnostic assessments, one at the beginning of the programme and another in the penultimate year, to monitor student knowledge. However, the fact that the training institutions are responsible for developing their own diagnostic instruments has resulted in a wide variety of instruments that are often not reliable enough to be able to make decisions (Giaconi et al., 2022). Martínez et al. (2019) studied whether students starting teacher education in Chile had a primary education level of mathematical knowledge and explored their beliefs on mathematics, its teaching and learning. They reported poor performances in algebra, statistics and probability, together with an arithmetic's perception of mathematics taken as a whole. Similarly, Rojas et al. (2021) showed that those who enter teacher education mostly have a medium or low mastery of school mathematics.

Fundamental Mathematical Knowledge

The mathematical knowledge that students already possess when they enter teacher education programmes is a topic scarcely researched even though their background mathematical knowledge may be predictive of their success in developing mathematical knowledge for teaching (Baumert et al. 2010). The variety of theories developed so far that describe the professional knowledge of mathematics teachers reflects the complexity of the knowledge and competences demanded by mathematics teachers' professional practice. Concepts such as *subject knowledge* – as a component of Pedagogical Content Knowledge, PCK, (Shulman, 1987), *common content knowledge* – as part of Mathematical Knowledge for Teaching, MKT, (Ball et al., 2008), or *knowledge of the topics* – within Mathematical Teacher Specialised Knowledge, MTSK, (Carrillo et al., 2018), guide much of the research being carried out today related to mathematics teachers' education.

However, none of the above frameworks indicate how subject knowledge, common content knowledge, or knowledge of the topics, is articulated or characterised when students enter initial teacher education. The Knowledge Quartet (Rowland et al. 2005), which is a much more knowledge-oriented framework for mathematics classroom management, introduces the *foundations dimension* which includes a knowledge and understanding of mathematics that informs the decisions of the future classroom teacher. Again, the mathematical knowledge with which prospective teachers arrive at initial teacher education is not covered, since the focus is on mathematical knowledge that is instrumental to classroom activity.

We therefore see that the relevance of student teachers' initial mathematical knowledge goes unmentioned in the theories that discuss the knowledge needed to teach mathematics at school. Consequently, the assessment of student teachers' mathematical knowledge, either as a precondition for admission or for diagnostic and training purposes at the beginning of their studies, has rarely been the subject of research. Moreover, the kind of prior mathematical knowledge that should be considered desirable and the methods that could be used to assess it has only been described tentatively up to now (Gorgorió, Albarracín, & Villarreal, 2017). This research is intended to contribute to the debate through an exploration of two instruments used for assessing students' mathematical knowledge at the beginning of their primary teaching degrees.

This study draws upon the idea of *Fundamental Mathematical Knowledge* (from now on FMK). In a first definition in Castro et al. (2014), FMK was presented as the disciplinary knowledge of mathematics needed to to successfully progress through courses of mathematics and its teaching, considering the requirements of professional practice and the mathematical competences of primary education. FMK is

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[1] https://www.educaweb.cat/continguts/educatius/estudis-universitaris/prova-aptitud-personal-pap/pap-educacio-infantil-educacio-primaria/

the desirable initial disciplinary knowledge which the student can draw on to develop the specialised and pedagogical content knowledge needed to work as a teacher.

Our understanding of FMK incorporated and developed elements of the deep knowledge discussed by Ma (1999), in that it can serve as the cornerstone that supports the didactics of mathematics and student teachers' learning during mathematics courses. The idea of FMK also ties in with the interpretation that Hiebert et al. (2019) make of common content knowledge as "(the) knowledge that people learn in school" (p. 6), when they specify the definition of Ball et al. (2008). It is also like the foundation content knowledge described by Linsell and Anakin (2012), which includes indivisibly linked conceptual and procedural knowledge. However, none of these references (Hiebert et al., 2019; Linsell & Anakin, 2012; Ma, 1999) explicitly specifies the mathematical content they refer to.

Specifying Fundamental Mathematical Knowledge

In the FMK specification process, we initially worked with two groups of experts. One was an international team of researchers from Finland, England, and Sweden (see Gorgorió, Albarracín, Ärlebäck, Laine, Newton & Villarroel, 2017), and the other made up of representatives of all the Catalan universities (see Gorgorió & Albarracín, 2020), because the definition of FMK was linked to the development of the specific aptitude test for admission to the teaching degrees, as explained above. Subsequently, the Chilean research team was enlisted for the development of this study, given their interest in the definition of mathematics diagnostic questionnaires for their institutions.

From a pragmatic standpoint, we adopted an approach like Ryan and McCrae (2005/06) and Linsell and Anakin (2012), who used the curricula of their respective countries' school systems as a framework for examining future teachers' initial knowledge. Although school mathematics may be bounded by its context and use different resources and procedures, the core concepts that lie at the root of elementary mathematics are universal. Therefore, our starting point was the mathematics syllabi set by the national curricula of schools in Catalonia (Departament d'Ensenyament, 2015), Finland (National Board of Education, 2016), Sweden (Skolverket, 2011) and England (Department of Education, 2013), limiting our attention to primary education in each case. The present study also incorporates the analysis of the Chilean primary education curriculum (Martínez et al. 2019; MINEDUC, 2012).

Considering that the aim in Gorgorió, Albarracín, Ärlebäck, Laine, Newton and Villarreal (2017) was to characterise or assess explicable knowledge in a written questionnaire designed for easy implementation in institutions, a content analysis of the different syllabi was conducted. The authors' first step was to identify the common areas, analysing the scope and importance given to each area, and then to decide which concepts and procedures should be predetermined when specifying FMK and which were important but not essential (see Gorgorió & Albarracín, 2020), thereby obtaining a concrete proposal for FMK.

The FMK content blocks chosen for the development of this study are those specified by Gorgorió, Albarracín and Villarreal (2017; p. 61):

- **Numeration and calculation**. Students must demonstrate an understanding of and ability to represent and use natural numbers, integers, and rational numbers in different situations; an understanding of the meaning and properties of operations and the relationships between them; a knowledge of the meaning of divisor; and a mastery of the skills needed to solve problems of factorisation and division of natural numbers.
- **Relationships and change**. Students must demonstrate an ability to identify and generalise patterns that are not necessarily numerical; to identify and interpret relationships of dependence between variables; to interpret and construct graphs that express relationships of change. It is also necessary to demonstrate an integrated understanding of the meanings of numerical proportionality and ratio and the ability to use these concepts to solve different problems.
- **Space and shape**. Students must demonstrate a knowledge of the characteristics and properties of two- and three-dimensional geometric figures and the ability to apply them in different situations; an understanding and ability to represent and use reflections,

rotations, and translations; an integrated understanding of the meanings of geometric proportionality, similarity, and scale; and the ability to use visualisation strategies to solve geometric and non-geometric problems.

- **Measurement**. Students must demonstrate a knowledge of the meaning of measurable magnitude (angle, length, area, volume, capacity, mass and time) and of measurement processes; a knowledge of the corresponding decimal and sexagesimal units of measurement and of the mechanisms required to solve problems with a change of units; and a mastery of the knowledge and skills needed to solve diverse problems related to the ideas of perimeter, area and volume.
- **Statistics and chance**. Students must demonstrate the ability to interpret, analyse, draw conclusions, and make predictions from statistical data; to interpret and construct statistical graphs; to interpret and calculate measures of centralisation; and to understand the meaning of chance.

This specification guided the development of the data collection instruments used in this study.

Initial Questionnaires

The knowledge of a discipline includes knowledge of a formal and explicit nature, together with tacit knowledge. Explicit knowledge can be assessed in various ways: through observation, interviews and written tests, among others. The instruments in this study were two written questionnaires that were easy to correct, since they responded to an institutional need, namely, to assess the explicit mathematical knowledge of students beginning their education as teachers. The questionnaires were designed taking the FMK specification as a reference. Since the goals of this research refer specifically to the questionnaires that have been used, we begin by presenting the questionnaires, and then describe the specific goals of the study.

The *Fundamental Mathematical Knowledge Questionnaire* was created in Catalonia and consisted of a set of 25 items, from O1 to O25, 23 of which were short-answer questions (for further detail, see Gorgorió & Albarracín, 2020). In Catalonia, students are familiar with this type of test, as it is often used for their assessment in schools. The *School Mathematical Knowledge Questionnaire*, based on the work of Martínez et al. (2019) in Chile, consisted of a set of 25 items, from C1 to C25, all of them multiple-choice. Each item has four possible answers of which only one is correct. Chilean students are used to doing multiple-choice tests both during their schooling and in university entrance exams. Throughout this paper, the Catalan questionnaire will be referred to as the *open questionnaire* and the Chilean one as the *multiple-choice questionnaire*.

The *open questionnaire* was part of the system of assessment of the Mathematics for Teachers course, studied in the first year of the primary teacher education programme at the Universtitat Autònoma de Barcelona (UAB). It assessed knowledge of mathematical content related to the curriculum of the Catalan compulsory education system: Numbers and arithmetic; Space, shape, magnitude and measurement; Relationships and change; and Statistics and randomness. In the questionnaire, the organisation of the content areas and the weight given to them (see Table 1) was deliberately linked to their weight and organisation in actual classroom practice. Answering the questions required an understanding of the concepts and procedures involved, and the results had to be interpreted within the context of each problem.

Table 1

Distribution of Items by Content Block in the Open Questionnaire

Content Block	Number of Items
Numbers and arithmetic	8
Space, shape, magnitude and measurement	9
Relationships and change	4
Statistics and randomness	4

The questionnaire content and structure were defined by a panel of experts from four Catalan universities within the framework of the *Estudi per a l'avaluació diagnòstica de les competències matemàtiques dels estudiants del grau en Educació Primària*[2] (Study for the Diagnostic Evaluation of the Mathematical Competences of Students of the Degree in Primary Education). The questionnaire was pilot tested on three occasions with students at the UAB, who provided feedback that resulted, among other aspects, in the correction of problematic wording of statements. Furthermore, the pilot-tests at the UAB served to contrast the reliability of the instrument (see Gorgorió et al., 2021) and confirm its criterion validity through a study of the correlation between the students' results in the questionnaire and the grades obtained in the Mathematics for Teachers course (for a complete description of the process, see Gorgorió & Albarracín, 2020). By way of an example, Figure 1 shows the O8 and O21 statements.

O8. Which natural number should Δ be substituted by so that the expression $\Delta \times \Delta = 3 \times 3 \times 7 \times 7$ is correct?

O21. Determine the area of the following figure:

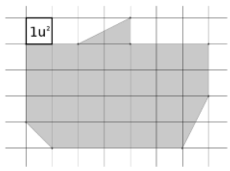

Figure 1. Example of two short-answer items.

Item O8 focused on the multiplicative structure of natural numbers. To solve it, students had to have a conceptual knowledge of prime factorisation, as they could not use any specific algorithm to solve the problem. In question O21, the content to be assessed was related to the measurement of surfaces; students had to understand the concept of unit of measurement and know that the diagonal of a parallelogram divides it into two triangles of equal surface area.

The *multiple-choice questionnaire* was based on a test developed by Martínez et al. (2019) that sought to measure the school mathematical knowledge acquired during primary education by those entering primary teacher education in Chile. The study drew on the official primary education curriculum (grades 1 to 6) of Chile (MINEDUC, 2012) to construct a set of items that covered the four thematic blocks, namely: Numbers and operations, Patterns and algebra, Geometry and measurement, and Data and probability. Following its validation by experts and its implementation with over 500 student teachers in their first year and the respective psychometric analyses, a validated scale with 40 items was obtained (for further details, see Martínez et al., 2019). Based on this validated instrument, 25 items were selected for this study, from C1 to C25, to equate it with the *open questionnaire*. Bearing in mind that

[2] 2014 ARMIF-00041, AGAUR, Agència de Gestió d'Ajuts Universitaris i de Recerca.

the *multiple-choice questionnaire* was to be applied in Catalonia as well as in Chile, various aspects were given consideration when selecting the 25 questions, including the curricular relevance of the items in the two contexts. The set of items selected for the *multiple-choice questionnaire* maintained the proportion per block of content of the Martínez et al. (2019) instrument, which was already constructed according to the curricular weight of each block, as shown in Table 2.

Table 2

Distribution of Items by Blocks of Content in the Multiple-choice Questionnaire

Content Block	Number of Items
Numbers and operations	12
Geometry and measurement	6
Patterns and algebra	3
Data and probability	4

In each of the multiple-choice items, the incorrect options were intended to be plausible answers based on students' misconceptions when trying to solve the problems. For example, the aim of item C11 (see Figure 2) was to find out if students knew how to compose and decompose numbers.

C11. Which of the following equalities is correct?

A. 30,000 + 2,000 + 50 + 700 + 2 = 32,572

B. 11,000 + 200 + 105 + 30 = 11,335

C. 60,000 + 5,000 + 30 + 1 = 6,531

D. 400 + 132 = 400,132

Figure 2. Example of a closed item.

Option A confused 50 and 500, with students being distracted by the place occupied by the 50 in the order of the sum instead of considering the positional value of the 5 in 32,572. Option C involved recognising the digits with the highest positional value while adding up and interpreting them as the digits that should be placed in the corresponding positions. Here, there was no grouping of hundreds and the positional value of 6 was not interpreted correctly. Finally, in option D the addends were joined in the order they appeared, without understanding them. This illustrated a common mistake when reading numbers, since it associates four hundred thousand one hundred and thirty-two with four hundred and one hundred and thirty-two, using the thousands separator comma as an operator to additively break up the number.

Study Goals

The concept of FMK fills a gap at a theoretical level and provides reflection around a specific need of teacher education institutions: the assessment of mathematical knowledge possessed by students on entering teacher education. Its specification allowed us to design instruments for its assessment. From now on, Initial Mathematical Knowledge (IMK) will refer to the knowledge that student teachers made explicit in the written questionnaires designed according to the specification of FMK.

Within this framework, the aim of this study was to validate the above-mentioned questionnaires and to find out what information they provided for the discussion of the IMK of prospective teachers. Therefore, our specific goals were as follows:

- Goal 1: To psychometrically validate the *open questionnaire* and the *multiple-choice questionnaire* as instruments to measure the IMK of students beginning their teacher education.
- Goal 2: To analyse whether the structure of the IMK exhibited by students beginning their teacher education matched the thematic grouping established theoretically when defining FMK.

Data Collection

This research considered two comparable populations in terms of curriculum, at two institutions where the authors taught on teacher education programmes. Therefore, the two questionnaires were administered to first-year students enrolled in primary teacher education programmes at the Universitat Autònoma de Barcelona (UAB), Spain, and the Pontificia Universidad Católica de Chile (PUC), Chile. A total of 283 students answered the questionnaires: 158 in Chile in August 2019 (2019 cohort) and 125 in Catalonia April 2019 (2018-2019 cohort). At the UAB, the data collected were part of the assessment system for the first-year subject Mathematics for Teachers. At the PUC, the study had the authorisation of the Ethics Committee for Social Sciences and Humanities. In both contexts, students gave their consent authorising the use of their anonymised responses to both questionnaires as data for the analysis. Each questionnaire was corrected by assigning a score of 1 or 0 to each question depending on whether the answer was correct or incorrect. The resulting sets of scores constitute the data of this study since they constituted the students' IMK as it had been defined.

Analysis and Results

To achieve our goals, a quantitative analysis of the data obtained was carried out with a double intention: on the one hand, to psychometrically validate the two questionnaires and, on the other hand, to obtain a measure of the students' IMK and to contrast whether the thematic grouping established when specifying the FMK was empirically confirmed. The analysis was carried out in three steps, which are schematised in Figure 3.

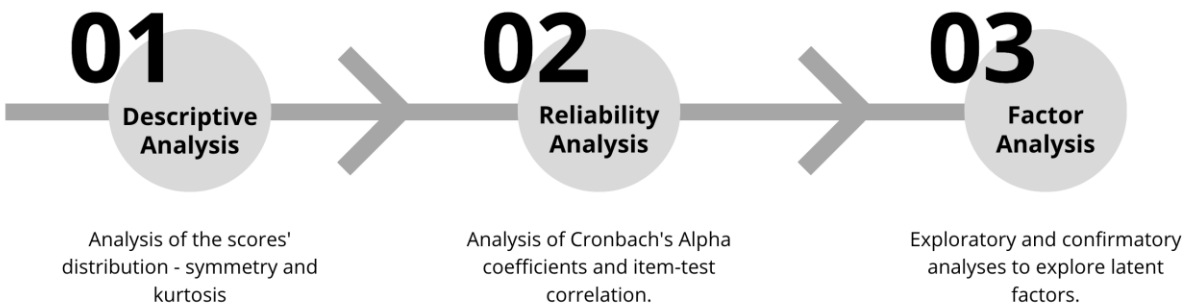

Figure 3. Steps of analysis.

The first step entailed a descriptive analysis of the number of correct answers to each item of the questionnaires, to understand their distribution. We were interested in its symmetry—whether the students' scores were skewed to one side or the other of the mean, and kurtosis—whether the students' scores of the questions were concentrated around the mean or not.

In the second step of the analysis, item reliability was studied using Cronbach's alpha coefficients, and test-item correlation was calculated using Pearson's *r* coefficients. Cronbach's alpha coefficients allowed for the determination of each question's suitability in measuring the students' IMK. The item-test correlation made it possible to assess the extent to which each item contributed to the assessment of students' IMK. Both coefficients were calculated for the entire questionnaires—taking FMK as a single theoretical construct—and for the questionnaires organised according to the content blocks that had been used to create them. Dealing with the entire questionnaires and eliminating from them those questions with low reliability and low item-test correlations, was a first step towards achieving Goal 1, validation of the questionnaires. The analyses of the questionnaires organised by mathematical content blocks suggested an answer to Goal 2, already indicating a single structure for the IMK.

The analyses carried out in the first and second steps indicated that the initial questionnaires behaved as single scales once some of their items had been eliminated. In the last stage of the process, factor analyses (both exploratory and confirmatory) of the two previously reduced questionnaires were carried out. The exploratory factor analysis served to find out to what extent the reduced questionnaires—and the grouping of the items by content blocks that made them up—were latent factors, i.e. whether they adequately represented FMK when taken as a whole, as well as by blocks. The content blocks could be understood as latent factors since their significance was proposed non-empirically. Confirmatory factor analysis was used to confirm the only latent structure identified in the previous steps, while trying to conserve as much variance as possible and determining the final set of items on the validated scale for each questionnaire. The third step of the analysis offered a definitive response both to Goal 1 by defining the set of items on the validated scale, and to Goal 2 because it enabled us to affirm that, once psychometrically adjusted, each of the questionnaires behaved as a single scale.

Descriptive Analysis: Distribution of Scores

From now on, the term "item score" will be used to refer to the number of correct answers collected for a given item. Table 3 shows the distribution of scores, symmetry, and kurtosis, with the two questionnaires and the two institutions considered separately.

Table 3
Score Distribution, Symmetry, and Kurtosis

	Questionnaire	Score		Symmetry			Kurtosis		
		mean	*sd*	mean	*sd*	interval	mean	*sd*	interval
UAB	Open	0.62	0.17	-1.33	1.61	[-6.15;0.51]	2.28	8.51	[-1.96;36.06]
	Multiple choice	0.83	0.14	-2.69	1.89	[-7.62;0.05]	8.79	13.58	[-2.02;56.55]
PUC	Open	0.58	0.21	-0.48	1.07	[-2.51;1.42]	-0.65	1.73	[-2.01;04.32]
	Multiple choice	0.71	0.25	-1.20	1.66	[-3.79;3.79]	2.13	4.48	[-1.77;12.42]

Table 3 shows that the group of UAB students performed better on average than the PUC students in both questionnaires. The symmetry analysis of the scores showed that it was negative for both instruments and both institutions, indicating that the distribution is skewed to the right. That the distribution was concentrated in the higher scores meant that the answers tended to be correct rather than incorrect. The kurtosis analysis showed that the number of right answers to the *open questionnaire* items was far from the mean in both institutions. This deviation indicates that students tended to have consolidated the knowledge that was part of this questionnaire. Conversely, the right answers to the *multiple-choice questionnaire* items were mainly concentrated around the mean of their answers.

Reliability Analysis: The Questionnaires as a Whole

Taking FMK as a single theoretical construct, Table 4 presents the Cronbach's alpha and test-item correlation coefficients for the *open questionnaire*. In parallel, Table 5 shows the coefficients obtained for the *multiple-choice questionnaire.* We consider the UAB and PUC results separately since alpha is a property of the scores obtained in a test by a specific sample of participants.

Table 4

Cronbach's Alpha Reliability Coefficients and Pearson's r *Item-test Correlations for the Open Questionnaire*

Item	UAB		PUC	
	Alpha	r.cor	Alpha	r.cor
O1	0.71	0.43*	0.74	0.42*
O2	0.70	0.47*	0.74	0.39*
O3	0.71	0.37*	0.73	0.43*
O4	0.70	0.46*	0.73	0.45*
O5	0.71	0.40*	0.74	0.37*
O6	0.71	0.38*	0.74	0.37*
O7	0.71	0.46*	0.75	0.11
O8	0.69	0.58**	0.73	0.52**
O9	0.72	0.13	0.75	0.24*
O10	0.71	0.33*	0.74	0.39*
O11	0.71	0.36*	0.74	0.30*
O12	0.72	0.25*	0.74	0.35*
O13	0.72	0.18	0.73	0.46*
O14	0.71	0.44*	0.73	0.44*
O15	0.72	0.19	0.73	0.44*
O16	0.73	0.15	0.74	0.29*
O17	0.72	0.19	0.73	0.50**
O18	0.71	0.34*	0.73	0.46*
O19	0.73	0.09	0.74	0.32*
O20	0.72	0.17	0.73	0.45*
O21	0.71	0.44**	0.73	0.55**
O22	0.73	0.01	0.74	0.30*
O23	0.72	0.23	0.75	0.29*
O24	0.70	0.52**	0.74	0.32*
O25	0.72	0.14	0.76	0.13
GLOBAL	0.72 (*SD* = 0.01)	0.31 (*SD* = 0.15)	0.75 (*SD* = 0.02)	0.37 (*SD* = 0.11)

Note: The Alpha of each item corresponds to the reliability of the scale after eliminating that item. It should lie between 0 and 1. In turn, r.cor corresponds to the item-test correlation, with **p* <0.05; ***p* <0.01.

Table 4 shows that in the case of the *open questionnaire* the Cronbach's Alpha coefficient was 0.72 for the UAB sample and 0.75 for the PUC, which is considered adequate reliability (Novick & Lewis, 1967). The same table also reports that in the case of the *open questionnaire* the scores in the UAB and PUC samples behaved differently. Despite these differences, the average item-test correlations were relatively high (Novick & Lewis, 1967), with a mean equal to 0.31 (*SD* = 0.15) for the UAB and 0.37 (*SD* = 0.11) for the PUC.

Looking at the correlations of the different items, we saw that items O9, O16, O19 and O22 had the lowest correlations with the total scale (*r.cor*) in the UAB sample and that the reliability (Cronbach's alpha) of the scale was maintained or increased when these items were excluded. Regarding the PUC sample, item O7 had the lowest item-test correlation (*r.cor*), with reliability (Cronbach's alpha) of the scale maintained or improved by excluding it. Also, item O25 showed very low correlations in both samples, with the respective reliabilities maintained or increased when the item was excluded.

Table 5

Cronbach's Alpha Reliability Coefficients and Item-test Correlation for the Multiple-choice Questionnaire

Item	UAB		PUC	
	Alpha	r.cor	Alpha	r.cor
C1	0.56	0.16	0.75	0.16
C2	0.56	0.12	0.74	0.30*
C3	0.56	0.10	0.74	0.26*
C4	0.56	0.14	0.73	0.45**
C5	0.54	0.38*	0.73	0.52**
C6	0.55	0.27*	0.73	0.39*
C7	0.57	0.08	0.75	0.07
C8	0.54	0.33*	0.76	-0.07
C9	0.53	0.38*	0.75	0.14
C10	0.54	0.32*	0.73	0.45**
C11	0.56	0.25*	0.74	0.33*
C12	0.56	0.22	0.73	0.44*
C13	0.57	0.17	0.72	0.58**
C14	0.54	0.41**	0.73	0.59**
C15	0.57	0.06	0.74	0.35*
C16	0.57	0.05	0.73	0.48**
C17	0.56	0.16	0.74	0.33*
C18	0.55	0.32*	0.73	0.38
C19	0.55	0.31*	0.73	0.41**
C20	0.55	0.25*	0.73	0.43**
C21	0.55	0.30	0.73	0.39**
C22	0.56	0.14	0.74	0.35*
C23	0.55	0.26*	0.73	0.55**
C24	0.53	0.36*	0.73	0.40*
C25	0.53	0.35*	0.76	-0.19
GLOBAL	0.56 (*SD* = 0.01)	0.05 (*SD* = 0.1)	0.75 (*SD* = 0.08)	0.11 (*SD* = 0.14)

Note: The Alpha of each item corresponds to the reliability of the scale after eliminating that item. In turn, r.cor corresponds to the item-test correlation, with $^{*}p<0.05$; $^{**}p<0.01$.

Table 5 shows that the items in the *multiple-choice questionnaire* exhibited totally different behaviours in the UAB and PUC samples. In the UAB, 15 items displayed a low item-test correlation that affected the Cronbach's Alpha coefficient, which did not exceed 0.57 while that of the complete scale was 0.56—considered very low for a scale (Novick & Lewis, 1967). In the PUC, on the other hand, the scale had very good item-test correlation indices (Novick & Lewis, 1967) in 19 of its items, as well as in the reliability coefficients, obtaining 0.72 as the lowest index and 0.75 for the complete scale. These results suggest that the items with this structure were better suited to the PUC sample than the UAB sample.

Reliability Analysis: The Questionnaires According to Content Blocks

The low reliability of items O7, O9, O16, O19, O22 and O25 suggested that they should be removed from the *open questionnaire* to ensure its homogeneity. However, before doing so, Cronbach's alpha and item-test correlation coefficients were determined by grouping the questions according to content blocks into which the FMK had been packaged (Tables 1 and 2), both for the *open questionnaire* (Table 6) and for the *multiple-choice questionnaire* (Table 7). It is important to note that, for this purpose, analogous content blocks from the curricula of Catalonia and Chile were assimilated and designated with a generic name. Therefore, for example, the GEO (geometry) block brought together those called

Space, shape, magnitude and measurement in Catalonia and Geometry and measurement in Chile, and the ALG (algebra) block brought together those called Relations and change in Catalonia and Patterns and algebra in Chile.

Table 6

Cronbach's Alpha Reliability Coefficients and Item-test Correlations Grouped by Content for the Open Questionnaire

Block	Item	UAB			PUC		
		Alpha_G	Alpha	r.cor	Alpha_G	Alpha	r.cor
NUM	O1	0.65	0.61	0.49*	0.55	0.51	0.38*
	O2		0.62	0.43*		0.48	0.48*
	O3		0.63	0.41*		0.51	0.38*
	O4		0.62	0.47*		0.49	0.46*
	O5		0.62	0.42*		0.53	0.31*
	O6		0.64	0.34*		0.54	0.26
	O7		0.61	0.49*		0.57	0.08
	O8		0.61	0.46*		0.48	0.48*
ALG	O9	0.32	0.23	0.35*	0.44	0.42	0.26
	O10		0.14	0.49*		0.32	0.46*
	O11		0.25	0.38*		0.38	0.37*
	O12		0.33	0.12		0.41	0.29
	O13		0.38	0.07		0.37	0.38*
GEO	O14	0.36	0.27	0.40*	0.61	0.57	0.44*
	O15		0.33	0.27		0.59	0.35*
	O16		0.31	0.34*		0.60	0.29*
	O17		0.34	0.22		0.58	0.38*
	O18		0.33	0.25		0.60	0.34*
	O19		0.39	0.06		0.60	0.32*
	O20		0.35	0.24		0.55	0.53**
	O21		0.33	0.24		0.54	0.57**
DATA	O22	0.23	0.31	-0.09	0.07	-0.02	0.31*
	O23		-0.01	0.42*		0.08	0.16
	O24		0.05	0.38*		0.03	0.11
	O25		0.22	0.14		0.14	0.05

Note: The Alpha_G corresponds to reliability by grouping, and the Alpha of each item corresponds to the reliability of the grouped block after eliminating that item. In turn, r.cor corresponds to the item-test correlation in the subscale, with *$p<0.05$; **$p<0.01$.

As shown in Table 6, the theoretical grouping of questions in the *open questionnaire* generated very low Cronbach's Alpha coefficients, so it was decided not to consider the theoretical grouping, accepting that all the questions measure a single construct – the students' IMK. This was supported by the assumption of tau-equivalence of the questions as regards the reliability coefficient and the high correlation of the items in the complete scale (Table 4).

Subsequently, to improve the reliability and correlation indices, we proceeded to eliminate items O7, O9, O16, O19, O22, O23 and O25 from the questionnaire in both samples in view of their low correlations with the total scale (Table 4). The elimination of those items enabled us to increase reliability from 0.72 to 0.75 and from 0.75 to 0.78 for the items of the *open questionnaire* in the UAB and PUC samples respectively, attaining a figure of 0.76 for the joint samples.

Table 7

Cronbach's Alpha Reliability Coefficients and item-test Correlations Grouped by Content for the Multiple-choice Questionnaire

Block	Item		UAB			PUC	
		Alpha_G	Alpha	r.cor	Alpha_G	Alpha	r.cor
NUM	C1	0.25	-0.10	0.40**	0.47	0.47	0.28
	C2		0.01	0.12		0.40	0.41**
	C3		0.12	-0.16		0.39	0.41**
	C4		0.03	0.04		0.40	0.41**
	C5		0.02	0.18		0.41	0.36**
ALG	C6	0.22	0.09	0.36*	0.44	0.01	-
	C7		0.14	0.22		-0.08	-
	C8		0.18	0.22		0.01	-
	C9		0.24	0.16		-0.18	-
GEO	C10	0.40	0.35	0.33*	0.61	0.69	0.46**
	C11		0.37	0.34*		0.71	0.33*
	C12		0.35	0.40**		0.70	0.39**
	C13		0.43	0.12		0.68	0.62**
	C14		0.37	0.30*		0.70	0.50**
	C15		0.39	0.22		0.71	0.38**
	C16		0.39	0.16		0.69	0.51**
	C17		0.43	0.14		0.72	0.31*
	C18		0.34	0.38*		0.70	0.39**
	C19		0.37	0.34*		0.69	0.47**
	C20		0.38	0.20		0.70	0.41**
	C21		0.40	0.22		0.71	0.34*
	C22		0.39	0.22		0.71	0.32*
DATA	C23	0.21	0.04	0.43**	0.07	0.00	0.36*
	C24		0.30	0.13		-0.21	0.50**
	C25		0.15	0.34*		0.43	-0.13

Note: The Alpha_G corresponds to the reliability per subscale, and the Alpha of each item corresponds to the reliability of the subscale after eliminating that item. In turn, *r.cor* corresponds to the item-test correlation in the subscale, with $*p<0.05$; $**p<0.01$.

Table 7 shows that considering grouping the items by content blocks in the *multiple-choice questionnaire* produced low indices of both item-test correlation and Cronbach's Alpha coefficient, the same as in the *open questionnaire*. Again, this prompts the consideration of the *multiple-choice questionnaire* as a single scale, given the accuracy of the assumption of tau-equivalence of the items for the reliability coefficient.

To conclude the reliability analysis, considering both samples individually and the multiple-choice questionnaire as a single scale, we eliminated the items that presented low item-test correlations (Table 5), namely items C3, C7, C8, C15, C16 and C25, thereby increasing reliability of the *multiple-choice* questionnaire to 0.57 for the UAB, 0.77 for the PUC, and 0.73 for the two samples together.

Exploratory Factor Analysis

In the final phase of the validation process, factor analyses (exploratory and confirmatory) were carried out on the questionnaires reduced to the remaining items after the reliability analysis. The purpose of the exploratory factor analysis was to uncover the underlying structure of the questionnaires. During this analysis, each content block was treated as a latent factor. This helps us understand how an item's responses are influenced by the construct being measured.

Before conducting the exploratory factor analysis, we had to confirm certain statistical assumptions for both questionnaires: calculating the Kaiser-Meyer-Olkin index (KMO) and the Bartlett χ^2 test of sphericity. KMO measured the adequacy of the data for this type of analysis and had to be greater than 0.6 to be acceptable. The Bartlett test of sphericity indicated that the variables were not correlated in the sample when a p-value (significance) of less than 0.05 was obtained (Tabachnick, Fidell, & Ullman, 2007). For both exploratory and confirmatory factor analysis, reference was made to Marsh, Balla and McDonald (1988) to determine which values could be considered acceptable. The values obtained are displayed in Table 8 for the *open questionnaire* and Table 9 for *the multiple-choice questionnaire.*

Table 8
Assumptions for the Factor Analysis of the Open Questionnaire

	KMO	Bartlett χ^2
UAB	0.64	(153, *n* = 125) = 311.48, *p*<.001
PUC	0.75	(153, *n* = 158) = 380.73, *p*<.001
Joint	0.82	(153, *n* = 283) = 585.35, *p*<.001

Table 9
Assumptions for the Factor Analysis of the Multiple-choice Questionnaire

	KMO	Bartlett χ^2
UAB	0.51	(171, *n* = 125) = 302.06, *p*<.001
PUC	0.76	(171, *n* = 158) = 483.98, *p*<.001
Joint	0.79	(171, *n* = 283) = 602.84, *p*<.001

The values displayed in Table 8 and in Table 9 indicated that it was appropriate to perform an exploratory factor analysis for both questionnaires.

For the exploratory factor analysis, the statistical indices—fit indices—used were the absolute χ2 and the relative Tucker-Lewis index (TLI), considered acceptable if greater than 0.9 and good if greater than 0.95. The root mean square error of approximation (RMSEA) indicated the error in the measuring the fit.

In the case of the *open questionnaire*, the exploratory factor analysis was performed with a PA (Principal Axis) extraction method and Varimax rotation. The number of factors to be retained was determined by the Kaiser-Gutman rule, the Cattell scree test and parallel analysis (Table 10).

Table 10
Fit Indices of the Single-factor Model for the Open Questionnaire

	χ^2	TLI	RMSEA
UAB	(153, *n* = 125) = 151.14, *p*>0.10	0.90	0.04
PUC	(153, *n* = 158) = 151.99, *p*>0.10	0.92	0.03
Joint	(153, *n* = 283) =1 45.68, *p*>0.20	0.97	0.02

For the *open questionnaire*, the Kaiser-Guttman rule (eigenvalues greater than 1.00) suggested the retention of a single factor, as did the scree test. As shown in Table 10 the single-factor model displayed good absolute and relative fit indices for the samples, both separately and jointly, independent of the university where the data was collected. In this process only one item – O23 – was eliminated.

The same procedure was then repeated for the *multiple-choice questionnaire*. Table 11 illustrates the fit indices obtained for the single-factor model.

Table 11

Fit Indices of the Single-factor Model for the Multiple-choice Questionnaire

	χ^2	TLI	RMSEA
UAB	(65, *n* = 125) = 154.48, $p<0.01$	0.47	0.07
PUC	(65, *n* = 158) = 116.99, $p<0.001$	0.87	0.05
Joint	(65, *n* = 283) = 139.36, $p<0.001$	0.90	0.04

As shown in Table 11, for the *multiple-choice questionnaire,* the Kaiser Gutman's rule and Cattell's scree test suggested the retention of a single factor. However, items with loadings below 0.3—C1, C2, C9, C11, C17 and C21—were eliminated to improve the relative and absolute fit indices (Lloret-Segura et al., 2014). Factor loading refers to the correlation coefficient that indicates the strength and direction of the relationship between a given item and a factor in factor analysis. It helps to determine how much the item contributes to a factor, providing insight into the underlying structure of data.

Confirmatory Factor Analysis

The analysis was concluded by performing a confirmatory factor analysis of the single-factor structure for each of the questionnaires. This analysis enabled the correlation – or shared covariance – of the items to be determined using the comparative fit index (CFI), the root mean square error of approximation (RMSEA) and the standardised root mean square residual (SRMR) as a measure of absolute fit. CFI is considered acceptable if greater than 0.9 and good if greater than 0.95. RMSEA with values less than 0.05 is considered a good fit, and between [0.05;0.08] a regular fit. When SRMR has a value of 0 it indicates a perfect fit and a value of less than 0.08 is good (Hu & Bentler, 1999).

To confirm the existence of a single factor in the *open questionnaire*, a confirmatory factor analysis was performed considering both samples together (UAB and PUC students), since the samples taken separately were rather small and the fit indices of the exploratory factor analysis were better for the joint sample (see Tables 10 and 11). To that end, the maximum likelihood extraction model and the *probit* link function were used—given that the variables were the scores 0/1 and therefore ordinal in nature. Table 12 shows that very good absolute and relative fit indices were obtained.

Table 12

Confirmatory Factor Analysis of the Open Questionnaire

	χ^2	CFI	TLI	RMSEA	SRMR
Joint	(135, *n* = 283) = 150.17, $p = 0.17$	0.97	0.96	0.02	0.05

Nevertheless, two of the items (O10 and O11) still exhibited absolute loadings lower than 0.3 so they were eliminated. After this, the model improved its fit indices as shown in Table 13.

Table 13

Confirmatory Factor Analysis of the Open Questionnaire After the Elimination of Two Items

	χ^2	CFI	TLI	RMSEA	SRMR
Joint	(104, *n* = 283) = 116.57, $p = 0.19$	0.97	0.97	0.02	0.04

The final loadings of the items in the *open questionnaire* in the one-factor model are shown in Table 14. It should be noted that in the *open questionnaire* what remained of the initial distribution by topic consisted of seven items on Numbers and Arithmetic, two on Relations and Change, six on Space, Shape and Measurement, and only one on Statistics and Randomness.

Table 14
Factor Loadings of the Open Questionnaire Items in the One-factor Model

Item	Factor Loading	Item	Factor Loading
O1	0.44**	O13	0.33**
O2	0.42**	O14	0.46**
O3	0.49**	O15	0.36**
O4	0.38**	O17	0.40**
O5	0.34**	O18	0.40**
O6	0.34**	O20	0.37**
O8	0.51**	O21	0.48**
O12	0.36**	O24	0.44**

Note: *$p< 0.05$; **$p< 0.01$.

Finally, a confirmatory factor analysis of the *multiple-choice questionnaire* was carried out using the maximum likelihood extraction model – once again taking the UAB and PUC samples together for the same reason as cited above. As can be seen in Table 15, acceptable absolute and relative fit indices were obtained.

Table 15
Confirmatory Factor Analysis of the Multiple-choice Questionnaire

	χ^2	CFI	TLI	RMSEA	SRMR
Joint	(78, n = 283) = 449.88, p = 0.00	0.91	0.90	0.04	0.05

As shown in Table 16, the final loadings of the *multiple-choice questionnaire* items in the single factor model were all higher than 0.3, so no items were eliminated at this stage. Note that what remained of the initial distribution by topic in the *multiple-choice questionnaire* consisted of eight items on Numbers and operations, two on Patterns and algebra, two on Geometry and measurement, and only one on Data and Probability.

Table 16
Factor Loadings of the Multiple-choice Questionnaire Items in the One-factor Model

Item	Factor Loadings	Item	Factor Loadings
C4	0.37**	C18	0.46**
C5	0.53**	C19	0.42**
C6	0.39**	C20	0.35**
C10	0.32**	C22	0.38**
C12	0.45**	C23	0.56**
C13	0.42**	C24	0.43**
C14	0.38**		

Note: *$p< 0.05$; **$p< 0.01$.

This last stage of the analysis provided a definitive answer to the two goals of our study. Two different psychometrically validated instruments were obtained by eliminating the items with poor reliability and low factor loadings from each of the initial questionnaires. Therefore, Goal 1 was met by two instruments containing the items in Tables 14 and 16, which are published here: https://ddd.uab.cat/record/257026?ln=ca. Regarding Goal 2, the reliability analysis suggested an initial structure of the students' IMK, indicating that the scale would be unidimensional. This finding was subsequently corroborated by the factor analysis. Therefore, it was observed that the way the students constructed their IMK was not separated by mathematical content blocks and presented a unidimensional structure.

Discussion and Conclusions

This study set out to validate two instruments designed to assess the mathematical knowledge of students accessing primary teacher education programmes in two different contexts. The realisation of these instruments had been based on the conceptualisation of Fundamental Mathematical Knowledge (FMK) (Castro et al., 2014; Gorgorió & Albarracín, 2020; Gorgorió, et al., 2021), understood as the mathematical knowledge necessary to successfully complete their training and acquire the mathematical knowledge needed for teaching. The results of the study allowed conclusions to be drawn at the empirical and theoretical level, discussing the implications for teacher education despite the limitations of the study due to the idiosyncrasies of the contexts in which it was defined.

At the empirical level, we have determined a validated set of questions that enable us to measure students' Initial Mathematical Knowledge, understood as the knowledge that student teachers made explicit in the written questionnaires designed according to the specification of FMK. Although the psychometric validation process of the *open questionnaire* and the *multiple-choice questionnaire* yielded fewer items than the original versions (16 and 13 questions respectively), this has allowed us to move from criterion-referenced instruments with 25 items each to psychometrically validated instruments. As in any knowledge measurement process, further development of questions is needed in order to discriminate between what the groups of students know, which raises new areas of research for the development of this type of instrument. Regarding this set of questions, it is important to point out that items in the Numbers block were predominant in both questionnaires, in accordance with the curricular weight of this content block in both the Chilean and Catalan curricula. However, this fact does not imply that the students' knowledge at the beginning of their education as teachers was based exclusively on number content, since other content blocks were also present in the validated instruments.

Our study opens up new opportunities for the development of tools for the study of students' mathematical knowledge when they enter teacher education programmes, complementing previous studies (Arce et al., 2017; Ingram & Linsell, 2014; Nortes & Nortes, 2013; Ryan & McCrae, 2005/06; Senk et al.; 2012; Tatto et al., 2008). However, it should be noted that the transferability of the results of this study requires differentiation. Regarding the validated instruments, necessary adaptations must be considered according to the curricular contexts, evaluative cultures, and training goals of the teacher education programmes in each country and context. Directly applying these instruments in other education systems appears to be a challenge.

On a theoretical level, our research contributes to filling a gap in the field of studies that examine the knowledge needed to teach mathematics. These studies considered the knowledge that is developed in initial or continuing education or in classroom practice (Ball et al., 2001; Ball et al., 2008; Ball et al., 2012; Carrillo et al., 2018; Cooney & Wiegel, 2003; Hiebert et al., 2002; Hill et al., 2007; Ma, 1999; Rowland et al., 2005, 2009; Shulman, 1987). However, to date there have been few studies that have looked at the knowledge needed to begin teacher education. The definition of Fundamental Mathematical Knowledge as that knowledge necessary for students to successfully follow mathematics-related and mathematics teaching subjects throughout their teacher education could be considered vague or noncommittal. However, the concretisation of such knowledge around curricular content allowed us to define and validate instruments not only to measure it but also, and more importantly, to study the structure of mathematical knowledge made explicit by students through these instruments.

In doing so, the reliability analysis carried out in our study suggested that the questionnaires for the realisation of fundamental mathematical knowledge behaved as single scales, despite being constructed from the different blocks of curricular content. In doing so, the content groupings were found to display low reliability with respect to the full scale, both in the *open questionnaire* and in the *multiple-choice questionnaire*. This finding suggested that the students' initial mathematical knowledge could not be explained in terms of the mathematical content blocks, implying that it was a single common underlying factor shared by the different items in each questionnaire (Segars, 1997). To test for latent uniqueness, confirmatory factor analyses of both questionnaires were carried out. The analyses confirmed the internal and external consistency and dependent structure of the items in each questionnaire, irrespective of the initial theoretical construct (Schreiber et al., 2006; Churchill, 1979; Jöreskog & Sörbom, 1989).

Therefore, it may be affirmed that the initial mathematical knowledge exhibited by UAB and PUC students is structurally unidimensional, and that fundamental mathematical knowledge is a single factor underlying the items in each questionnaire. Although adaptations of the instruments may be required, in the light of our results, it is reasonable to assume that the structure of students' initial mathematical knowledge in other educational contexts may also be unidimensional. However, this hypothesis needs to be tested and deserves to be tested given its importance in relation to teacher education.

The results of our study may also have implications for teacher education, since instruments for assessing the mathematical knowledge of students entering teacher education may have different uses. On the one hand, given that these instruments measure initial mathematical knowledge, they could be used to establish admission requirements for teacher education programmes based on the attainment of a certain level of mathematical knowledge. Considering the use of the instruments to regulate admission, the negative skew of the distribution of scores showed that the percentage of correct answers was high in most cases at both universities. This would suggest that, although students enrolling in teaching degrees at these institutions show some deficiencies, their demonstrated initial mathematical knowledge could be considered sufficient in relation to FMK. However, it would be up to each education system or institution to determine what is the minimum performance required when students begin their training as teachers. In addition, the *multiple-choice questionnaire* displayed different behaviour between the two universities, with lower reliability indices for UAB students than for PUC students. This could be related to the assessment culture of each context, since multiple choice questionnaires are much more widely used in the Chilean context. Therefore, before transferring the assessments instruments to other contexts or institutions, it would be necessary not only to adapt their content but also their format prior their validation.

On the other hand, since the validated instruments inform us about the structure of the initial mathematical knowledge of student teachers, they could also be used as diagnostic tools to reorganise mathematics courses and mathematics teaching courses. The unidimensional structure of their initial mathematical knowledge suggested that during their pre-university education they had developed a body of interwoven mathematical knowledge whose content could not be dissociated. This suggests that teacher training courses should integrate different types of mathematical knowledge rather than compartmentalising them by topics, which would make teacher education even more challenging given the limited curricular time available and the traditions of university teaching.

Finally, we would like to address the methodological limitations of the study, which are related to both the type and number of participants and the format of the instruments. These aspects are key issues that merit consideration in future studies that seek to gain an in-depth understanding of the mathematical knowledge of those entering teacher education. On the one hand, the participants in this study came from two highly selective universities in their respective contexts. This circumstance may have had an impact on the fact that the adjustment indices of the factor analyses increased when analysing the samples jointly as compared to analysing each of the groups of students separately. To make sure this does not skew the validity of the results (homogeneity of the type of participant), researchers who aim to develop an assessment instrument similar to those studied in this research but adapted to the educational system of their region should guarantee variability within the samples, and it may be preferable to use data from students registered at different universities or on different types of programmes. This would allow for a larger sample size, thereby strengthening the reliability of the validated instruments. On the other hand, this study only contemplated the use of "pencil and paper" type data collection instruments, which means that it was only possible to explore certain aspects of their mathematical knowledge, those that can be expressed in this format. It would be interesting to extend this study to other types of instruments and strategies, such as digital tests, performance simulations, and interviews, to obtain a deeper understanding of students' initial mathematical knowledge and its structure.

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References

Albarracín, L., Rojas, F., Chandia, E., Ubilla, F. M. & Gorgorió, N. (2021). Unidimensionalidad del conocimiento matemático inicial de estudiantes para maestro. [Unidimensionality of initial mathematical knowledge of students for teachers] In P. D. Diago, D. F. Yáñez, M. T. González-Astudillo & D. Carrillo (Eds.), *Investigación en Educación Matemática XXIV* (pp. 117–124). SEIEM.

Arce, M., Marbán, J. M. & Palop, B. (2017). Aproximación al conocimiento común del contenido matemático en estudiantes para maestro de primaria de nuevo ingreso desde la prueba de evaluación final de Educación Primaria. [Approach to common knowledge of mathematical content in students for new elementary school teachers from the final evaluation test of Primary Education] In J. M. Muñoz-Escolano, A. Arnal-Bailera, P. Beltrán-Pellicer, M. L. Callejo & J. Carrillo (Eds.), *Investigación en Educación Matemática XXI* (pp. 119–128). SEIEM.

Association of Mathematics Teacher Educators. (2017). *Standards for preparing teachers of mathematics*. https://amte.net/standards

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449–466. https://doi.org/10.1086/461626

Ball, D. L., Blömeke, S., Delaney, S., & Kaiser, G. (2012). Measuring teacher knowledge: Approaches and results from a cross-national perspective. *ZDM Mathematics Education*, *44*(3), 223–455.

Ball, D. L., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching*, (4th ed., pp. 433–456). Macmillan.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389−407. https://doi.org/10.1177/0022487108324554

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M. & Tsai, Y. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*, 133–180. https://doi.org/10.3102/0002831209345157

Beswick, K., & Goos, M. (2012). Measuring pre-service primary teachers' knowledge for teaching mathematics. *Mathematics Teacher Education & Development, 14*(2), 70–90.

Carrillo, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, A., Ribeiro, M. & Muñoz-Catalán, M. C. (2018). The Mathematics Teacher's Specialised Knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236–253. https://doi.org/10.1080/14794802.2018.1479981

Castro, Á., Mengual, E., Prat, M., Albarracín, L., & Gorgorió, N. (2014). Conocimiento matemático fundamental para el grado de Educación Primaria: Inicio de una línea de investigación. [Fundamental mathematical knowledge for primary education: Beginning of a line of research] In M. T. González, M. Codes, D. Arnau & T. Ortega (Eds.), *Investigación en Educación Matemática XVIII* (pp. 227–236). SEIEM.

Churchill Jr, G. A. (1979). A paradigm for developing better measures of marketing constructs. *Journal of Marketing Research*, *16*(1), 64–73.

Cooney, T. J., & Wiegel, H. G. (2003). Examining the mathematics in mathematics teacher education. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 795–828). Kluwer.

Departament d'Ensenyament (2015*). Currículum d'educació primària.* http://portaldogc.gencat.cat/utilsEADOP/PDF/6900/1431926.pdf

Department of Education. (2013). *The national curriculum in England: Key stages 1 and 2 framework documents.*

Feuer, M. J., Floden, R. E., Chudowsky, N., & Ahn, J. (2013). *Evaluation of teacher preparation programs: Purposes, methods, and policy options.* National Academy of Education.

Giaconi, V., Gómez, G., Jiménez, D., Gareca, B., Durán del Fierro, F., & Varas, M. L. (2022). Initial diagnostic assessment in pre-service teacher training in Chile and its relationship with institutional contexts. *Pensamiento Educativo, Revista de Investigación Latinoamericana (PEL), 59*(1), 1–16. https://doi.org/10.7764/PEL.59.1.2022.4

Gorgorió, N., Albarracín, L., & Villareal, A. (2017). Examen de competència logicomatemàtica en la nova prova d'accés als graus de mestre. *Noubiaix, 39*, 58–64.

Gorgorió, N., Albarracín, L., Ärlebäck, J., Laine, A., Newton, R. & Villarreal, A. (2017). *Fundamental mathematical knowledge: Progressing its specification.* Linköping University Electronic Press. https://www.diva-portal.org/smash/record.jsf?pid=diva2%3A1318541&dswid=-7867

Gorgorió, N., Albarracín, L., Laine, A., & Llinares, S. (2021). Primary education degree programs in Alicante, Barcelona and Helsinki: Could the differences in the mathematical knowledge of incoming students be explained by the access criteria? *LUMAT*, 9(1) 174–207. https://doi.org/10.31129/lumat.9.1.1468

Gorgorió, N., & Albarracín, L. (2020). El conocimiento matemático previo a la formación inicial de los maestros: Necesidad y concreción de una prueba para su evaluación. [Mathematical knowledge prior to initial teacher training: The need for and specificity of a test for its assessment] In E. Badillo, N. Climent, C. Fernández & M. González-Astudillo (Eds.), *RED8-Educación Matemática y Formación de Profesores* (pp. 111–132). Ediciones Universidad de Salamanca.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2002). Knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher, 31*(5), 3–15.

Hiebert, J., Berk, D., Miller, E., Gallivan, H. & E. Meikle (2019). Relationships between opportunity to learn mathematics in teacher preparation and graduates' knowledge for teaching mathematics. *Journal for Research in Mathematics Education, 50*(1), 23–50. https://doi.org/10.5951/jresematheduc.50.1.0023

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge. What knowledge matters and what evidence counts? In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111–156). Information Age Publishing.

Hu, L. T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, 6(1), 1e55. https://doi.org/10.1080/10705519909540118

Ingram, N., & Linsell, C. (2014). Foundation content knowledge: Pre-service teachers' attainment and affect. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice.* Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia (pp. 712–718). MERGA.

Jöreskog, K. G., & Sörbom, D. (1986). *LISREL VI: Analysis of linear structural relationships by maximum likelihood and least square methods*. Scientific Software.

Law 20.903 (2016 April 1). *Sistema de Desarrollo Profesional Docente* [Teacher professional development system]. Biblioteca del Congreso Nacional de Chile. http://bcn.cl/2py3i

Linsell, C., & Anakin, M. (2012). Diagnostic assessment of pre-service teachers' mathematical content knowledge. *Mathematics Teacher Education and Development, 14*(2), 4–27.

Linsell, C., & Anakin, M. (2013). Foundation content knowledge: What do pre-service teachers need to know? In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow.* Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia (pp. 442–449). MERGA.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Erlbaum.

Marsh, H. W., Balla, J. R., & McDonald, R. P. (1988). Goodness-of-fit indexes in confirmatory factor analysis: The effect of sample size. *Psychological Bulletin*, *103*, 391–410.

Martínez, M. V., Rojas, F., Ulloa, R., Chandía. E., Ortíz, A., & Perdomo-Díaz, J. (2019). Beliefs and mathematical school knowledge at the beginning of pre-service primary teacher education. *Pensamiento Educativo*, *56*(2), 1–19. https://doi.org/10.7764/PEL.56.2.2019.9

Miller-Levy, R., Taylor, D., & Hawke, L. (2014). Maintaining the boundaries: Teacher preparation program admission criteria for screening quality candidates. *Administrative Issues Journal: Education, Practice, and Research, 4*(1), 40–49. https://doi.org/10.5929/2014.4.1.8

MINEDUC. (2012). *Bases Curriculares para la Educación Básica*. Ministerio de Educación. Chile. https://www.curriculumnacional.cl/

National Board of Education. (2016). *National core curriculum for basic education 2014*. Finnish National Board of Education.

Nortes, A., & Nortes, R. (2013). Formación inicial de maestros: un estudio en el dominio de las matemáticas. Initial teacher training: a study in the domain of mathematics] *Profesorado: Revista de Currículum y Formación del Profesorado, 17*(3), 185–200.

Nortes, R., & Nortes, A. (2018). ¿Tienen los futuros maestros los conocimientos matemáticos elementales? [Do future teachers have basic mathematical knowledge?] In L. J. Rodríguez-Muñiz, L. Muñiz-Rodríguez, A. Aguilar-González, P. Alonso, F. J. García-García & A. Bruno (Eds.), *Investigación en Educación Matemática XXII* (pp. 397-406). SEIEM.

Novick, M. R., & Lewis, C. (1967). Coefficient alpha and the reliability of composite measurements. *Psychometrika 32*, 1–13.

Rojas, F., Seissus, E. & Jiménez, D. (2021). Diagnóstico inicial docente en matemática en programas de pedagogía en educación básica. [Initial teaching diagnosis in mathematics in basic education pedagogy programs] In D. M. Gómez, C. Cornejo & M. V. Martínez (Eds.). *Actas de las XXV Jornadas Nacionales de Educación Matemática*. UOH—SOCHIEM.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The Knowledge Quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*, 255–281. https://doi.org/10.1007/s10857-005-0853-5

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching: Reflecting on practice with the Knowledge Quartet*. SAGE Publications.

Ryan, J., & McCrae, B. (2005/06). Assessing pre-service teachers' mathematics subject knowledge. *Mathematics Teacher Education and Development, 7*, 72–89.

Schreiber, J. B., Nora, A., Stage, F. K., Barlow, E. A., & King, J. (2006). Reporting structural equation modelling and confirmatory factor analysis results: A review. *The Journal of Educational Research, 99*(6), 323–337. https://doi.org/10.3200/joer.99.6.323-338

Segars, A. H. (1997). Assessing the unidimensionality of measurement: A paradigm and illustration within the context of information systems research. *Omega*, *25*(1), 107–121.

Senk, S. L., Tatto, M. T., Reckase, M., Rowley, G., Peck, R., & Bankov, K. (2012). Knowledge of future primary teachers for teaching mathematics: An international comparative study. *ZDM Mathematics Education, 44*(3), 307–324. https://doi.org/10.1007/s11858-012-0400-7

Shulman, S. (1987). Knowledge and teaching: Foundations of the new reforms. *Harvard Educational Review, 57*(1), 1–22. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Skolverket. (2011). *Curriculum for the compulsory school, preschool class and the recreation centre, 2011.* Fritzes.

Tabachnick, B. G., Fidell, L. S., & Ullman, J. B. (2007). *Using multivariate statistics (Vol. 5)*. Pearson.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher education and development study in mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework*. Teacher Education and Development International.