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ORIGINAL PAPER

Exploring the role of assumptions in mathematical modeling teacher training using Fermi problems

Lluís Albarracín[1] · Jonas Bergman Ärlebäck[2]

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Abstract
This study explores the assumptions made by pre-service primary teachers during mathematical modeling activities using Fermi problems. While mathematical modeling is emphasized in curricula worldwide, little attention has been given to how teachers can effectively support students in making the necessary assumptions to solve these problems. This research addresses this gap by analyzing 100 Fermi Problem Activity Templates produced by 35 pairs of pre-service teachers, focusing on the assumptions made during model formulation, mathematical processing, and strategic choices. Results reveal participants' challenges in identifying key quantities and constructing coherent models, particularly in distant and complex contexts. While accessible activities like guesstimation and data searching dominate, strategic decisions like experimentation and statistical polling remain underutilized. Findings highlight the potential of FPATs in teacher training to foster modeling competencies and address contextual problem-solving skills.

1 Introduction

Mathematical modeling—broadly understood as the application of mathematics to solve real-world problems—has become a prevalent curriculum content in many countries, with objectives such as illustrating how mathematics can describe complex phenomena and enhance critical thinking skills (Niss & Blum, 2020). However, mathematical modeling activities have yet to be established as a regular content taught in mathematics classrooms in most countries, with exceptions like Germany and South Korea (Borromeo Ferri, 2021). This situation contrasts with the maturity of research on mathematical modeling in classrooms (Schukajlow et al., 2018), making the study of how to facilitate teachers' incorporation of modeling activities in the classroom a priority. One of the key factors in promoting the incorporation of mathematical modeling in classrooms is teachers (Niss et al., 2007), who need to develop a wide range of teaching competencies to foster mathematical modeling in students (Geiger et al., 2022). Finding and using meaningful contexts that are productively aligned with curriculum content is also challenging for teachers (Blum, 2015; Burkhardt, 2006). The inherent characteristics of mathematical modeling tasks often lead to a reduction in the level of control teachers can exercise, as these activities inherently provide students with greater autonomy in their decision-making processes (Garfunkel et al., 2021). Consequently, it becomes essential to design targeted teacher training programs and develop accessible, well-suited resources to support the effective integration of mathematical modeling into classroom instruction. To support students, their teachers need to have developed subject-specific competencies (Kunter et al., 2013). Regarding mathematical modeling there are different dimensions of such competencies: (1) a theoretical dimension; (2) an instructional dimension; (3) a diagnostic dimension; and (4) a task dimension that includes cognitive task analysis (Borromeo Ferri, 2017). One aspect that cuts across all three dimensions that teachers need to consider, and in which students need to develop competences in, is making the assumptions needed to solve modeling problems. The nature, role and function of assumptions in mathematical modeling is an area of educational research that has not until recently received much explicit attention (Chang et al., 2020; Galbraith & Stillman, 2001). In part, this is because the notion of assumption can be connected to several aspects of a modeling activity. For example, it can refer to making the first assumptions about what aspects of the situation or

✉ Lluís Albarracín
lluis.albarracin@uab.cat

1 Universitat Autònoma de Barcelona, Cerdanyola del Vallès, Barcelona, Spain

2 Linköping University, Linköping, Sweden

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problem modelled are important, to what assumptions can be made to simplify the problem, as well as what mathematical knowledge and pre-existing models can be assumed to be useful to employ in the problem at hand. Recent research has increasingly focused on the role of assumptions in mathematical modeling (Chang et al., 2020; Krawitz et al., 2018), highlighting that identifying unknown quantities and formulating numerical assumptions about them represent significant cognitive challenges in tackling open modeling tasks (Schukajlow et al., 2023). This study extends this existing body of work by examining the role and significance of assumptions in mathematical modeling through the lens of Fermi problems, by more broadly investigating assumptions made in model formulation, to facilitate mathematical processing, and in strategic choices in the solution process.

Fermi problems are a type of open-ended miniature modeling problems (cf. Ärlebäck, 2009; Robinson, 2008) that focus on identifying the necessary quantities and estimating numerical values that allow the problem to be solved. As such, from a methodological point of view, Fermi problems provide a more straightforward problem capturing crucial aspects of modelling problems (Albarracín & Gorgorió, 2014; Ärlebäck, 2009). In this paper, we use a sequence of Fermi problems to investigate how pre-service primary teachers (from here on referred to as Participants) solve and analyze Fermi problems using a specific framework known as Fermi Problem Activity Templates (FPATs) (Albarracín & Ärlebäck, 2019). FPATs explicitly capture solvers' strategic choices, revealing both assumptions underpinning the structure of the solution and the activities to be enacted to obtain the needed numerical data, hence enabling the study of the nature of solvers' assumptions more nuanced than previous studies (Chang et al., 2020; Krawitz et al., 2018; Schukajlow et al., 2023). So far, FPATs have only been used for the design of classroom activities and in pilot studies (Albarracín & Ärlebäck, 2022). In this first comprehensive empirical study, we characterize 100 FPATs produced by 35 pairs of participants in the teaching sequence, with the aim to gain insights into the assumptions made that lead to the solutions, as manifested in the FPATs.

2 Background, theoretical foundation and research questions

2.1 Mathematical modeling

Early mathematics education research on mathematical modeling discussed the applications of mathematics, resulting in a theoretical framework for mathematical modeling that situates the modeling process by distinguishing between the mathematical domain and the rest of the world (Pollak, 1969a, 1969b, 1979). This separation necessarily leads to a process of mathematization of a real phenomenon or situation by making assumptions to connect reality to the mathematical domain. The solver then works in the mathematical domain and generates a model that provides an answer that needs to be reinterpreted and validated in the real context.

Approaches that describe the cognitive processes required to complete a modeling task typically consist of a sequence of key processes and stages in idealized modeling cycles (Blum & Leiss, 2007; Galbraith & Stillman, 2006). During the modeling process, the solvers attempt to solve the task by going through different stages and successively reassessing their work, made assumptions, and the situation under investigation in a cyclical manner. This allows the solver to improve the models and solutions found for the problem they are working on, adapting them to the requirements of the problem statement (Blum & Borromeo Ferri, 2009).

Among the various theoretical perspectives on mathematical modeling (Kaiser & Sriraman, 2006), our research on Fermi problems and mathematical modeling is framed within the Models and Modeling Perspective (Lesh & Doerr, 2003). From this perspective, models are conceptual systems (consisting of elements, relations, operations and rules governing interactions) that are expressed by external notation systems and are used to construct, describe or explain the behavior of other systems, in other to manipulate or predict them appropriately (Lesh & Doerr, 2003). Fundamentally, models are sense-making systems regardless of the nature of the situation or problem at hand, be it real, authentic, hypothetical or fiction (Efthimiou & Llewellyn, 2007)—as long as it is relatable to the solver (Lesh & Harel, 2003); this means that models can vary greatly in complexity (Brady et al., 2020). By engaging in modeling activities, learners develop, modify, extend and revise their models through multiple cycles of interpretations, descriptions, connections, explanations and justifications (Doerr & English, 2003)—by making and drawing on different types of assumptions (Galbraith and Stillman (2001).

2.2 Research on assumptions in educational research on mathematical modeling

The literature shows that students' ability to make assumptions is crucial in both mathematical modeling (Djepaxhija et al., 2015; Niss & Blum, 2020) and mathematics in general (Komatsu et al., 2024; Stylianides & Stylianides, 2023). However, Galbraith and Stillman (2001) observed that "the role of assumptions in modeling activities has been oversimplified" (p. 301). Chang et al. (2020) further noted that the role of assumptions in modeling is an area of educational research that has received limited attention. Pre-service primary teachers often struggle with making appropriate assumptions due to a lack of experience with modeling and estimation problems, leading to errors in the simplification and structuring stages of the modelling process (Segura & Ferrando, 2021).

Galbraith and Stillman (2001) discuss assumptions with respect to (1) model formulation, (2) mathematical processing, and (3) strategic choices in the solution process. These instances of assumptions depend in different ways on the context of the problem. During the model formulation phase, assumptions are typically made about what quantities and data are relevant and needed. During mathematical processing, assumptions help map the context to mathematical concepts and structures. When making strategic choices, assumptions focus on what and how to evaluate the models and the results obtained. Similarly, Seino (2005), in discussing a teaching principle called "the awareness of assumptions", highlights three stages of the modeling process where assumptions come to the fore: the formulation phase of a model; the working mathematical phase; and the interpretation and evaluation phase of the mathematical conclusion in relation to the real-world problem. Seino's research shows that students can recognize and be aware of the role and function of assumptions in all three phases. He argues that it is crucial for students engaged in modeling to realize and appreciate the importance of assumptions throughout the modeling process. Similarly, Komatsu et al. (2024) emphasize the importance of students understanding the role of both local and global assumptions in mathematical processes, particularly in recognizing that conclusions are contingent on the assumptions that support the argument, and that explicitly stating these assumptions are essential to achieving agreement on the conclusion.

While some assumptions are explicitly stated in the modelling process, others may be implicitly embedded. Explicit assumptions are crucial for reaching consensus, as they make the underlying reasoning transparent (Komatsu et al., 2024). Implicit assumptions, however, can lead to oversight and may result in inadequate models if not identified. Related to the model formulation process, Chang et al. (2020) distinguish between numerical and non-numerical assumptions. Non-numerical assumptions pertain to "situational conditions, which require realistic considerations and extra-mathematical knowledge" (p. 61). Numerical assumptions involve missing values of quantities, which, in addition to extra-mathematical knowledge and realistic considerations being made, also "mathematical estimation skills are necessary" (p. 61). One line of research focusing on students' abilities to make adequate and realistic assumptions uses *open-* or *missing information and value problems* (Chang et al., 2020; Krawitz et al., 2018; Schukajlow et al., 2023). Comparing the abilities of German and Taiwanese students to make numerical and non-numerical assumptions when solving modeling problems, Chang et al. (2020) indicates that the educational system and culture influence student' abilities. However, it is possible to improve students' ability to make numerical assumptions by using task-specific prompts (Schukajlow et al., 2023). Another aspect that influences the nature and success of students' assumptions is their personal interest in the context of the problem at hand. Research shows that when students are invested and interested in the context of the problem, that (1) the assumptions made are more realistic; (2) students are better able to justify their assumptions; (3) the solutions are more complete yet also often more complicated (Kämmerer, 2023, 2024). Interestingly, Kämmerer's work also highlights how students' assumptions can be seen as either context-specific or more general, which has some similarities to Komatsu et al.'s (2024) distinction between local and global assumptions.

2.3 Fermi problems as mathematical modeling activities

According to Ärlebäck (2009), Fermi problems are "open-ended, non-standard problems that require students to make assumptions about the problem situation and estimate relevant quantities before often performing simple calculations" (pp. 331–332). Following Thompson (1994), quantities are here understood as measurable attributes of an object or phenomenon that can be compared, measured, and reasoned about. Thus, solving Fermi problems involves making assumptions about, and rough but informed estimates of, quantities, often based on incomplete data (Efthimiou & Llewellyn, 2007). Robinson (2008) points out that, to solve a Fermi problem, students need to synthesize a model, examine the physical principles and conditions at work in the phenomenon under investigation, identify the most relevant constraints, and decide on the level of simplicity that can be adopted while maintaining the required realism. Consistent with the development of mathematical models, this involves making assumptions to decompose the original problem into simpler subproblems (Carlson, 1997) to arrive at a solution. The solving process requires making assumptions how to simplify and mathematize reality by encouraging students to generate mathematical models based on their prior knowledge of the real world (Henze & Fritzlar, 2010), through reasonable estimates or educated guesses (Sriraman & Knott, 2009). Therefore, Fermi problems can act as model-promoting activities in the sense of Lesh et al. (2000) and have been found to be effective in introducing students to the modeling process at the elementary (Haberzettl et al., 2018; Peter-Koop, 2009) and secondary (Albarracín & Gorgorió, 2014; Ärlebäck, 2011) levels.

Some features of Fermi problems make them particularly interesting as activities for introducing mathematical modeling in the classroom, as they encourage discussion, allow a clear connection between reality and mathematical content, and are accessible to students (Ärlebäck, 2009). The accessibility is due to the fact that it is possible to create multiple approaches of different complexity to solve a given Fermi problem (Albarracín & Gorgorió, 2014), depending on the solver's prior mathematical knowledge and ability to make assumptions to connect to the real world. In real-life situations and contexts, complexity arises from the interaction of numerous interrelated elements. Estimation can be used as a facilitator to reduce this complexity to a level that the solver can handle (Ärlebäck & Albarracín, 2024). Indeed, Fermi problems are a type of problem in which assumptions about reality and the situation at hand are a key aspect of the solution. Indeed, identifying the relevant quantities, identifying the relationships established between them to support a model and the values obtained by making estimates are essential aspects of solving a Fermi problem.

2.4 Fermi problems activity templates

The *Fermi estimation method* is the classic approach to solving Fermi problems, in which assumptions are made to decompose the original problem into simpler subproblems that are solved separately using reasoned estimates based on knowledge of the real world. Studies indicate that this approach can be effectively taught to secondary school students using direct instruction, enabling them to address a diverse range of Fermi problems (Kothiyal & Murthy, 2018). To represent the structure of the solution of a Fermi problem and make it explicit for students, Anderson and Sherman (2010) proposed a simple geometric representation of the solution (see Fig. 1).

Although the original method proposed by Enrico Fermi himself (Allison et al., 1955) and most of the subsequent research on Fermi problems has generally emphasized the use of estimation, it has been suggested that this method of obtaining numerical results for subproblems can be replaced by other mathematical activities (Sriraman & Knott, 2009). Investigating this further, Ärlebäck and Albarracín (2019) presented a review of the use of Fermi problems in different areas of education and found that Fermi problems have been used with multiple and diverse didactic objectives, and that there are proposals to improve the models and results generated by the students by introducing new activities that allow them to go beyond mere estimated values.

Albarracín and Ärlebäck (2019) propose a theoretical framework to support the solving of Fermi problems based on four mathematical activities that connect with the curricular learning goals of compulsory school mathematics: *guesstimation*; *experimentation* (usually referring to measurement activities); *looking for data*; and *polling and statistical data collection*. In this way, solving a Fermi problem can be extended from a short activity, in which a simple model is generated, and partial values are determined from estimations to larger and more substantial projects, in which some key subproblems are studied as small investigations (Albarracín, 2021).

Building on the work of Anderson and Sherman (2010) and Albarracín and Ärlebäck (2019) integrated the four different types of activities and the structure of the solution of a Fermi problem into a framework called *Fermi Problem Activity Templates—FPATs*. In a FPAT, the intended activity to solve a subproblem is indicated by a specific geometric shape: Guesstimation (ellipse); Experimentation (trapezoid); Looking for data (rectangle); and Polling or Statistical data collection (hexagon). In addition, each subproblem is delimited using square brackets. For each solution to an FP, the corresponding FPAT can be constructed; see Fig. 2 for an example of a possible FPAT for the solution (by Anderson & Sherman, 2010) in Fig. 1, with specified activities for solving each subproblem.

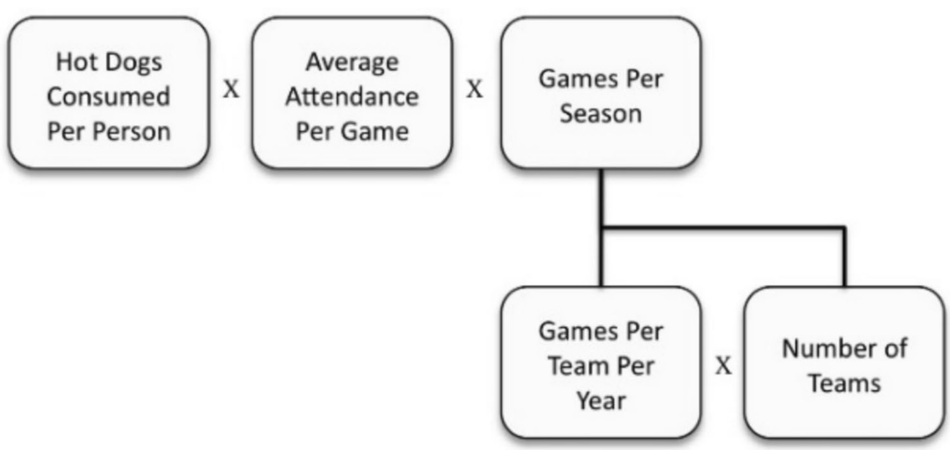


Fig. 1 The solution structure for estimating the number of hot dogs consumed in a Major League Baseball season (Anderson & Sherman, 2010, p. 38)

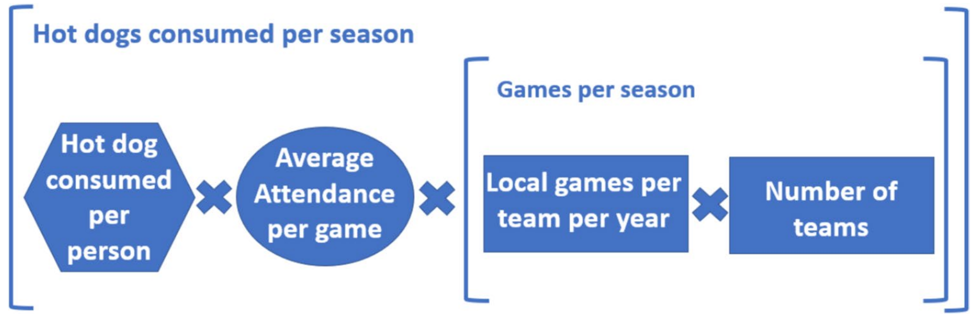


Fig. 2 A FPAT based on the solution structure presented in Fig. 1

2.5 Theoretical framework and research question

To gain insight into the different assumptions made in a modeling process (Galbraith & Stillman, 2001), we use Fermi problems as miniature modeling problems (Ärlebäck, 2009; Robinson, 2008) as instantiations of modeling problems (cf. Lesh & Doerr, 2003). This allows us to analyze assumptions made with respect to (1) model formulation, (2) mathematical processing, and (3) strategic choices in the solution process (Galbraith & Stillman, 2001), in the sense that solving a Fermi problems requires making assumptions about: (1) the real world (Djepaxhija et al., 2015), which quantities (Thompson, 1994) that are relevant (Niss & Blum, 2020), and to decompose the initial problem into subproblems (Carlson, 1997); and (2) how to map the context onto mathematical concepts and structures, i.e. to formulate an adequate model (Albarracín & Gorgorió, 2014). These two aspects (1) and (2) of assumption are captured by the graphical representation of FPATs (Albarracín & Ärlebäck, 2019) in terms of *complexity* (number of quantities and subproblems; see Sect. 3.2.1) and *quality of the assumptions* (see Sect. 3.2.2). In addition, the FPATs also include proposed mathematical activities (*guesstimation*, *experimentation*, *searching for data*, and *polling or statistical data collection*) to obtain the numerical information needed to solve each subproblem identified from the literature (Ärlebäck & Albarracín, 2019). This last component of the FPAT, *activities* (see Sect. 3.2.3), captures assumptions related to strategic choices (e.g. Galbraith and Stillman's (2001) no. (iii)).

Therefore, in order to shed light on what assumptions are made in the solving of Fermi problems in terms of (1) model formulation, (2) mathematical processing, and (3) strategic choices in the solution process (Galbraith & Stillman, 2001), the research questions we address in this paper are: *What characterize the FPATs produced by the Participants in terms of the (a) complexity of the solutions; (b) quality of proposed assumptions; and (c) proposed mathematical activities?*

3 Methodology

3.1 Settings of the study, participants, design and data collected

The data analyzed in this study were obtained from the initial weeks of a one semester third-year course "Mathematics for understanding the world" taught by the first author in a 4-year pre-service primary teacher program at the Universitat Autònoma de Barcelona (Spain). The study is framed in a naturalistic-qualitative tradition and uses a classroom-based research strategy (Moschkovich & Brenner, 2000). The teacher, in the role of a researcher, used non-participant observation, just guiding the implementation of the activities to and gathering information about the work and practices that emerged in the classroom in a way that was as genuine as possible. The Participants had previously studied three courses in mathematics and mathematics education, none of which included any modeling or real-life problem solving. The course is designed to teach mathematical modeling activities in general, and sequences of Fermi problems and FPATs as didactic tools are the first contents taught. The data was collected from three 3-h sessions centered around three Fermi problems (see below) mixing (1) short lectures/instruction; (2) individual- and pair problem solving; and (3) whole-class discussions. The course was repeated three times (2022–2024), with 22, 24 and 24 students participating in each iteration (a total of 53 females and 17 males with an average age of 22.3 years). Thus, we have data from 35 pairs of Participants working on the following three Fermi problems:

- *Paper problem* How many rolls of toilet paper are needed at school during the school year? How much space would they occupy?
- *Car problem* If a city council wanted to buy all the cars in your town, how much money would it have to spend?
- *Ambulance problem* How many ambulances do we need if we want to attend any emergency happening at any place in Catalonia in less than 8 min?

Each problem can be approached in multiple ways, resulting in models that require identifying, making assumptions about, and combining different quantities during the solving process. It is well established that real-life contexts used in Fermi problems enable students to make more authentic estimates (Ärlebäck, 2011) and that varying the proximity of the problem contexts to the student's prior experiences is a pedagogical strategy that encourages students to generate richer models (Pla-Castells & Ferrando, 2019). Hence, the *Paper problem* is situated within a well-known context for the Participants, whereas *Car problem* is set within their own city but involve a novel quantity which the participants not likely have previously considered: the combined value of all the cars in a city. This quantity is of economic interest to the insurance market (UNESPA, 2021) and relevant to the discussion on sustainable mobility (Dans, 2016). In contrast, the *Ambulance problem* presents a more complex scenario, with which the Participants have less experience. All three contexts of the problems are relatable to the Participants but posed on different levels with respect to what previous knowledge can be applied depending on the Participants proximity to each context.

Table 1 shows the implementation of the three sessions divided into three whole class activities and five phases of individual and pair work on the three Fermi problems by the Participants.

All documents produced during the implementation were collected, but for this study, we focus only on the 100 FPATs created in Phase IV for the three Fermi problems (35 FPATs for the *Paper problem*, 33 for the *Car problem* and 32 for the *Ambulance problem*). Not all pairs provided an FPAT for each problem; in some cases, some participants did not manage to construct a complete FPAT and chose not to submit it for collection.

3.2 Analysis

To answer the research question, the analysis of the FPATs focused on the mathematical structure of the Participants' solutions and the relevant quantities involved (cf. Thompson, 1994) that the Participants chose to include in their models through the FPAT. To this end, we collaborative developed and revised the preliminary analytic approach from Albarracín and Ärlebäck (2022). Given the nature of the FPAT, the Participants provide the data in a format that allows the researchers to clearly identify and quantify the sub-problems and the proposed activities. The quality of the assumptions to reach an adequate solution to the problem was evaluated independently by the two researchers. Discrepancies in the coding were reported in three of the 100 FPATs collected and were resolved through discussion.

The analysis resulting in the categorization of each of the 100 FPATs was saved in a spreadsheet. Table 2 provides an illustrative example of the characteristics identified for the FPAT from Fig. 6.

3.2.1 The complexity of the solution

Regarding the complexity of the solutions, broadly understood as a measure of the number of elements of which the model developed is composed, there is no specific proposal in the literature on how to measure the complexity of a solution to a Fermi problem (or any modeling problem generally). We follow the proposal of Greefrath and Frenken (2021), who consider both the number of subproblems, and

Table 1 The implementation of the three sessions of Participants' work on the three Fermi problems

Implementation steps	Participants…
Introduction Fermi problems (30 min) *Whole class*	…are introduced to Fermi problems and two examples of how to solve Fermi problems provided
Phase I (60 min) *Individually*	…write down their action plan to solve each of the three Fermi problems (given to the participants at the same time)
Phase II (90 min) *Pairs*	…discuss their solution plans in pairs and write down their joint action plans for the three Fermi problems
Phase III (60 min) *Pairs*	…discuss and document the primary school mathematics curricular contents connected to their joint solution plans
Introduction to FPATs (30 min) *Whole class*	…are introduced to the FPAT framework, highlighting (i) the need of decomposing a problem into subproblems; and (ii) determining activities to get the needed numerical information for answering the subproblem
Phase IV (90 min) *Pairs*	…create the FPATs for their solutions of the three Fermi problems
Phase V (90 min) *Pairs*	…revisit the products generated in phases II and III to check their own initial solution plans and the evolution of their own ideas
Summary discussion (90 min) *Whole class*	…discuss and highlight the differences from all type of solutions in the three Fermi problems

Table 2 Characterization of pair #22's FPAT for *Car problem*

Analysis dimensions	Characteristics	# yes/no
Complexity	How many subproblems?	3
	How many quantities are involved? (dimensions)	3
Quality of assumptions	Are key quantities identified?	Yes
	Are they connected adequately in order to solve the problem?	Yes
Activities	How many *guesstimation* subproblems are?	0
	How many *experimentation* subproblems are?	0
	How many *statistics polling* subproblems are?	2
	How many *looking for data* subproblems are?	1

quantities (cf. Thompson, 1994) involved as an indicator of a solution complexity (Table 2). It is important to note that the mere use of a high number of subproblems in a solution does not necessarily qualify as a complex solution to a Fermi problem. In some cases, students' solutions may present multiple subproblems that are related in a complicated way to identify and estimate a simple and uncomplicated quantity. One illustrative example is calculating the number of class hours of a year-long school course by traversing years-months-weeks-days-hours. Such cases illustrate an *artificial complexity* that manifests itself in the decomposing and posing of several subproblems that center around a single quantity in what could be characterized as unnecessarily complicated.

3.2.2 The quality of assumptions

A second dimension of the analysis concerns the quality of the assumptions made by the Participants, in terms of the extent to which these, potentially or realized, results in a model that solves the Fermi problem based in the mathematical processes manifested in the FPAT. In categorizing this aspect of the Participants' FPATs, we employed the three-tiered notion of Chang et al. (2020) in their study of the quality of students' assumptions when solving modeling problem but only focusing on if "the non-numerical assumptions … are appropriately made for solving the task" (p. 65). In other words, we consider a FPAT to represent either a *correct*, *incomplete* or *incorrect* solution.

To apply this categorization to FPATs we consider two specific characteristics. On the one hand, we identify whether the FPAT presents a set of key quantities that allows us to solve the problem. Secondly, we identify if the structure of the FPAT is adequate to solve the problem, i.e. if the operations and relations between those quantities allow solvers to reach a suitable estimation. Thus, if the two characteristics are coded as fulfilled, the FPAT is considered to be *correct* and based on appropriate key quantities that are successfully related to each other in a model to provide an answer to the Fermi problem. If an appropriate set of key quantities is identified but there are structural errors in the relationship between them, we characterized the FPAT as *incomplete*. In all other cases we considered the FPAT as *incorrect*.

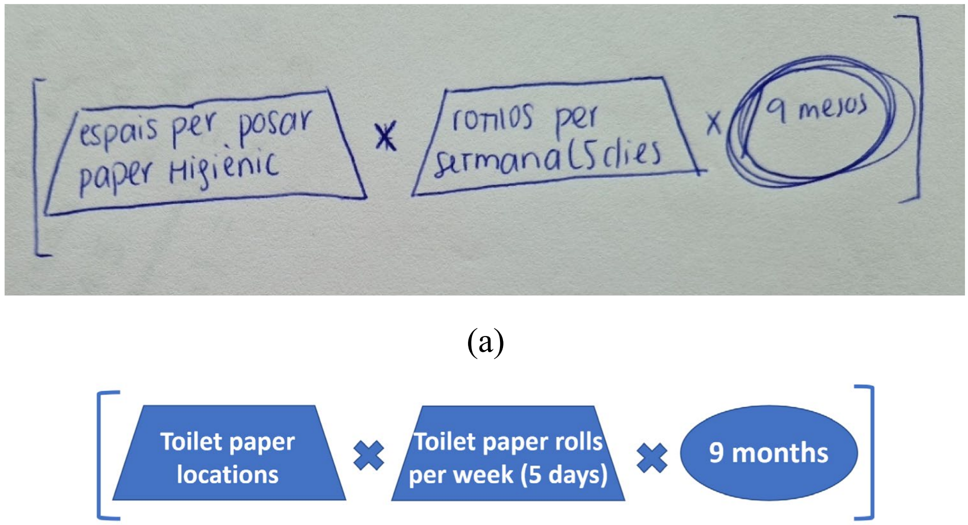
3.2.3 The FPAT activities

The third aspect of the analysis focuses on the inherent feature of selecting the activities to employ to obtain the numerical information required to solve the subproblem when producing a FPAT giving access to the strategic choices and assumptions in the suggested solution process. In certain cases, an estimation may be deemed an appropriate activity, given that the quantity in question is commonly known and within the students' grasp. However, there are instances where students may lack the requisite experience to justify a reasoned estimate. In such cases, students are expected to be aware of alternative means of accessing the required information, such as using an accessible database, designing a particular experiment or conducting a survey. Adjusting the activity appropriately demonstrates that the Participants understand the nature of the subproblem under study and can make appropriate assumptions. In the analysis we register and count all the activities (*Guesstimation, Experimentation*, *Looking for data*, *Polling or Statistical data collection*) in the FPATs produced by the Participants.

3.2.4 Examples of data and analysis

The following section presents and discusses the analytical techniques employed in the data analysis process, illustrated by examples taken from the dataset.

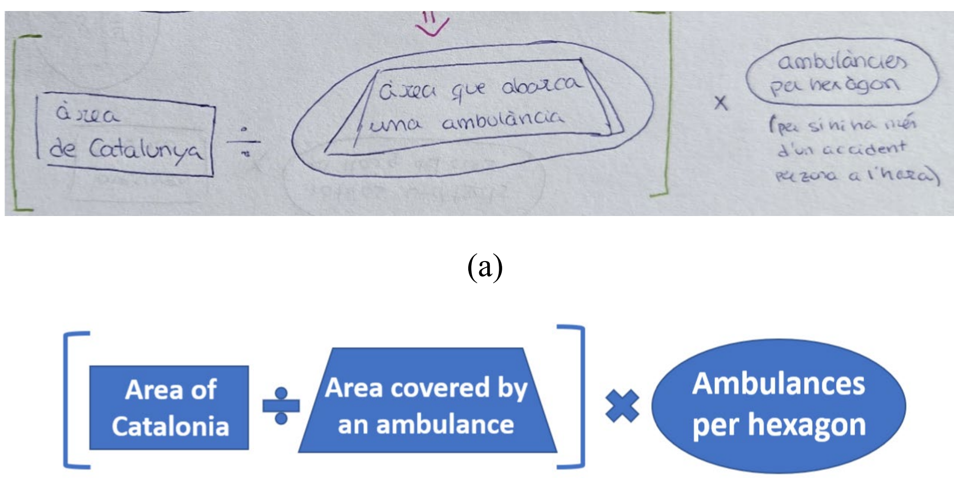
Figure 3 illustrates an example of a Participant who produced a FPAT for the *Paper problem* with the objective of estimating the number of rolls of toilet paper required in a school year. It shows three identified key quantities related to three corresponding subproblems. Hence, the complexity of the FPAT is characterized by three subproblems and their corresponding quantities, all with a different unit of measurement: number of locations, number of rolls and time. The three subproblems are necessary to obtain the requisite information, but from a conceptual point of view, these



(a)

(b)

Fig. 3 **a** Pair #16's original produced FPAT for the *Paper problem*; and **b** a recreated and translated version of the same FPAT



(a)

(b)

Fig. 4 **a** Pair #2's FPAT for the *Ambulance problem* and **b** a recreated and translated version of the same FPAT

quantities are not adequately interconnected, as the second experiment seeks to find the weekly paper consumption, while the time (the length of a school year) is expressed in months. Consequently, this proposal does not solve the problem and is categorized as incomplete. The students propose a direct count of the number of locations where toilet paper is used, e.g. number of toilets (*experimentation*). A further week-long *experiment* is proposed to determine the number of rolls consumed in a specific location. The duration of the school year is estimated to be 9 months (*guesstimation*), and the model consists of multiplying these three values together.

In the solution to the *Ambulance problem* (Fig. 4), the entire area of Catalonia (*looked up from a data source*) is divided by the area that can be covered by an ambulance (determined by *experimentation*). Additionally, the solution involves multiplying by the estimated number of ambulances per area (a *guesstimation*). In our analysis, we characterized this FPAT as correct by considering the essential aspects that contribute to the generation of a model that leads to a reasonable rough estimate. The central idea is based on the area an ambulance can cover in 8 min, and the solution consists of three subproblems and two key quantities (area and number of ambulances). However, further refinement of this solution is possible by introducing additional subproblems and providing clearer distinctions between the specific types of activities proposed to achieve more accurate results.

The two FPATs above (Figs. 3, 4) provide illustrative examples of how the FPATs in the dataset have been analyzed. However, in some FPATs the students fail to construct a comprehensive model by (1) failing to establish

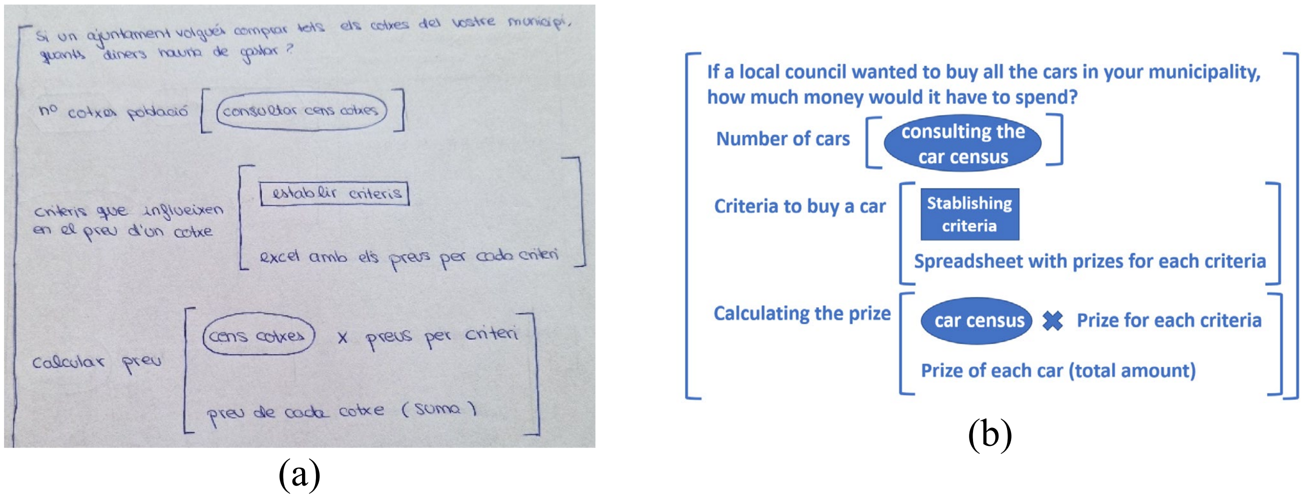


Fig. 5 **a** Pair #2's original FPAT for the *Car problem*; and **b** a recreated and translated version of the same FPAT

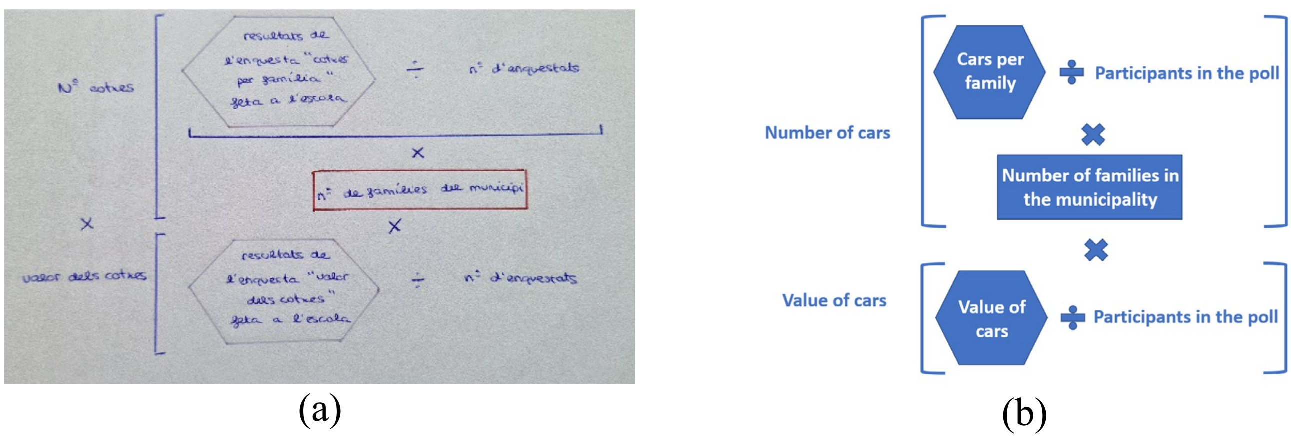


Fig. 6 **a** Pair #22's FPAT for the *Car problem*; and **b** a recreated and translated version of the same FPAT

Table 3 The average numbers of quantities and subproblems considered in the three problems

	Paper problem	Car problem	Ambulance problem
Number of subproblems	3.80	3.36	3.88
Number of quantities	2.57	2.70	2.59

appropriate relationships between the identified quantities (and accompanying subproblems) as illustrated in Fig. 5; or (2) by offering a too simplistic solution (Fig. 6).

4 Results

We now present the results of the analysis of the 100 collected FPATs. First, we discuss the complexity of the solutions. Next, we examine the quality of the assumptions. Finally, we explore the types of activities proposed in the FPATs.

4.1 The complexity of the solutions

In this section we present various approaches to measuring the complexity of the collected FPATs. First, Table 3 shows the complexity of the FPATs in terms of the average number of identified quantities and subproblems for each problem. On average, the same number of quantities and subproblems were considered in the three problems, and a non-parametric

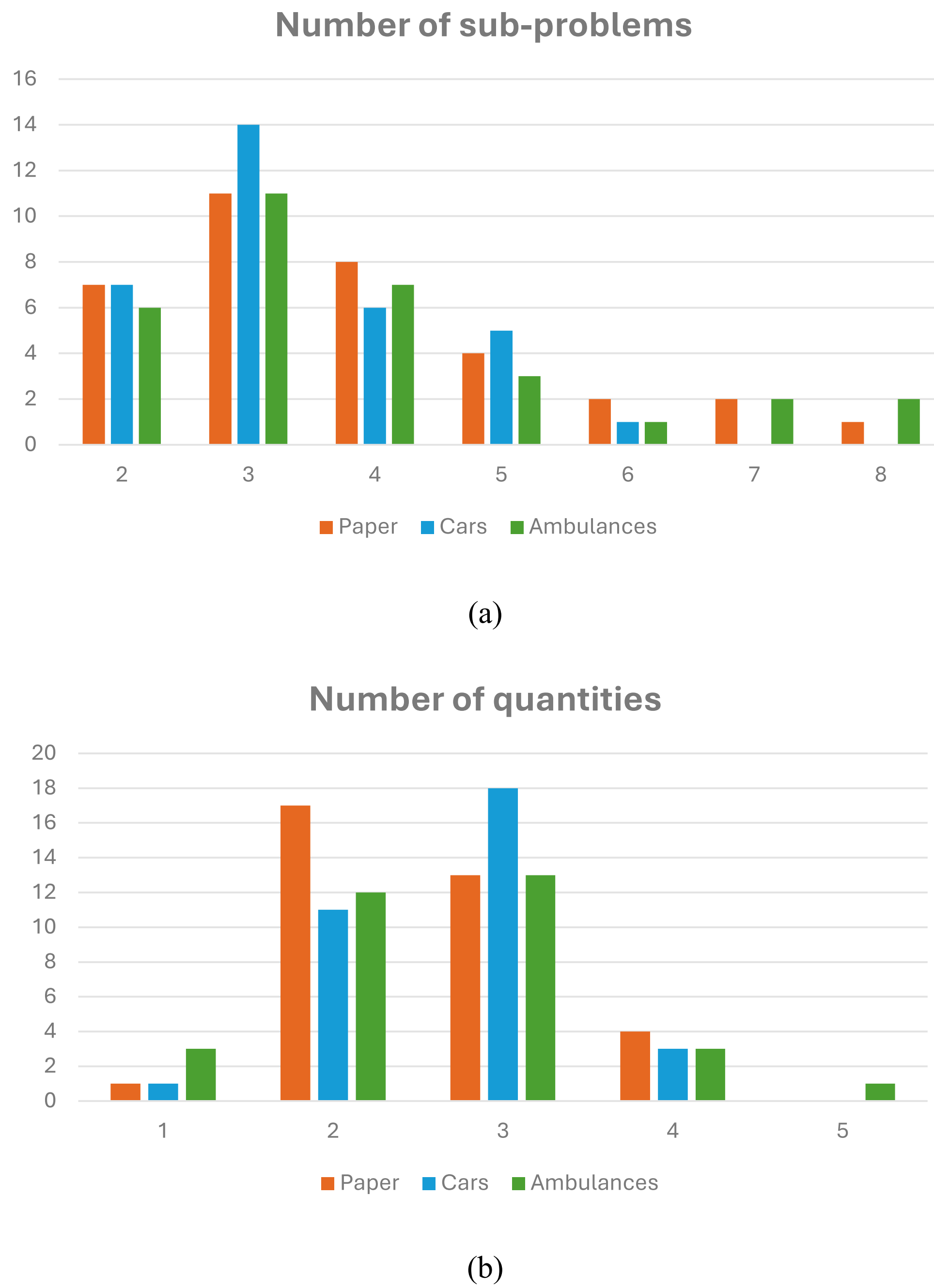


Fig. 7 Distribution of the number of subproblems (**a**) and quantities (**b**) per problem

Kruskal–Wallis test (for quantities, $\chi^2 = 0.863$; $p = 0.650$; for subproblems $\chi^2 = 1.18$; $p = 0.55$) shows that there are no significant differences in this respect. Thus, the results indicate that, on average, the complexity of the proposed FPATs does not show significant variations with respect to the proximity of the problem contexts, nor with respect to the level of complexity in terms of the assumptions that the Participants have made for the model formulation of the three phenomena.

To gain a more detailed understanding of the complexity of the FPATs, Fig. 7 shows the distribution of the number of subproblems, and quantities present in the FPATs, and Table 4 shows the overall breakdown of the analysis.

The result indicates a variation in the complexity of the FPATs developed by the Participants, as they range from simple solutions (two subproblems) to complex solutions (8 subproblems). Here, the *Paper problem* (2–8) and the *Ambulance problem* (2–7) were found to have a wider range of subproblems than the *Car problem*

Table 4 Cross-table over the identified quantities and subproblems in the 100 analyzed FPATs

		Number of quantities in a FPAT					Total number FPATs proposing # subproblems
		1	2	3	4	5	
Number of sub problems in a FPAT	2	1/3/1 (5)	6/3/6 (15)	0/0/0 (0)	0/0/0 (0)	0/0/0 (0)	7/6/7 (20)
	3	0/0/0 (0)	7/4/2 (13)	4/7/12 (23)	0/0/0 (0)	0/0/0 (0)	11/11/14 (36)
	4	0/0/0 (0)	2/3/2 (7)	6/4/4 (14)	0/0/0 (0)	0/0/0 (0)	8/7/6 (21)
	5	0/0/0 (0)	1/0/1 (2)	1/1/2 (4)	2/2/2 (6)	0/0/0 (0)	4/3/5 (12)
	6	0/0/0 (0)	0/1/0 (1)	1/0/0 (1)	1/0/1 (2)	0/0/0 (0)	2/1/1 (4)
	7	0/0/0 (0)	0/1/0 (1)	1/1/0 (2)	1/1/0 (2)	0/1/0 (1)	2/4/0 (6)
	8	0/0/0 (0)	0/0/0 (0)	0/0/0 (0)	1/0/0 (1)	0/0/0 (0)	1/0/0 (1)
Total number FPATs proposing # quantities		1/3/1 (5)	16/12/11 (39)	13/13/18 (44)	5/3/3 (11)	0/1/0 (1)	35/32/33 (100)

(2–6). However, the mode for the number of subproblems in the FPATs is the same for all three problems (3) and the underlying general distributions are all right-skewed (see Fig. 7a). Only a limited number of FPATs represent solutions with a high number of subproblems. On the other hand, the distribution of quantities identified in the FPATs varies between 1 and 5 (see Fig. 7b and Table 4), while the mode of the number of quantities presented in the FPATs is 2 for the *Paper problem* and 3 for the *Car problem* and the *Ambulance problem*. This could be since the *Paper problem* presents less difficulty for the Participants because it has a familiar context and involves known quantities.

To explore the relationship between the number of subproblems and the number of quantities, we have compiled the cross-table shown in Table 4.

Five FPATs were found with the simplest possible characterization (1 quantity and 2 subproblems). However, FPATs with 2–4 quantities typically have a varied number of subproblems distributed around the diagonal in Table 4. This distribution indicates a possible correlation between the number of proposed quantities and subproblems. There are 38 FPATs where the number of subproblems and quantities is the same, and 40 FPATs where the number of subproblems is one less (n − 1) than the number of quantities (n). The former is the more frequent, especially in the case of the FPATs with two or three proposed quantities. The results show that the number of FPATs with artificial complexity due to unnecessary transversing of units is small. Overall, this analysis shows that the models formulated by Participants are mostly based in 2 or 3 quantities and 2 to 4 subproblems and only 12 solutions involve working with 4 or more quantities. In many cases, the Participants use a large number of subproblems to obtain an estimate of a simple and uncomplicated quantity.

Table 5 The distribution of the quality of the solutions

	Paper problem	Car problem	Ambulance problem
Unanswered	0 (0%)	3 (9%)	2 (6%)
Incorrect	1 (3%)	0 (0%)	7 (20%)
Incomplete	5 (14%)	6 (17%)	15 (43%)
Correct	29 (83%)	26 (74%)	11 (31%)
Total	35	35	35

4.2 The quality of the assumptions made

The quality of the assumptions made by the Participants regarding relevant quantities and their role in the model constructed are presented in Table 5. In the case of the *Paper problem*, all pairs of students provided a FPAT, even if some were *incomplete* (14%) or *incorrect* (3%). This is not the case for the *Car problem* and *Ambulance problem*, which are set in more distant contexts. In these cases, 9% of the PSPT pairs were unable to produce an FPAT for the *Car problem*, and 6% were unable to do so for the *Ambulance problem*. This shows a higher accessibility for the *Paper problem* compared to the other two problems.

In the case of the *Paper problem*, 83% of the Participants provided a *correct* FPAT, showing that they were able to identify the relevant quantities and adequately connect them to synthesize a model that could be used to solve the FP. In the case of the *Car problem*, this percentage drops to 74% and is only 31% in the *Ambulance problem*, making the latter Fermi problem the most challenging for the Participants. Furthermore, it can be observed that those Participants who provided a FPAT were able to identify the relevant quantities in all cases for the *Car problem* and in all but one case for the *Paper problem* (97%, considering incomplete and correct answers), but that only 74% of the Participants managed to do so for the *Ambulance problem*.

4.3 Activities proposed in the FPATs

Table 6 summarizes the analysis of which activities were made by the Participants as strategic choices to obtain numerical information were suggested in the FPATs. The $\#_{FPAT}$-column shows the number of FPATs in which the specific activity was proposed by the Participants. The $\#_{tot}$-column shows the total number of subproblems in which specific activity was suggested in each problem. The μ-column presents the average of the number of subproblems that were proposed to be solved with a specific activity for each problem, while the μ^*-column presents the average when only the FPATs that did propose at least one instance of the activity in question are considered (hence $\mu \leq \mu^*$).

Table 6 shows that the most frequently proposed activities in all three Fermi problems were *looking for data* (47% of all identified activities) and *guesstimation* (34% of all identified activities). These two activities are, by their nature, more accessible to Participants because they do not involve the added difficulty of designing an experiment or collecting data. In the case of using *experimentation* to achieve the required values, this activity is most frequently suggested in the FPATs of the *Paper problem* when comparing the three Fermi problems, indicating that students could take advantage of a known context and set up realistic experiments. As evidenced by Table 6, a mere handful of Participants suggest the use of *statistical data collection* in their FPATs. This may indicate that participants do not consider the use of statistical data collection techniques as a viable means of supporting their estimates.

Closer examination of the μ^*-column in Table 6 demonstrates differences between the three problems when the Participants decided to make use of a specific activity in their FPATs. One difference is the tendency that once a particular activity is used once, it is more or less likely to be used multiple times in the same FPAT. In the *Paper problem* and the *Ambulance problem* this trend is strongest for using *Looking for data* ($\mu^*_{PA} = 1.72$; $\mu^*_{PC} = 2.19$), followed by *Experimentation* ($\mu^*_{PA} = 1.63$; $\mu^*_{PC} = 2.11$) and then *Guesstimation* ($\mu^*_{PA} = 1.45$; $\mu^*_{PC} = 1.79$). For the *Car problem* on the other hand, the trend is *Guesstimation* ($\mu^*_{PB} = 1.85$), followed by *Experimentation* ($\mu^*_{PB} = 1.67$) and then *Statistical polling* ($\mu^*_{PB} = 1.60$).

5 Discussion and conclusions

In this article we have investigated Participant's assumptions when engaging in proposing solutions in terms of FPATs to three Fermi problems with varying complexity and context proximity. The analysis focused on assumptions made as manifested in the complexity of the solutions, the quality of the assumptions and the mathematical activities proposed.

The analysis of the proposed solutions to the three Fermi problems shows that the assumptions made did not create FPATs of very high complexity. In terms of the identified subproblems and quantities, the complexity is low and often provides a close to minimal structure to solve the problem, demonstrating a limited proficiency in making assumptions supporting model formulation, and without reaching a level of detail that would allow greater precision in the answers and nuanced mathematization of the phenomena in question. The results suggest that the number of quantities and the number of subproblems in the FPATs are correlated. For Participants to be able to design modeling activities and help their students to solve modeling problems (Blum, 2015; Burkhardt, 2006), both in simple and complex ways (Borromeo Ferri, 2021), it is desirable that they have various competences that facilitate them to make assumptions linking real-world phenomena

Table 6 Activities proposed in the FPATs

	Paper problem				Car problem				Ambulance problem			
	$\#_{FPAT}$	$\#_{tot}$	μ	μ^*	$\#_{FPAT}$	$\#_{tot}$	μ	μ^*	$\#_{FPAT}$	$\#_{tot}$	μ	μ^*
Guesstimation	29	42	1.20	1.45	28	34	1.52	1.85	19	50	1.06	1.79
Experimentation	19	31	0.89	1.63	3	19	0.15	1.67	9	5	0.59	2.11
Statistical polling	4	5	0.14	1.25	5	2	0.24	1.6	2	8	0.06	1.00
Looking for data	31	55	1.57	1.72	31	70	1.42	1.52	32	47	2.19	2.19
FPATs collected	35				32				33			

and mathematics (Geiger et al., 2022). It seems necessary for teacher training to include approaches that facilitate the development of such competences as well as their flexibility in contextualized problem solving (Segura & Ferrando, 2023). This study illustrates that Fermi problems and FPATs can be a tool for addressing such Participant's learning goals.

Solving a contextualized problem requires making assumptions about the particularities of the reality studied. In solving Fermi problems assumptions are made regarding the quantities involved, the relationship between them, and FPATs manifest the way in which the solver intends to achieve the partial results, reflecting the rigor and effectiveness of the suggested or actualized mathematical processing. Our study shows, by jointly considering the categories of incomplete and correct solutions expressed in the FPATs, that all Participants were able to identify and make assumptions about the relevant quantities for the *Paper problem*, 81% for the *Car problem* and 74% for the *Ambulance problem*. Hence, we can conclude that the Participants were able to identify the relevant quantities even in Fermi problems set in contexts distant from their experience, but that distant context makes it more difficult to create complete models and to adequately connect the relevant quantities. In this context, the didactical approach of upscaling contexts proposed by Pla-Castells and Ferrando (2019) can be considered both appropriate and essential for teacher training. As in the case of "missing information/value"-problems it makes sense to differentiate between context-specific assumptions (price of a car) and general assumptions (days in a year, seconds in an hour). This result confirms that the ability to transfer extra-mathematical knowledge is key to solve Fermi problems (Ärlebäck, 2009), and the possibility to improve solvers' assumptions by carefully choosing contexts (Kämmerer, 2024).

The results show that all four FPAT-activities are proposed by the Participants, primarily *guesstimation* and *looking for data* which together account for 81% of the proposed activities. This may occur because these activities are more accessible to Participants and require little elaboration. *Experimentation* or *statistical polling* activities account for 19% of the proposed activities, and 12 of the Participants did not propose either of these for any of the three Fermi problems. For the *Paper problem* and the *Car problem*, the main activity suggested is *looking for data*, whereas it was the second most suggested activity for *Ambulance problem*. This could be interpreted as the Participants are often relying on assumptions based on the accessibility to reliable and open sources to obtain the information they need, which generally is not the case. This fact can become a barrier to solving Fermi problems but also open modeling problems in general (Schukajlow et al., 2023), in the sense that Participants could ascribe to the belief that all numerical information always is directly accessible, and hence there is no need to model the problem in the first place. Although it is necessary to encourage the search for existing information, it is also important to provide Participants with examples of interesting situations that are not pre-quantified or answerable by direct information searches.

The fact that only 19% of the proposed activities are *experimentation* or *statistical polling* is a notable result demonstrating a limited proficiency in making assumptions informing strategic decision-making. Although the students know the didactics related to these activities as content being in the last year of their training as Participants, it does not seem that they have been able to plan to implement this content in solving the three Fermi problems. This provides an interesting area for future research, namely, to investigate the possibility of improving Participants' ability to make numerical guesses using *experimentation* or *statistical polling* using task-specific prompts (cf. Schukajlow et al., 2023). This approach may be particularly productive to develop measurement in real-world contexts (Sarama et al., 2022).

From a methodological point of view this study shows that the FPAT (cf. Albarracín & Ärlebäck, 2019) has proved to be a useful tool to investigate Participants made assumptions engaged in modeling with respect to (1) the model formulation when solvers decide what quantities and subproblems are relevant; (2) the mathematical processing when working on simplifying the problem; and (3) the strategical choices in the solution process when choosing the method to obtain the needed numerical values (cf. Galbraith & Stillman, 2001). The Participants successfully used the theoretical framework to manifest the structure their made assumptions underlying their proposed solution to the Fermi problems in terms of their subproblem structure (Anderson & Sherman, 2010; Carlson, 1997) and the mathematical activities they would use to achieve the necessary estimates (Ärlebäck & Albarracín, 2019).

A limitation of this study is that it focuses exclusively on the Participants' construction of FPATs as a representation of solution plans, while neglecting their implementation to fully solve the problems. The implementation of the solution plan may allow for further identification of difficulties and the promotion of changes in the approaches to solving the problems and revising the assumptions made. In future research, it would be useful to analyze the actual implementation of the proposed models, in order to ascertain how the assumptions made affect the development of the solutions and what adjustments are made in the process.

A further limitation of the study is that it does not observe whether the prospective teachers' ability to do or teach mathematical modelling is improved by working with assumptions in the context of Fermi problems. This limitation calls for a subsequent follow-up study on the prospective teachers' teaching competences, in order to assess whether the participants transfer this knowledge to their future modelling work and teaching practice. Therefore, we consider it necessary to continue exploring the assumptions that prospective teachers make based on completely solving the problems and reflecting on the implemented mathematical model.

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