

From problem-solving to problem-posing: Are the ways in which 11–12-year-old students solve pattern problems related to the types of problems they pose?

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ABSTRACT

We understand pattern problems as non-routine problems that require finding implicit regularities and mathematical structures to generalize beyond immediate evidence. Our study examines in what ways 11- and 12-year-old students' approaches to solving pattern problems are related to the types of pattern problems they pose in an unstructured task. Data were collected during four sessions in which fifty students first solved a sequence of pattern problems requiring near and far generalization and later engaged in a problem-posing task. We analyzed students' written productions to characterize the strategies they used and the extent to which they were able to generalize. We also categorized the problems they posed into null, incomplete, variation of a solved problem and new problem. Results reveal that those who did not pose any problems, or who posed null or incomplete problems, in the problem-solving task had mainly resorted to strategies that had little to do with generalization, such as extending the given drawing. In contrast, students who posed variations of previously solved problems or entirely new problems, had mainly used strategies oriented toward generalization, such as mixed strategies and the continuation of arithmetic progression. Although our findings are context-specific, they contribute to ongoing efforts to better understand how problem-solving experiences relate to students' ability to pose new problems. We conclude that research seeking to explore problem-posing processes should provide students with opportunities not only to pose problems, but also to discuss and solve them, thereby facilitating a more comprehensive understanding of their reasoning processes.

1. Introduction

In recent decades, problem solving has been recognized as a central focus in mathematics education (Lester & Cai, 2016; Schoenfeld, 1992). However, it was not until the 1990s that problem posing, i.e., the creation of problems or reformulation of given problems (Silver, 1994), began to gain prominence in the literature (Liljedahl & Cai, 2021). Currently, curricula point out that the goal of education is to prepare students for diverse types of knowledge application. Many of them establish that problem posing should have

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an important weight as part of mathematics education (Cai et al., 2015; Zhang et al., 2025). However, problem-posing statements are still an uncommon practice in classrooms (Zhang & Cai, 2021), which might limit the development of students' mathematical competence.

Problem-posing research in mathematics education has adopted multiple approaches, among which the study of its relationship with problem solving stands out (Cai et al., 2015).

While both theoretical and empirical research has provided substantial evidence of a relationship between problem solving and problem posing (Cai et al., 2015; Singer et al., 2011), much less is known about the specific mechanisms through which students' activity during problem solving informs their subsequent problem-posing activity. In particular, prior studies have paid limited attention to how the strategies students use when solving problems, together with the level of success they achieve, may later function as resources for generating new problems, especially in unstructured problem-posing situations and within elementary algebraic contexts.

Beyond considering whether students obtain correct answers when solving problems or the difficulty of the problems they create (Cai & Hwang, 2002), analyzing students' problem-solving strategies offers a more fine-grained perspective on their mathematical activity and may provide key insights into their capacity to pose new problems (Cai & Rott, 2023; Liljedahl & Cai, 2021). However, relatively few studies have examined students' solving strategies beyond classical heuristics (Chanudet, 2023; Joaquin, 2023), and even fewer have explored how these strategies are connected to problem posing under unstructured conditions, that is, situations in which students are required to generate problems without relying on given models or explicit guidelines (Stoyanova & Ellerton, 1996). Such contexts place higher cognitive demands on students and require greater autonomy and creativity (Baumanns & Rott, 2021), making them particularly suitable for investigating how problem-solving behavior supports or constrains problem-posing activity.

The present study examines the relationship between students' problem-solving behavior, understood as the strategies they use and their level of success, and their problem-posing performance in primary school within the specific context of pattern problems. We focus on pattern problems because they elicit distinctive families of strategies that are characteristic of this type of task (Montenegro et al., 2018), and because they are widely recognised as a key gateway to early algebraic thinking. In line with prior work showing the potential of pattern problems to foster both problem solving and problem posing (Cai & Hwang, 2002), our analysis focuses on examining how students' approaches to a sequence of pattern problems (in terms of the strategies they used and their level of success) relate to the types of problems they posed under unstructured instructions. In contrast with Cai and Hwang's study (2002), which primarily addressed success in problem solving and problem posing, our approach examines not only whether students succeed but also how they solve the problems and how they pose new problems. Furthermore, we categorize the problems posed by students under an unstructured demand, so we offer students the opportunity to generate new pattern problem scenarios that can be related (or non-related) to the tasks they are solving.

In this study, pattern problems are understood as non-routine problems that require finding implicit regularities and mathematical structures to generalize that go beyond immediate evidence. We situate the study within the context of pattern problems because of their central role in introducing and fostering algebraic thinking from an early age. Tasks that require students to establish relationships between quantities, identify structures and regularities, and generalize patterns offer an effective means of supporting the development of algebraic thinking in school mathematics (NCTM, 2000). This focus is also consistent with the Spanish primary education curriculum, where pattern problems are explicitly framed as part of algebraic or pre-algebraic thinking (LOMLOE, 2020).

Against this curricular and theoretical background, the study was conducted in a Spanish primary school through an educational intervention involving 50 students aged 11–12 years. During the intervention, students first worked on a sequence of four pattern problems and subsequently participated in an unstructured problem-posing task.

2. Theoretical framework

In the following four sections, we establish the conceptual basis that supports the research and rationales of the study. We first frame problem solving as a core component of mathematical competence and key curricular goal. We then examine problem posing, focusing on its typologies, degrees of structuring, and metacognitive demands, and review research on its relationship with problem solving. Finally, we define pattern problems as the specific context of this study.

2.1. Problem solving

Understanding that mathematics goes beyond content development is neither a new nor an experimental trend since the work of Pólya (1945), who considered problem solving as the central axis of mathematical activity. The PISA assessment framework (OECD, 2017) conceptualizes mathematical competence as "an individual's ability to formulate, use and interpret mathematics" (p. 66). Leow and Kaur (2024) emphasize the need to improve mathematical competence to train future citizens to be effective in dealing with the problems of their daily lives. This vision, based on the development of mathematical competence, has influenced not only a large part of educational curricula, but also the approaches of organizations focused on the teaching and learning of mathematics (NCTM, 2000; NGACBP & CCSSO, 2010; OECD, 2017). Thus, these organizations understand that learning mathematics requires the development of key mathematical processes, among which they place problem solving (Niss & Højgaard, 2019).

When we refer to problem solving, we can distinguish between routine and non-routine problems. Determining whether a problem is routine or non-routine may depend on the individual solver (Baumanns & Rott, 2021). Non-routine problems are those situations in which the solver lacks a previously known or automated method to solve them, since neither the situation, nor its instructions, provide an obvious solving strategy. The resolution of non-routine problems requires logical reasoning, advanced cognitive skills and creative

or flexible thinking (Leow & Kaur, 2024; Pólya, 1945). In contrast, routine problems, commonly called exercises, can be solved by the direct application of algorithms, and often the solver already knows a method or procedure for solving them. Since problem solving is characterized as a situation in which the individual lacks an immediate strategy to reach a solution (Schoenfeld, 1985), from this point on, in this text, we will use the term problem to refer to non-routine problems unless otherwise specified.

Numerous studies indicate that non-routine problem-solving fosters students' mathematical competence (Lee et al., 2014; Öztürk et al., 2020), and its absence may lead learners to equate problem solving with mere algorithmic application (Suseelan et al., 2022). We conceive non-routine problem-solving strategies as cognitive practices that enable students to engage meaningfully with mathematical situations, going beyond mechanical procedures (Santos-Trigo, 2024) and involving conjecturing, exploring alternative approaches, using multiple representations, and constructing arguments. In this way, problem solving becomes a key context for developing mathematical thinking habits such as modelling, generalisation, and validation of results.

2.2. Problem posing

In recent years the focus on problem solving has evolved while diversifying by incorporating problem posing (Cai et al., 2015). In this study we understand problem posing as the activity in which the student must generate new problems or reformulate given situations, either before, during or after a problem-solving sequence (Silver, 1994).

Baumanns and Rott (2021) allow us to establish which elements define a problem-posing task, determining three aspects:

- a. what we mean by problem posing: generation of new problems or reformulation of given problems
- b. what type of problems are required: routine problems or non-routine problems
- c. what the metacognitive demand is like: from highest to lowest

According to Silver (1994), problem-posing tasks can respond to two different situations: the generation of new problems or the reformulation of given problems. The generation of new problems can occur before or after a problem-solving task. To create new problems from an open-ended task, mathematical knowledge, skills and concepts from previous experience must be applied. The reformulation of given problems can occur during or after a problem-solving task. In this case, the task demands varying the conditions of a concrete or previously solved problem.

Stoyanova and Ellerton (1996) distinguish three types of problem-posing situations: unstructured, semi-structured, and structured. Unstructured situations invite problem creation in open contexts; semi-structured situations provide a problem and require the formulation of related questions; and structured situations ask solvers, after answering given questions, to generate new ones that modify the problem's difficulty (Cai & Hwang, 2002). While unstructured and semi-structured situations foster the creation of new problems, structured situations primarily elicit reformulations of existing ones.

According to Baumanns and Rott (2021), the metacognitive demand of a problem-posing task can be more or less high. We understand metacognitive demand as the level of demand of the task in terms of a) planning: the identification and selection of appropriate strategies or resources to solve it, b) monitoring: attention and awareness of understanding, and c) evaluation: assessment of learning processes and products. In this study, we understand that the less structured the problem-posing task is, the greater its metacognitive demand.

2.3. Problem solving and problem posing

There are several trends of research on students' ability to face problem-posing tasks. Among them, some studies examine the relationship between student performance in mathematics and the ability to pose new problems. Authors such as Cai and Hwang (2020) or Elgrably and Leikin (2021) point out that the relationship between students' mathematical performance and their ability to pose problems is complex and cannot be reduced to a direct or linear association. Therefore, further research should explore the range of factors that may influence students' ability to pose problems, including how they engage in problem solving (Cai et al., 2015). Ran et al. (2025) point out the importance of carefully considering the clarity of context and instructions when presenting a problem-posing task, as students may divergently interpret what is expected of them.

On the other hand, several studies have tried to examine the relationship between problem-solving and problem-posing tasks (Baumanns & Rott, 2019; Cai & Hwang, 2020; Ellerton, 1986). There is a general agreement that the ability that students show when solving a problem is directly related to the ability to pose a new problem (Singer et al., 2013; Xie & Masingila, 2017). However, various factors have been identified that influence this relationship. Baumanns and Rott (2018) found a close relationship between the type of tasks that students solved and the type of tasks they posed. After solving non-routine problems, students tended to generate new non-routine problems in a variety of contexts and with great mathematical richness. Kontorovich and Koichu (2014) identified cognitive patterns that reveal how problem-posing experts organize their knowledge. Aspects such as connections with prior knowledge, creativity or the search for novelty, contrast with those of novice students, who tend to reproduce or slightly vary known problems without modifying their deep structure (Kontorovich et al., 2012). For their part, Xie and Masingila (2017) found that good problem-solvers tended to be able to pose more complex mathematical problems.

2.4. Pattern problems

Sarama and Clements (2009) define pattern problems as contexts of searching for regularities and mathematical structures to order,

cohere, and anticipate apparently unorganized situations, facilitating their generalization. According to Radford (2010) the purpose of pattern problems would be to introduce students to algebraic generalization, understood as the coincidence observed in some terms of a sequence S that is applicable to all terms of S , and that can be used to provide a direct expression for any term of S . Pattern generalization involves three actions: identifying a common property, generalizing that property to all the terms of the progression, and using it to determine a rule to find any term of the sequence (Dreyfus, 1991).

Pattern problems can be characterized according to two criteria: the type of generalization they require and the type of progression they include. Stacey (1989), referring to the type of algebraic generalization required by pattern problems, distinguishes between immediate generalization, near generalization and far generalization. Statements require immediate generalization when the demand refers to an immediate element or step to the information given in the statement. Generally, this type of demand can be answered by counting or drawing elements. Near generalization demands refer to items or elements relatively close to the information given in the mathematical problem statement. Near generalization tasks can also be solved through counting or drawing, although in certain cases these strategies may lead to error. Far generalization statements demand positions or elements far away from those of the information given in the statement. Finding such elements by drawing or counting step by step requires a lot of execution time and, again, often leads to error (Torregrosa & Albarracín, 2026). Thus, far generalization demands require solving strategies linked to arithmetic or algebra.

Vergel (2015) states that young students, such as those participating in our study, when faced with far generalization demands sometimes fail to reach an algebraic generalization, since their proposal does not allow them to compute any term of the sequence. Therefore, in our study, we use the term arithmetic generalization understood as a generalization applicable to some terms of S without corroborating its extension to all of them (Vergel, 2015).

The progressions underlying pattern problems can be arithmetic or geometric (El Mouhayer & Jurdak, 2013; Radford, 2010). Arithmetic progressions are those successions of numbers in which the difference between consecutive terms is constant. Geometric progressions are those sequences of numbers in which each term is obtained by multiplying the previous one by a constant. Geometric progression pattern problems require a more advanced level of generalization, since the relationship between consecutive terms is not additive, but multiplicative.

3. Research question

The present study aims to extend existing research on the relationship between problem solving and problem posing when the focus is on pattern-problems within the context of early algebraic thinking. Pattern problems offer a privileged setting for examining how students engage with generalization and structure, making visible the mathematical ideas they mobilize and reorganize when transitioning from problem solving to problem posing, especially under unstructured task conditions.

The research question guiding this paper is therefore the following: In what ways are students' approaches to solving pattern problems related to the types of problems they later pose in an unstructured problem-posing task?

4. Methodology

4.1. Design

Our study is situated within the interpretive paradigm. Its aims motivated the adoption of a qualitative research approach, and its design was shaped by the specific features of the research question, which focuses on pattern problems. Previous studies examining relationships between students' approaches to problem solving and problem posing have employed a wide range of methodological designs (e.g., Kontorovich & Koichu, 2014; Ran et al., 2025; Rott, 2011; Xie & Masingila, 2017). Given the absence of a common methodological framework in this line of research, we designed our study essentially taking contextual constraints into account, ensuring that data collection could be smoothly integrated into the school curriculum and aligned with the time deemed appropriate by the participating teachers.

The empirical part of the study had two blocks: one structured around the solving strategies used by the students when dealing with pattern problems and the other around the pattern problems posed by the students when the task demand was unstructured. The data of the study consisted of the documents written by a group of 50 students, ages 11–12 years old, in response to different tasks in mathematics class, around problem solving and problem posing in the specific case of pattern problems. To address the research question, we performed a qualitative content analysis (Mayring, 2015) of the collected documents. The analysis was carried out in three stages. First, the analysis of the strategies that students used when approaching the pattern problems allowed us to outline students' behavior as problem-solvers. Next, the analysis of the problems posed by the students allowed us to determine problem typologies and their behavior as problem-posers. Finally, a relational analysis of the results obtained so far enabled us to establish associations between the strategies used by the students when solving a sequence of pattern problems and the type of problems they posed.

4.2. Instruments

The first decision regarding data collection instruments was to select the typology of problems we would present to students in the problem-solving tasks: the generalization required (Stacey, 1989) and the progression underlying the problems (El Mouhayer & Jurdak, 2013). We focused on demanding near and far generalizations by including two questions in each problem statement.

Immediate generalizations ask for an item or step consecutive to the information given in the statement and can often be addressed through direct counting (Stacey, 1989; Radford, 2010). For this reason, we did not include immediate generalizations in our problem statements as they do not necessarily require students to identify underlying regularities or to engage in more structured forms of reasoning about the pattern. Near generalizations, while still relatively close in position within the sequence, tend to require greater structuring of the pattern and activate strategies similar to those used in immediate generalizations, but with an additional degree of planning. We considered the questions in our problems to demand near and far generalizations requiring students to work beyond the next element in the sequence and, in the case of far generalizations, to compute terms located at a considerable distance from the given information, which cannot be efficiently obtained through counting or drawing and instead call for strategies involving abstraction and arithmetic reasoning. It is important to mention that in our study we did not intend to see whether the students were able to recognize the generalization of the problem-solving strategies they used or whether they understood the meaning of generalization as a way of thinking, given that the students were between 11 and 12 years old.

Also, due to the age of the participants, we decided to focus on arithmetic progressions in terms of the type of progression underlying the problems (El Mouhayer & Jurdak, 2013). In addition, the Spanish educational curriculum (LOMLOE, 2020) for primary education focuses on working with pattern problems in arithmetic progressions as part of the introduction to algebraic thinking. We also decided that we would pose four problems to the students since we were limited by the time agreed with the teaching staff, conditioned, in turn, by the weight of algebraic thinking in the curriculum. Fig. 1 shows the four problems used in the study:

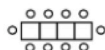
The problems were ordered by increasing complexity, established a priori based on the difficulty of determining the general term of the progression and the mathematical skills required to solve it, and this ordering was subsequently confirmed empirically through the results obtained in the study. The statements of all problems comprised a pictorial representation to facilitate their understanding (Samson & Schafer, 2007). Although the first three problems focused on arithmetic progressions, we decided to include a fourth problem, based on a geometric progression. This choice allowed for the emergence of strategies closer to arithmetic generalization (Villalonga & Deulofeu, 2017).

In relation to the problem-posing task, and based on Stoyanova and Ellerton (1996), we decided to propose a task under unstructured instructions. This type of task offers more room for creativity, enhances connections with prior knowledge, and has a high

PROBLEM 1: THE DINNER PARTY

Source: Adapted from Arbona, Beltrán, Gutiérrez & Jaime (2017)

We want to organize a dinner party with our friends but we are not sure how many tables we need. What we know is this: we want to set tables in a single row and we want our friends to sit as shown in the drawing.



- If we are 40 friends, how many tables will we need?
- What if we are 100?

PROBLEM 2: PINES AND ORANGE TREES

Source: Adapted from Morera, Chico, Badillo & Planas (2012)

A farmer wants to plant orange trees following a square shape and around it he wants to plant pine trees. He imagines the following plan for 1, 2 and 3 rows of orange trees:



- How many pines will he need if he wants to plant 4 rows of orange trees?
- What if he wants to plant 10 rows? How would you do it?

PROBLEM 3: THE NUMBER GRID

Source: Adapted from FEEMCAT, SCM, Generalitat de Catalunya (2000-2018). Problemes a l'esprint

July has written in a table the natural numbers 1, 2, 3, 4, 5... putting them as shown in the picture:

1	4	5	8	9
2	3	6	7	10
11	14	15	18	19
12	13	16	17	20
21	24	25	28	...

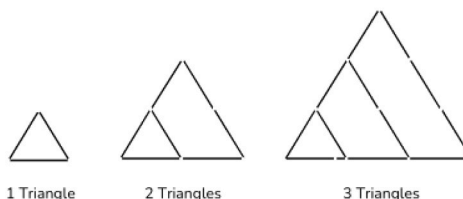
Then she has decided to add the numbers that have a common side with 10 ($9+7+19=35$). Then she added the numbers that have a common side with 100 but she didn't need to write them all down to know what they were. Could you explain how July did it?

- What result did she get when she added the numbers that have a common side with 100?
- And with 1000? How would you do it?

PROBLEM 4: THE CHOPSTICKS

Source: Adapted from Zapatera (2018)

Anne has been playing at making shapes with chopsticks. First she decided to make triangles in the following way:



Then she has asked herself:

- How many toothpicks do I need if I want to make 10 triangles following the same pattern?
- What if I want to make 20? How would I do it?

Fig. 1. Problems used in the study.

cognitive demand (Baumanns & Rott, 2021). We decided to implement the problem-posing task after the problem-solving task, since Ayllón et al. (2011) argue that both the content addressed and the types of problems solved prior to a problem-posing activity influence the coherence and difficulty of the problems students create. Likewise, Baumanns and Rott (2018) found a close relationship between the type of tasks students solved and the type of tasks they posed. They observed that after solving non-routine problems, students tended to pose new non-routine problems in a variety of contexts and with great mathematical richness. For all these reasons, after completing the problem-solving sequence, we gave the students a sheet of paper with the following problem-posing task presented in Fig. 2.

4.3. Participants

The data were collected in a public school in the metropolitan area of Barcelona. In this school, during the entire primary education stage (from 6 to 12 years old), mathematics is taught following a traditional methodology, based on textbooks and routine problem solving. Fifty students (68% girls and 32% boys) in the last year of primary school, aged between 11 and 12 years, participated in the study. Authors such as Cai and Hwang (2020), Elgrably and Leikin (2021), or Silber and Cai (2021) point out that there is no direct relationship between students' good grades in mathematics and a greater capacity or ability to pose problems. Therefore, we did not consider students' grades as variable in our study.

The students were unfamiliar with non-routine problem solving and, even more, with the design and implementation of their own problem-solving strategies. Furthermore, the school's teachers did not use problem-solving tasks, either in math classes or in other subjects. In this way, we minimized the influence of previous activities, other than those we presented to them, on problem-solving tasks (Ayllón et al., 2011).

4.4. Data collection

The data were collected in four two-hour class sessions taught by the first author of this paper who is also a teacher herself. During the first hour of each session, the problem statement was read together and the students addressed it individually in writing. During the solving process, when students encountered difficulties or asked for help, they were given answers such as "reread the question", "keep thinking about what you could do" or "think about what you have done", to avoid influencing the strategies that students developed (Jacobse & Harskamp, 2012).

During the second hour of the first three sessions, students presented their solving strategies to the whole class. The role of the researcher leading the sessions was to encourage students to share their solving processes, validate the proposed solutions and paraphrase the contributions, articulating them when necessary. It was considered essential for students to understand what the correct solution to the problem was and to see the variety of strategies proposed by their peers to reach the solution. While guiding the sessions, the researcher did not introduce any concept that had not emerged in the students' contributions. It is important to note that in none of the sessions the students pointed out the importance of identifying the repetition pattern, which meant that the teacher did not explicitly discuss the characteristics of pattern problems either. In the fourth session, after the hour devoted to solving the problem, half an hour was devoted to group sharing. The remaining half hour was devoted to the problem-posing task: *Write two new pattern problems that could be solved by one of your classmates.*

The written responses of the 50 students (S1, S2, S3... S50) constitute our primary data. On the one hand, we have the documents the students submitted in response to the four problems, with a total of 188 productions containing text, drawings, tables, calculations and other mathematical elements. For the first problem we have 48 answers, for the second 50, for the third 47, and 43 for the last one, since not all the students were present in all the sessions. Of the 43 students present during the problem-posing task, 5 did not pose any problem, 24 posed only one problem, and 14 posed two problems, meaning that we collected a total of 52 problems. Throughout the process, the first researcher, who led the lessons, took notes on the students' comments and the interactions in the large group discussion. These field notes, taken both during and after the sessions, serve as [supplementary information](#) that helps us interpret the students' work.

4.5. Data analysis

We carried out a content analysis of two sets of data, the problems solved and the problems posed by the students, considering as indivisible units of meaning each of the different students' productions. The analysis was carried out in three stages. In the following sections, we begin by presenting the analysis of the strategies used when solving the problems and the students' behavior as problem-solvers. Next, we describe the analysis of the problems posed by the students and their behavior as problem-posers. Finally, we present a relational analysis of the results obtained in the first and second phases of the analysis, which enabled us to answer our research question.

Write two new pattern problems that could be solved by one of your classmates.

Fig. 2. Problem-posing task.

4.5.1. Analysis of the students' problem-solving strategies

Based on the documents with the students' solutions to the problems, we characterized their strategies through a deductive process. To this end, we adopted the categories previously developed and validated by Torregrosa et al. (2021) who had established them in a process similar to that of Magiera and Zawojewski's (2011) study. These categories were selected because they reflect a potential progression from more concrete strategies, typically associated with immediate and near generalization, to increasingly abstract strategies which are closer to a possible generalization and commonly linked to far generalization. These categories are:

- Inadequate strategies*: this category includes the strategies of students who approach the problem through proportional reasoning and, therefore, are incorrect when the problems refer to arithmetic progressions (Fig. 3).
- Extension of the given drawing*: this category includes the responses of students who extended the drawing that appeared in the statement until they reached the requested position (Fig. 4). This strategy would be the furthest from algebraic thinking.
- Continuation of the arithmetic progression*: in this category we place the responses in which the student abstracted the progression underlying the drawing presented in the problem statement and arithmetically determined the requested position (Fig. 5). The strategies in this category would indicate that the student is approaching algebraic thinking and the ability to generalize.
- Mixed strategies*: this category includes students' responses in which the drawing was extended, and at the same time, arithmetic reasoning was used to determine the solution. It includes both responses in which students used the continuation of the arithmetic progression to confirm a result initially obtained through extending the drawing, and responses in which students relied on the drawing for the initial cases of the progression and subsequently worked arithmetically on more complex cases corresponding to far generalizations (Fig. 6).

Once the strategies used by the participants were characterized, we determined their frequency of use and the success of each of them for each problem, which told us which strategy was the most efficient for each problem (Jiang et al., 2014). Furthermore, the analysis enabled us to observe how the use of strategies changed throughout the sequence of problems and how the strategies were distributed when solving each problem.

At the individual level, we identified which type of strategy students used to solve each problem and to what extent they succeeded in solving the problem by using it, in terms of the level of generalization attained. From here, we characterized the students in terms of the strategies used and the success achieved in solving the problems, with which we can outline their behavior as problem-solvers. We used this characterization for the subsequent relational analysis.

4.5.2. Analysis of the problems posed by the students

The 43 students who participated in the problem-posing task generated 52 problems that we analyzed based on the categories developed and validated previously by Torregrosa et al. (2021). We adopted these categories because they enable us to distinguish qualitative differences in students' problem-posing task. Specifically, they capture differences in the mathematical nature of the

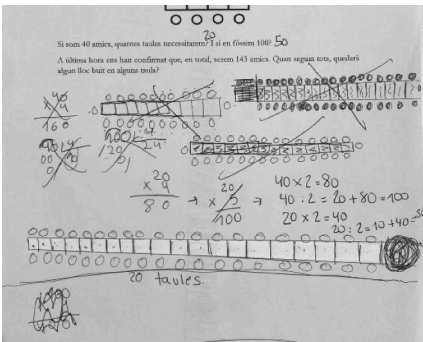
Original student's worksheet	Description of the worksheet and translation of the text
	<p>The student made several drawings and calculations at the top of the sheet and crossed them all out. At the bottom of the sheet, she drew 20 tables for 40 people (without keeping the pattern of the original drawing) and wrote: <i>20 tables</i>.</p> <p>On top of the drawing of the 20 tables, the student wrote, without crossing them, the following calculations:</p> $40 \times 2 = 80$ $40 \div 2 = 20 + 80 = 100$ <p>(Based on the field notes, we understand that this student began by trying to relate the 20 people in the first question to the 100 in the second, although he expressed the relationship with an incorrect concatenation of signs =)</p> $20 \times 2 = 40$ $20 \div 2 = 10 + 40 = 50$ <p>(Here she applied the same calculations she had made for people to the number of tables, mistakenly assuming a ratio of 1 table to 2 people.)</p>

Fig. 3. Inadequate strategy used by student S26 when addressing problem 1 (The dinner party).

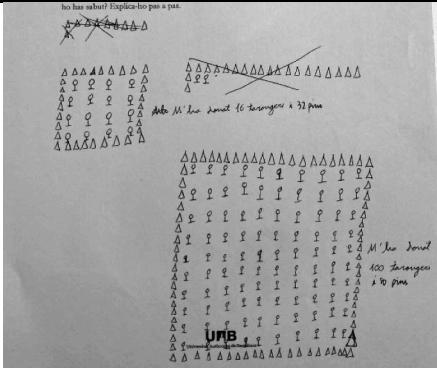
Original student's worksheet	Description of the worksheet and translation of the text
	<p>In the upper left corner, the student draws 4 rows of orange trees (16 orange trees) and 32 pines. Next to it the student writes: [crossed out word] <i>I got 16 orange trees and 32 pines.</i></p> <p>At the bottom center the student draws 10 rows of orange trees (100 orange trees) and 80 pines. Next to it the student writes: <i>I got 100 orange trees and 80 pines.</i></p>

Fig. 4. Extension of the given drawing strategy used by student S17 when addressing problem 2 (Pines and orange trees).

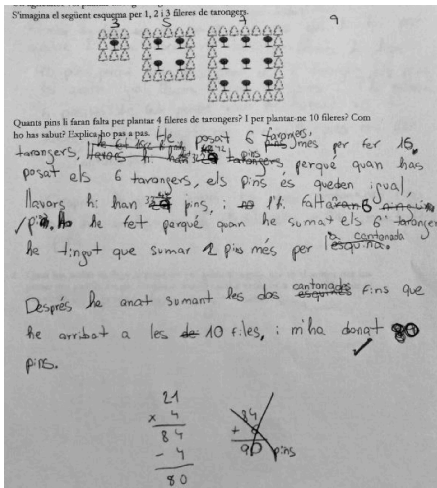
Original student's worksheet	Description of the worksheet and translation of the text
	<p>Above each of the drawings in the statement, the student writes the number of pins on each side of the squares determined by the pine trees (3, 5, 7, 9).</p> <p>Next, the student writes: <i>I have placed 6 more orange trees to have 16 orange trees</i> [meaning that he has added 6 orange trees to the 9 orange trees in step 3 to find the orange trees in the 4 rows in step 4]. <i>Then I calculated 16×2, and I got 32 because when you put the 6 orange trees, then the pines stay the same, so there are 32 pines. I did it this way because when I added the 6 orange trees, I had to add 2 more pines per corner</i> [meaning that he increased the number of pines 8 times - 2 pines per corner multiplied by 4 corners].</p> <p><i>Then I added the two corners until I reached 10 rows, and I got 80 pines.</i> [At the top of the sheet he writes the operations $21 \times 4 = 84$; $84 - 4 = 80$ meaning that for 10 rows of orange trees, there are 21 pines per side of the square made by the pines, but since he is counting the corners twice, he subtracts 4 to get the total of 80 pines].</p>

Fig. 5. Continuation of the arithmetic progression strategy used by student S43 when addressing problem 2 (Pines and orange trees).

problem, whether or not involves pattern structure, the completeness and solvability of the task, which reflects the student's ability to structure a coherent mathematical situation and the degree of transformation, ranging from minor contextual variations of previously solved problems to the creation of entirely new problems with unseen progressions. The categories were:

- *Null*: includes those problems that are coherent and solvable but are not pattern problems, since their statement do not suggest that there is any progression underlying the problem.
- *Incomplete*: includes those pattern problems that, due to lack of data or information, are unsolvable.
- *Variation of a solved problem*: includes those pattern problems that start from the progression of one of the problems that had been solved but modify the context of the problem, and those that start from the context of one of the solved problems and modify the progression involved.
- *New problem*: includes those pattern problems that can be solved with a progression that did not appear during the problem-solving task and are presented in a totally different context than the solved problems.

In Table 1 we present an example for each of the categories created to organize the problems posed by the students.

Once the problems posed by the students had been characterized, we calculated the frequency of occurrence of each of the categories, at the global level. At the individual level, we determined the number of problems posed by each student and their typology, thus establishing students' behavior as problem-posers. Specifically, we distinguished between students who did not pose problems,

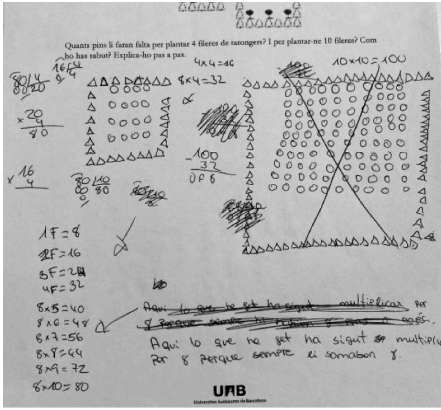
Original document	Description of the worksheet and translation of the text
	<p>In the upper left part of the sheet the student draws the pines needed (32) for the 4 rows of orange trees. In the upper right part of the sheet the student tries to draw the pines needed for 10 rows of orange trees but crosses out the drawing. We notice that he makes several calculations such as 16×4, 20×4, 8×4 or $100 - 32$. At the bottom left the student makes a correspondence between rows of orange trees and pines indicating:</p> <p>1F [row] = 8 2F [row] = 16 3F [row] = 24 4F [row] = 32 8×5 [F row] = 48 ... 8×10 [F row] = 80</p> <p>On the lower left side, the student writes: <i>Here what I have done</i> [points out with an arrow the correspondence between rows of orange trees and pine trees] <i>has been to multiply by 8 because they always added up to 8.</i></p>

Fig. 6. Mixed strategy used by S18 when dealing with problem 2 (Pines and orange trees).

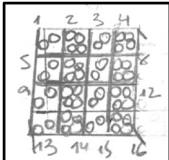
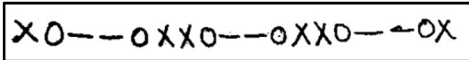
posed null problems, incomplete problems, variations of solved problems, or new problems.

4.5.3. Relational analysis between students' problem-solving and problem-posing behaviors

Based on the results obtained from the analyses of students' problem-solving and problem-posing behaviors, we carried out a relational analysis with the aim of identifying connections and shared patterns between the results obtained in the two previous phases of the analysis. We examined potential relationships between the strategies students used and their success when problem solving and the types of problems they posed. To this end, we grouped the students considering whether they posed no problems, null problems, incomplete problems, variations of solved problems, or new problems. Next, for each group, we examined their problem-solving

Table 1

Examples of the categorization of the problems posed by the students.

Category	Example
Null	At a party 10,500 people are celebrating a birthday, how many tables do you need for everyone if each table seats 15 people? (the exercise would be solved by dividing)
Incomplete	In a safe the code starts with 1,22. It has 10 numbers, what code is it? (there is not enough information to determine the repeating pattern)
Variation of a solved problem	How many more columns are needed to have 20 squares in a column? <div data-bbox="327 1308 521 1447" data-label="Image">  </div> <p>(keeps the progression of problem 1)</p> <p>There is a table and the table has tiles. On each tile there are 2 or 4 plates and each square has a number (picture). On tile 30 there are 4 plates or 2? And on tile 100?</p> <div data-bbox="327 1526 496 1685" data-label="Image">  </div> <p>(keeps the context of problem 3 and changes the progression)</p>
New problem	In a park, some children have drawn a series on the ground. If this series reaches 100 figures, how many of each will there be? What if it reaches 200? <div data-bbox="327 1765 797 1824" data-label="Image">  </div>

behavior in terms of the strategies they had used throughout the sequence of pattern problems and their overall success in tackling near and far generalization demands. We observed that qualitative trends emerged, allowing us to establish relationships between students' approaches to solving pattern problems and the types of problems they posed.

5. Results

5.1. Strategies used across the sequence of problems

In this section, we present the strategies used by the 50 students when tackling the four pattern problems across the sequence of problems and their success (Table 2). We distinguish different levels of success: Success 0 when the student did not manage to reach any answer using that strategy or the answer was wrong; Success A when the strategy allowed the student to reach the correct answer for the first question of the problem - near generalization - and Success A&B when the student correctly answered the two questions of the problem - near and far generalization.

The first row of Table 2 shows, for each problem (P1, P2, P3 and P4) the number of students who attempted it, whether or not they managed to solve it. The subsequent rows list the different types of strategies used by the students: inadequate, extension of the given drawing, mixed strategies and continuation of arithmetic progression. The four columns into which each strategy is divided present, for each problem, the percentage of students who used that strategy to solve it, and the strategy's success rates (0, A or A&B). The success rate is defined as the number of students who achieved success 0, A, or A&B on this problem divided by the total number of students who used the strategy to address the problem.

Table 2 offers two complementary interpretations which, taken together, reveal how the use of strategies and their success evolved throughout the sequence of problems. On the one hand, when we consider each type of strategy across the four problems, we observe that strategies based on the extension of the given drawing show a marked decline across the sequence: they account for 50% of those used in P1, 54% of those in P2, 19.1% in P3 and 20.9% in P4. At the same time, the proportion of students who achieved both near and far generalizations using this strategy falls significantly, from 37.5% in P1–26.0% in P2, and down to 0% in P3.

We also noted that mixed strategies displayed a progressive increase in frequency from P1 (25%) and P2 (24%) to P3 (42.6%), before slightly decreasing in P4 (34.9%). This increase is accompanied by a marked improvement in the effectiveness of the strategy across the sequence of problems. The rate of A&B success associated with mixed strategies rises from 33% in P1–66.6% in P2, reaches 70.0% in P3 and remains above 50% in P4 (60%).

We finally observed that strategies based on the continuation of the arithmetic progression follow a similar pattern, albeit a more pronounced one. Although only 12.5% of students used them in P1, their frequency reaches 22% in P2, 38.3% in P3, and 37.3% in P4. At the same time, their success rates increase significantly, with success A&B rate rising from 50% in P1, 54.5% in P2–77.8% in P3, and reaching 87.5% in P4.

On the other hand, when we consider the distribution of strategies used and the success achieved, for each problem simultaneously, we observe that when addressing P1, half of the students (50%) relied on the extension of the given drawing; however, 54.2% of them only reached near generalization. We also noted that when addressing P2, again, extension of the given drawing was used by around half of the students (54%). However, 55.5% of the students who used this strategy only achieved success A (near generalization), while only 26% achieved success A&B. In P3, mixed strategies become the most common and lead to higher levels of success: 70% of the students who used them achieved success A&B. Finally, in P4, the strategies used are distributed between mixed strategies and the continuation of arithmetic progression, both of which are associated with high rates of success A&B: 60% for the former and 87.5% for the latter.

Table 2

Strategies used across the sequence of problems and their level of success.

		P1	P2	P3	P4
Number of students		48	50	47	43
Inadequate strategies	Frequency of use	12.5%	0%	0%	6.9%
	Success 0	100%	0%	0%	100%
	Success A	0%	0%	0%	0%
	Success A&B	0%	0%	0%	0%
Extension of given drawing	Frequency of use	50%	54%	19.1%	20.9%
	Success 0	8.3%	18.5%	66.6%	33.3%
	Success A	54.2%	55.5%	33.3%	55.6%
	Success A&B	37.5%	26%	0%	11.1%
Mixed strategies	Frequency of use	25%	24%	42.6%	34.9%
	Success 0	16.7%	16.7%	15%	33.3%
	Success A	50%	16.7%	15%	6.7%
	Success A&B	33%	66.6%	70%	60%
Continuation of arithmetic progression	Frequency of use	12.5%	22%	38.3%	37.3%
	Success 0	33.3%	18.2%	16.7%	6.25%
	Success A	16.6%	27.3%	5.5%	6.25%
	Success A&B	50%	54.5%	77.8%	87.5%

5.2. Individual students' behavior as problem-solvers

In this section, we present individual students' behavior as problem solvers, characterized through the strategies they used when tackling each of the four pattern problems and the level of success they achieved (0, A, or A&B) when applying those strategies. Table A.1 in Appendix A provides a complete overview of this information for each student, showing, for every problem, the strategy employed and the corresponding success level.

Based on the information summarized in Table A.1, we constructed a four-dots graph for each student to visually represent their problem-solving behavior across the sequence of problems. We placed each problem (P1, P2, P3, and P4) on the horizontal axis. On the vertical axis, we placed the strategies: Inadequate (I), Extension of the given drawing (D), Mixed Strategies (M) and Continuation of the arithmetic progression (P). The segments between the dots are merely for visual aid. To indicate the success of each strategy, we colored the dots of each pair (problem, strategy) as follows: red (success 0), orange (success A) and green (success A&B). This form of representation allows us to visually summarize the behavior of each of the 50 students across the sequence of problems (see Fig. B.1 in Appendix B).

As an example of how to interpret the four-dots graphs, we next present Fig. 7 which illustrates the behavior of some students as problem-solvers.

In Fig. 7 we see, for example, that student S5 addressed P1, P2, and P4 using the extension of the given drawing strategy, reaching only near generalization (success A), whereas P3 was addressed using a continuation of the arithmetic progression strategy, resulting in far generalization (success A&B). Student S11 dealt with P1, P2 and P3 using the extension of the given drawing strategy, that led to no solution for P1 (success 0), and only near generalizations for P2 and P4. S14 addressed P1 and P2 using a mixed strategy that led to near generalization for P1 and to far generalization to P2 and used a strategy of continuation of the arithmetic progression to reach far generalization in P3 and P4. S28 reached far generalization in all problems using the continuation of the arithmetic progression strategy, except for P3 which he solved through a mixed strategy. S44 only solved the near generalization question of P2 through the extension of the given drawing strategy.

The graphs thus elaborated allowed us to outline the students' behavior as problem-solvers, established in terms of the strategies they had employed and their success in solving the problems. Once the students' behavior was established, we grouped them according to their overall success, defined in terms of the number of problems in which the student reached both near and far generalization (Success A&B). Accordingly, students were grouped as follows:

- Very high overall success (VHS): This cluster grouped students who correctly found both the near and the far generalization (A&B) when solving the four problems. Their behavior graph had four green dots.
- High overall success (HS): In this cluster we included those students who correctly found both the near and far generalization (A&B) in 2 or 3 of the problems. Their behavior graphs had two or three green dots, but not four, with the rest of the dots being of any color.
- Medium overall success (MS): Students in this cluster were those who reached both near and far generalization (A&B) in only one problem. Their behavior graphs had a single green dot, and the other dots were any color.
- Low overall success (LS): This cluster included those students who did not reach far generalization (A&B) in any problem, and reached, at least, near generalization (A) in one of the problems. Their behavior graphs had no green dots and at most three red dots.
- Very low overall success (VLS): We placed in this cluster those students who did not manage to reach the near generalization (A) even in one of the problems. Their graphs had four red dots.

Considering that 7 students did not attend all the sessions, it is important to make explicit the decisions made to place them in some category. We placed those who addressed three problems in the category for which we had sufficient information to assign them.

Thus, S45 with two green dots and one yellow dot on his graph, was characterized as a high overall success student (HS), since this category includes students whose graph had two or three green dots, regardless of the color assigned to the remaining dots. S1 and S48 addressed only two problems, but their graphs contained one green dot, allowing us to characterize them at least as students with medium overall success (MS). Similarly, we characterized S25 and S47 as students with low overall success (LS), S26 as being a student

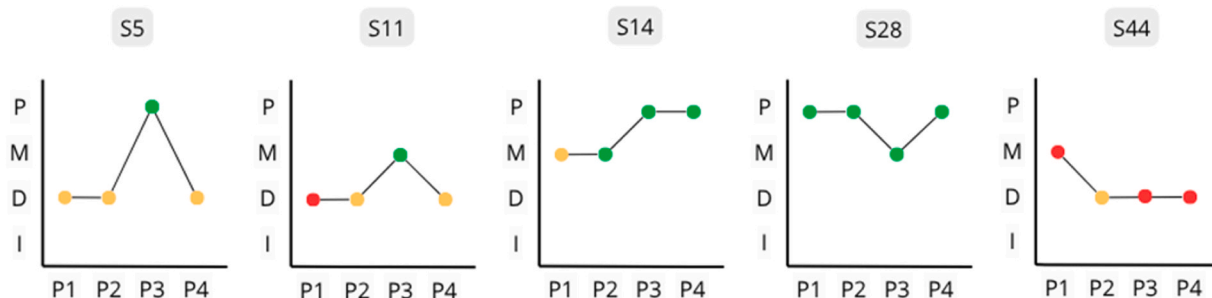


Fig. 7. S5, S11, S14, S28, and S44 behavior as problem-solvers.

with very low overall success (VLS) and characterized S39 and S40 as NA (not assigned). Table 3 shows, for each overall success, the students assigned to the cluster, their number and the proportion they represent with respect to the total number of students.

Given that this was the students' first experience with pattern problems, the fact that almost half of them achieved a high or very high overall success, and that approximately one quarter reached a medium overall success, suggests that, on the whole, they were able to meaningfully engage with the tasks despite the lack of prior experience.

5.3. Typology of problems posed and students' behavior as problem-posers

In this section, we present the findings from the characterization of the problems posed by the students, both globally and individually. Thus, Table 4 shows the count and frequency of occurrence (%) of each of the categories characterizing the problems posed by the students.

Once the problems had been characterized, we characterized individual students' problem-posing behavior based on the number and type of problems posed by each student, as shown in the graph in Fig. 8. The horizontal axis shows the students (from S1 to S50). The vertical axis indicates whether the student posed no problem, only one problem (P1) or two problems (P1 and P2). The type of problem posed is indicated by colors: red (Null problem), orange (Incomplete problem), light green (Variation of a solved problem) and dark green (New problem). For example, if a student has a red dot in P1 and in P2, it means that this student generated two problems and both are null. Students who do not have any dot on the graph (S16, S36, S37, S48 and S50) did not pose any problem. Students who do not appear on the graph (S25, S26, S39, S40, S45, S47 and S49) were absent during the problem-posing task.

5.4. Relationship between student's behaviors as problem-solvers and as problem-posers

In this section, we present the findings that enable us to establish associations between students' behavior as problem-solvers and their behavior as problem-posers. In Appendix C, we present 5 figures (Figures C.1 – C.5) showing the problem-solving behavior (strategies used and levels of success) of those students who posed each type of problem (no problems, null problems, incomplete problems, variations of a solved problem, or new problems). A joint analysis of the five figures presented in Appendix C, together with the interpretations provided there, enables us to compile Table 5.

Thus, for each type of problem posed, Table 5 reports the number and percentage of students who posed that type of problem (frequency of occurrence), their overall success, the types and frequencies of strategies they used across the four pattern problems, and the generalization achieved, that is, the extent to which they were able to generalize.

When consulting the table, readers should note that strategies are abbreviated as follows: I (Inadequate), D (Extension of the given drawing), M (Mixed Strategies), and P (Continuation of the arithmetic progression). Similarly, strategy success is coded as 0 to indicate no success, A to indicate near generalization, and A&B to indicate both near and far generalization. Finally, it should be noted that the counts of strategies and the generalization achieved by these strategies are aggregated for each problem-posing type.

Table 5 shows that students who did not pose any problems (11.4% of the group), when solving the sequence of problems, had mostly used strategies based on extending the given drawing (52.6%) and mixed strategies (36.8%), and none had used strategies involving the continuation of arithmetic progression. Accordingly, their success was very limited: 52.6% failed to solve the problems and only 36.8% managed to reach a far generalization. Furthermore, all students who did not pose any problems had medium, low or very low levels of overall success when solving the sequence of problems. None of the students with high or very high overall success failed to pose problems.

Table 5 also shows a similar pattern of problem-solving behavior among students who posed null problems (22.8%) and those who posed incomplete problems (13.7%). Their behavior during problem solving was characterized by a strong reliance on the extension of the given drawing strategy (66.7% of those who posed null problems and 70.8% in the case of incomplete problems). In contrast, strategies based on continuation of the arithmetic progression appear only marginally (5.1% and 8.3%). Although around half of the students who posed null problems (51.3%) or incomplete problems (54.2%) could achieve near generalization, the way in which they approached problem solving rarely allowed them to reach far generalization (20.5% and 16.7%). Both students who posed null problems and those who posed incomplete problems were students with low overall success (30% and 33.3%) or medium overall success (50% and 66.6%). Only one student with high overall success posed a null problem, whilst no student with high or very high overall success posed null or incomplete problems.

In Table 5 we also see that the students who posed variations of a solved problem (36.4% of the group) exhibited a problem-solving behavior that was very different from that of the previous groups. These students essentially used mixed strategies (42.2%) or

Table 3

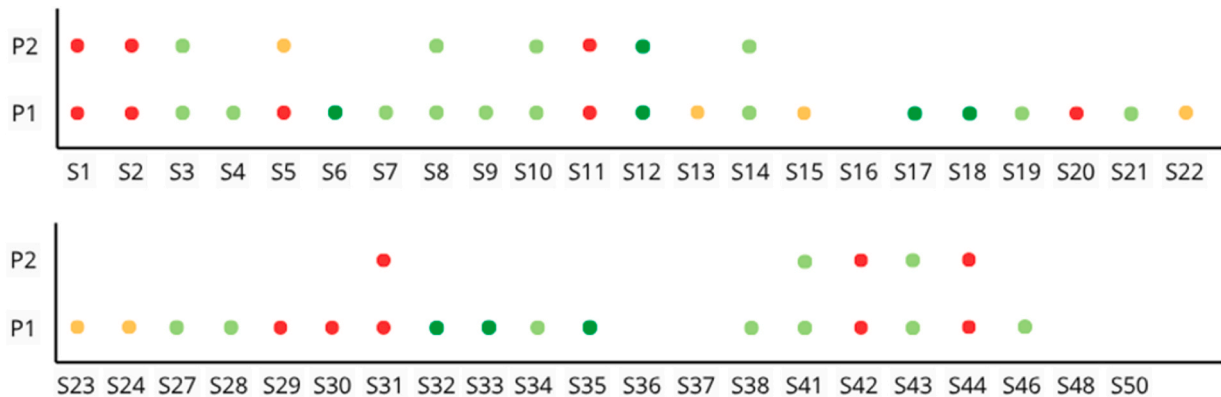
Students grouped by overall success and number (percentage) of students per group.

Overall success	Students according to their overall success	Number of students (%)
Students with very high overall success (VHS)	S3, S6, S8, S18, S19, S21, S27, S28, S33, S35, S43	11 (22%)
Students with high overall success (HS)	S4, S7, S9, S10, S12, S14, S17, S30, S32, S38, S41, S45, S46	13 (26%)
Students with medium overall success (MS)	S1, S2, S5, S11, S13, S15, S24, S29, S34, S37, S48, S49	12 (24%)
Students with low overall success (LS)	S16, S20, S22, S23, S25, S31, S36, S44, S47	9 (18%)
Students with very low overall success (VLS)	S26, S42, S50	3 (6%)
Not assigned (NA)	S39, S40	2 (4%)

Table 4

Count and frequency of occurrence (%) of the different categories of problems posed.

	Null	Incomplete	Variation of a solved problem	New problem
Count (%)	16 (30.8%)	6 (11.5%)	22 (42.3%)	8 (15.4%)

**Fig. 8.** Students' problem-posing behavior (number and type of problems posed).**Table 5**

For each type of problem posed: frequency of occurrence, the count (%) of students' overall success, the type and number (%) of strategies they used, and the type of generalization they achieved.

Type of problem posed	Frequency occurrence	Students' overall success	Strategies used*	Generalization achieved*
None	5 (11.4%)	1 VLS (20%) 2 LS (40%) 2 MS (40%) 0 HS (0%) 0 VHS (0%)	2 I (10.5%) 10 D (52.6%) 7 M (36.8%) 0 P (0%)	10 0 (52.6%) 7 A (10.5%) 2 A&B (36.8%)
Null	10 (22.8%)	1 VLS (10%) 3 LS (30%) 5 MS (50%) 1 HS (10%) 0 VHS (0%)	3 I (7.7%) 26 D (66.7%) 8 M (20.5%) 2 P (5.1%)	11 0 (28.2%) 20 A (51.3%) 8 A&B (20.5%)
Incomplete	6 (13.7%)	0 VLS (0%) 2 LS (33.3%) 4 MS (66.6%) 0 HS (0%) 0 VHS (0%)	0 I (0%) 17 D (70.8%) 5 M (20.8%) 2 P (8.3%)	7 0 (29.1%) 13 A (54.2%) 4 A&B (16.7%)
Variation	16 (36.4%)	0 VLS (0%) 0 LS (0%) 1 MS (6.2%) 8 HS (50%) 7 VHS (43.8%)	1 I (1.5%) 10 D (15.6%) 27 M (42.2%) 26 P (40.6%)	4 0 (6.2%) 10 A (15.6%) 50 A&B (78.1%)
New	7 (15.9%)	0 VLS (0%) 0 LS (0%) 0 MS (0%) 3 HS (42.9%) 4 VHS (57.2%)	0 I (0%) 4 D (14.3%) 9 M (32.1%) 15 P (53.6%)	1 0 (3.6%) 4 A (14.3%) 23 A&B (82.1%)

* For some types of problems, the total number of strategies used and the count of the success of the strategies (generalization achieved) is not equal to 4 times the number of problems as would be expected since, as mentioned above, there were students who did not address all 4 problems.

strategies based on the continuation of an arithmetic progression (40.6%), and only a few used strategies involving extending the given drawing (15.6%). This behavior was accompanied by significantly higher success rates than in the previous groups, as more than three quarters of them achieved a far generalization (78.1%). The students who posed variations of a problem had high overall success (50%) or very high overall success (43.8%) as problem-solvers. In this group, there were no students with very low or low overall success and only one with medium overall success.

The most consistent performance was observed among students who posed new problems (15.9% of the group). These students mainly used strategies involving the continuation of arithmetic progressions (53.6%) or mixed strategies (32.1%). The vast majority

achieved far generalizations (82.1%) and very few were unable to solve any problems (3.6%). All students who posed new problems had high (42.9%) or very high (57.2%) overall success.

Overall, the results shown in Table 5 suggest that the use of structured strategies aimed at generalization, and consequently higher overall success in problem solving, is associated with the ability to pose mathematically coherent and well-structured problems.

6. Discussion

Our study contributes to research examining links between students' behavior as problem solvers and their behavior as problem posers showing that students' behavior to solving pattern problems (considering both strategy use and success) may be associated with the types of pattern problems they pose in an unstructured context (Singer et al., 2013; Xie & Masingila, 2017).

The characterization of the strategies used by individual students and their success allowed us to outline students' behavior as problem solvers. When establishing success in solving a problem, we considered whether they managed to answer correctly the far generalization question of the problem, since far generalization brought them closer to algebraic thinking (Stacey, 1989). In this way, we organized the students into 5 groups, depending on whether their overall success in the sequence was very high, high, medium, low or very low. Thus, we observed that almost half of them had a high or very overall success while solving the problems, while a quarter had a low or very low overall success. Considering that students participating in our research had not previously addressed pattern problems, their success in solving the problems is noteworthy.

We observed that the problem-solving behavior of students with high or very high overall success was characterized by the frequent use of strategies closely related to generalization, such as mixed strategies and the continuation of the arithmetic progression strategies. We also observed that throughout the sequence of problems, students progressively abandon concrete thinking strategies linked to an image in favor of more efficient strategies, which allowed them to get closer to a possible arithmetic generalization (Vergel, 2015). Overall, the findings regarding how students approached the sequence of problems suggest that engaging them in sequences of pattern-based problems provided a productive context for fostering and supporting effective problem-solving behavior, even in the early stages of algebraic thinking (Montenegro et al., 2018).

By characterizing the problems posed by the students, we were able to group them into four categories: null, incomplete, variation of a problem, and new. Based on this categorization we observed that more than a fifth of the problems collected were null problems, since did not meet the conditions of the task, as they were not pattern problems. This finding points to a possible confusion between the application of arithmetic calculations and the development of algebraic thinking, as emphasized by Thompson and Harel (2021).

Moreover, just over a tenth of the problems posed were incomplete since they provided insufficient information to be solvable, because the underlying repetition pattern could not be identified. We interpret this as evidence that students were not systematically required to make pattern structures explicit during earlier problem-solving sessions. As a result, some posed problems lacked sufficient information, revealing a gap between recognizing a pattern for solving purposes and articulating it clearly when posing a new problem. This gap highlights an important limitation that underscores the metacognitive demands of problem posing (Cai & Rott, 2023).

We also observed that even though the task was unstructured (Stoyanova & Ellerton, 1996), students most often produced variations of problems they had encountered before. We interpret this result as potentially related to the ambiguity of the term "new" in the task statement (*Write two new pattern problems that can be solved by one of your classmates*).

This ambiguity might help explain why fewer genuinely new problems arose than might have been expected. In particular, the task may have been interpreted as a problem reformulation activity (Silver, 1994), rather than as an invitation to generate new problems, since the requirements of the task were not made fully explicit, as Ran et al. (2025) point out.

Finally, by integrating all the above results, we were able to address the research question guiding this study: in what ways students' approaches to solving pattern problems are related to the types of problems they later pose in an unstructured problem-posing situation. Thus, when analyzing the problem-solving behavior of students who posed problems of the same type, we observed that those who did not pose any problems, or who posed null or incomplete problems, in the problem-solving task had mainly resorted to strategies that had little to do with generalization, such as extending the given drawing. Consequently, they demonstrated low overall success throughout the entire problem-solving sequence. In contrast, students who posed problems in line with the task requirements, that is variations of previously solved problems or entirely new problems, had shown a clear tendency to use strategies more closely aligned with generalization, such as mixed strategies and the continuation of arithmetic progression. As a result, most of them achieved a high or very high overall success when solving pattern problems.

In summary, we were able to associate the students' approaches to solving pattern problems to the types of problems they later posed in an unstructured problem-posing situation. Our findings are consistent with previous research indicating that "stronger problem-solvers" tend to pose more complex mathematical problems (Xie & Masingila, 2017) and support the view that problem-solving success, together with the strategies employed, is a factor influencing problem-posing performance (Cai et al., 2015).

7. Conclusions

The present study set out to examine whether the ways in which 11–12-year-old students solve pattern problems are related to the types of pattern problems they subsequently pose in an unstructured situation. By characterizing students' written solutions in terms of both strategy use and success, we captured how students approached near and far generalization demands across a sequence of pattern tasks. In parallel, we categorized the problems students posed according to their mathematical nature and completeness, distinguishing null and incomplete productions from coherent pattern problems that were either variations of previously solved tasks or genuinely new problems. Taken together, these two analytic lenses provide a more fine-grained account of how mathematical ideas are

transferred from solving to posing problems than approaches focused solely on correctness or task difficulty, thereby contributing to current efforts to link problem solving and problem posing (Cai & Rott, 2023).

Across the sample, the students' productions revealed a clear association between their behavior as problem solvers, in terms of the strategies they used and their overall success, and the nature of the problems they posed. These results are in line with the findings of Singer et al. (2013) and Xie and Masingila (2017), emphasizing that, when appropriate analytical frameworks are used, it is possible to identify relationship between problem solving and problem posing. Although our findings are context-specific and not generalizable, they contribute to ongoing efforts to better understand how problem-solving experiences relate to students' ability to pose new mathematical tasks.

The absence of interviews or recordings of classroom interactions, due to context-related constraints, limited our analysis to students' written productions, supported only by field notes, and prevented deeper insight into why many students posed non-pattern problems or created tasks with indeterminate patterns. However, one plausible interpretation is that the previous instruction had prioritized obtaining solutions rather than explicitly articulating the underlying structures, allowing students to operate with patterns procedurally without developing the means to use these structures clearly when posing new problems. This suggests a gap between recognizing a pattern when solving a problem and being able to generate problems that embody patterns. This gap could be bridged through teaching approaches that explicitly connect problem solving and problem posing as complementary practices.

On the other hand, we would like to emphasize that, in research designs focused on problem solving, tasks should be presented with clear and well-contextualized instructions. This is particularly relevant when trying to understand how students generate new problems, as suggested by Ran et al. (2025). At the same time, research designs that seek to explore problem-posing processes should provide students with opportunities not only to create problems, but also to discuss and solve those they pose, verifying their completeness and coherence. Such opportunities could facilitate a more comprehensive understanding of students' reasoning and decision-making processes. In this way, researchers could better interpret whether difficulties arise from conceptual understanding, the ability to formulate the problem clearly, or other factors involved in the problem-solving process.

To conclude, we would like to stress the importance of continuing to research the components involved in problem posing, beyond the analysis of solving strategies and their effectiveness (Cai & Rott, 2023). A shared, validated theoretical framework, comparable to those in problem solving, would clarify how students think when posing problems and strengthen the case for integrating problem posing into curricula. Such evidence could also support teachers in giving greater prominence to both problem-posing and problem-solving activities in mathematics classrooms.

CRediT authorship contribution statement

Núria Gorgorió: Writing – review & editing, Writing – original draft, Supervision, Formal analysis. **Alba Torregrosa:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Lluís Albarracín:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

In Table A.1 we present the strategies used by each of the 50 students when tackling the four problems, as well as the level of success (0, A or A&B) they obtained when applying them. For ease of reading, the strategies have been abbreviated as: Inadequate (I), Extension of the given drawing (D), Mixed Strategies (M) and Continuation of the arithmetic progression (P). When a student misses a session and, therefore, does not address a particular problem, a dot (·) appears in the corresponding box.

Table A.1

Strategies used by the students to address the problems and their level of success

	Problem 1		Problem 2		Problem 3		Problem 4	
Student n°	Strategy	Success	Strategy	Success	Strategy	Success	Strategy	Success
S1	D	A	D	A	P	A&B	·	·
S2	D	A	D	A	M	A&B	D	A
S3	D	A&B	M	A&B	M	A&B	M	A&B

(continued on next page)

Table A.1 (continued)

	Problem 1		Problem 2		Problem 3		Problem 4	
S4	P	A	P	A	P	A&B	P	A&B
S5	D	A	D	A	P	A&B	D	A
S6	D	A&B	P	A&B	M	A&B	M	A&B
S7	M	A	P	A&B	P	A&B	P	A&B
S8	D	A&B	D	A&B	M	A&B	P	A&B
S9	D	A	D	A	P	A&B	P	A&B
S10	M	A&B	M	A&B	P	A	M	A&B
S11	D	0	D	A	M	A&B	D	A
S12	P	0	P	A	P	A&B	P	A&B
S13	M	A	D	A	D	A	D	A&B
S14	M	A	M	A&B	P	A&B	P	A&B
S15	D	0	D	A	M	A&B	P	A
S16	D	A	D	0	M	0	M	A
S17	D	A	D	A&B	M	A&B	P	A&B
S18	D	A&B	M	A&B	M	A&B	P	A&B
S19	M	A&B	M	A&B	P	A&B	P	A&B
S20	D	A	D	A	D	A	D	A
S21	D	A&B	M	A&B	M	A&B	M	A&B
S22	M	A	D	A	M	A	D	0
S23	D	A	D	0	D	0	D	0
S24	D	A	D	A&B	D	0	M	0
S25	.	.	D	A	P	0	.	.
S26	I	0	M	0	M	0	.	.
S27	M	A&B	M	A&B	M	A&B	P	A&B
S28	P	A&B	P	A&B	M	A&B	P	A&B
S29	D	A	D	A&B	M	A	I	0
S30	D	A&B	D	A&B	M	A&B	D	A
S31	M	A	D	A	D	0	M	0
S32	M	A	P	A	P	A&B	P	A&B
S33	P	A&B	P	A&B	P	A&B	P	A&B
S34	I	0	D	0	P	0	M	A&B
S35	M	A&B	M	A&B	P	A&B	M	A&B
S36	D	A	D	A	M	A	M	0
S37	D	A	D	A&B	D	A	M	0
S38	D	A&B	D	A	M	A&B	M	A&B
S39	I	0	P	0	.	.	.	.
S40	P	0	P	0	.	.	.	.
S41	D	A	P	A&B	P	A&B	M	A&B
S42	I	0	M	0	D	0	I	0
S43	P	A&B	P	A&B	P	A&B	P	A&B
S44	M	0	D	A	D	0	D	0
S45	D	A&B	M	A	P	A&B	.	.
S46	M	0	M	A	M	A&B	M	A&B
S47	I	0	D	A	.	.	.	.
S48	.	.	D	A&B	M	0	I	0
S49	D	A&B	D	0	P	0	P	0
S50	I	0	D	0	D	0	M	0

Appendix B

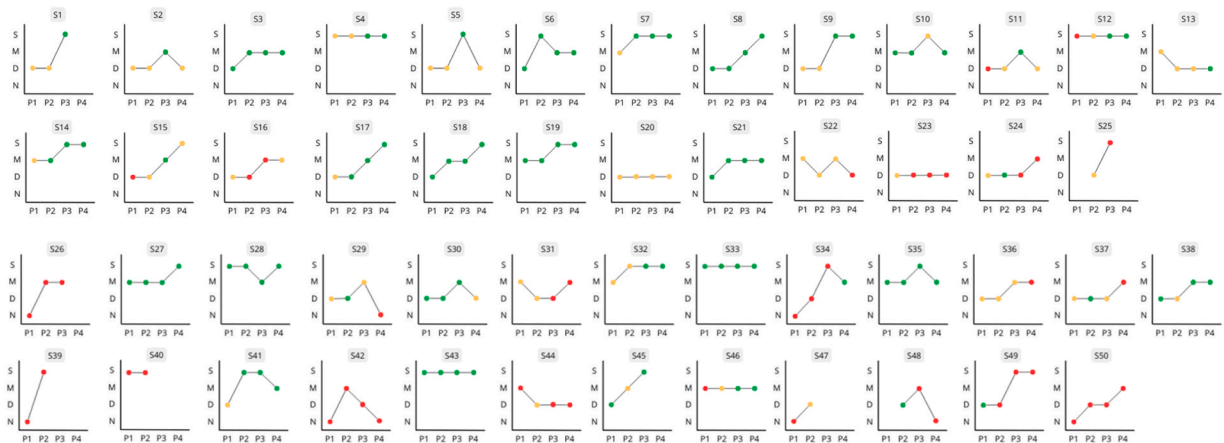


Fig. B.1. Solving behavior of each student (S1 to S50)

Appendix C

Below, we present 5 figures (Figures C.1, C.2, C.3, C.4 and C.5) showing the problem-solving behavior (strategies used and levels of success) of students grouped according to the type of problems they posed: no problems, null problems, incomplete problems, variations of a solved problem, or new problems.

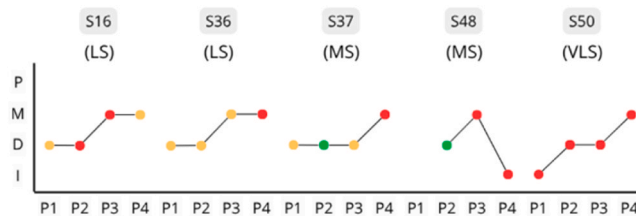


Fig. C.1. Problem-solving behavior of the students who did not pose any problems

In Figure C.1 we observe that the 5 students that posed no problems were rooted in the extension of the given drawing strategy and, when they approached a problem with mixed strategies, they failed to solve it, except in two cases that reached near generalization. None of the students who did not pose any problem had used the continuation of arithmetic progression strategy. Moreover, none of the 5 students who did not pose any problem had a high (HS) or very high (VHS) overall success as problem-solvers.

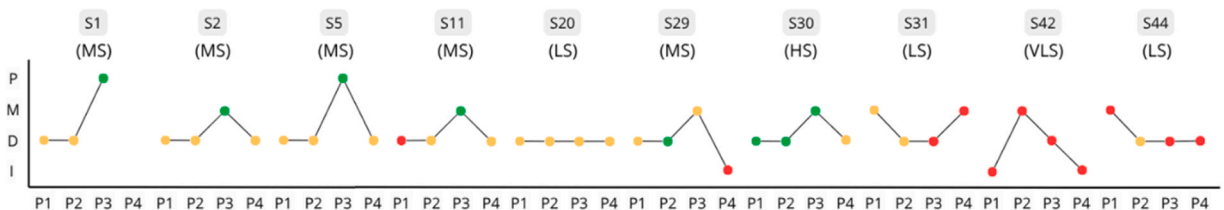


Fig. C.2. Problem-solving behavior of the students who posed null problems

There were 10 students who posed null problems, that is, problems that were coherent but did not involve a pattern structure. In Figure C.2, we observe that, when solving the pattern problems, these students tended to rely on the extension of the given-drawing strategy. On the few occasions on which they used mixed strategies (8 instances in total), they managed to solve the problem completely in only three cases. Only two students used the continuation of the arithmetic progression strategy, both in problem P3, achieving both near and far generalization. In terms of overall success, these students were distributed across low levels, ranging from very low (VLS) to medium (MS), with only one student reaching a high overall success (HS). No student in this group achieved a very high overall success (VHS).

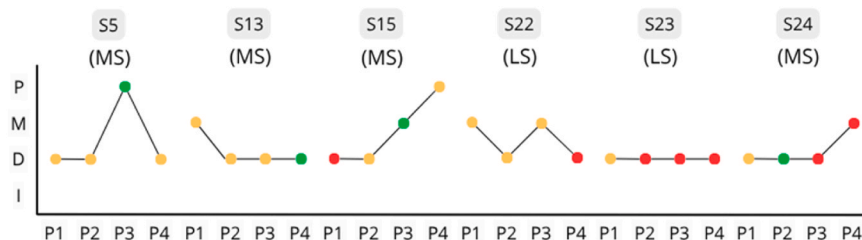


Fig. C.3. Problem-solving behavior of the students who posed incomplete problems

Again, in Figure C.3, we observe that students who posed incomplete problems relied predominantly on the extension of the given drawing strategy. On the few occasions in which they used mixed strategies, they reached near generalization in three cases, and the problem was completed in only one case. Only two students used the continuation of the arithmetic progression strategy, each in one problem, reaching near generalization in one case and far generalization in the other. Among the six students who posed incomplete problems, most showed low to medium overall success levels: two were classified as having low overall success (LS) and four as having medium overall success (MS). No students in this group achieved a high (HS) or very high (VHS) overall success.

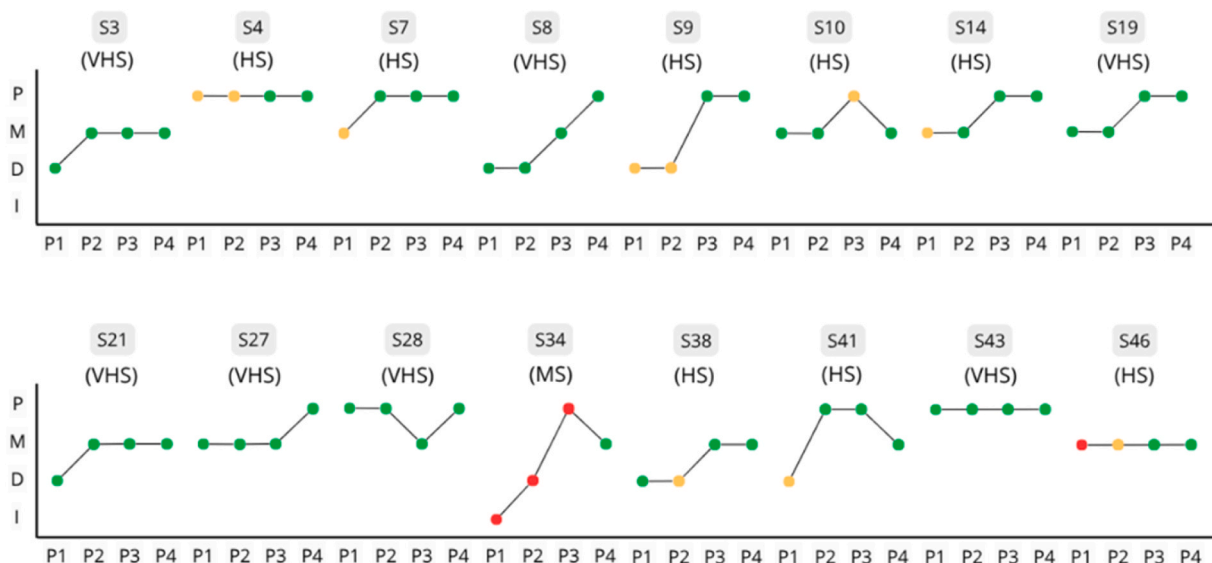


Fig. C.4. Problem-solving behavior of the students who posed variations of a solved problem

Figure C.4 shows that, although some students used the extension of the given drawing strategy, this strategy was limited to the first two problems, with all of these students switching to a mixed strategy when solving the remaining problems. Two-fifths of the problem-solving strategies used by this group (26 out of 64) were mixed strategies, which led to far generalization in all but two cases. Another two-fifths of the strategies (26 out of 64) corresponded to continuation of the arithmetic progression strategies, which resulted in far generalization in most cases, except for three instances of near generalization and one instance in which the problem was not solved. As regards the success achieved, we note that among the 16 students who posed variations of a solved problem, eight had achieved a high overall success (HS) and seven achieved a very high overall success (VHS) when solving the problems. Only one student in this group was classified as having a medium overall success (MS), and none had low (LS) or very low overall success (VLS).

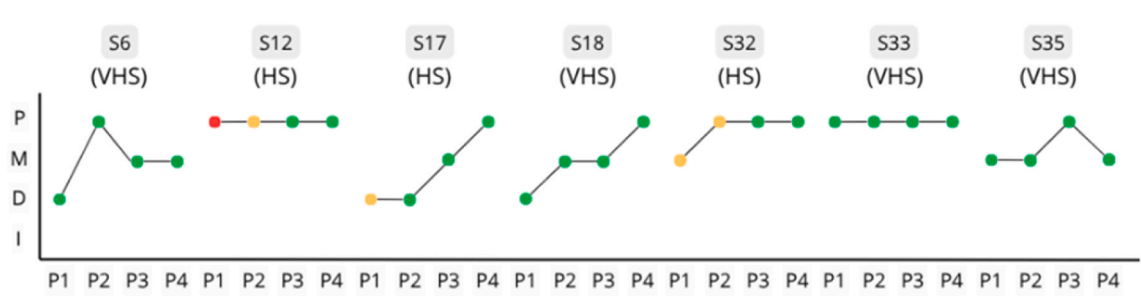


Fig. C.5. Problem-solving behavior of the students who posed new problems

Finally, figure C.5. shows that, as in the case of students who generated variations of a solved problem, the extension of the given drawing strategy had little presence. In contrast, more than half of the strategies used by these students were based on the continuation of the arithmetic progression, and in all but three cases these strategies allowed students to reach both near and far generalization. As regards the success achieved, we note that among the seven students who posed new problems, three achieved a high overall success (HS) and four achieved a very high overall success (VHS).

Appendix D. Supporting information

Supplementary data associated with this article can be found in the online version at [doi:10.1016/j.jmathb.2026.101348](https://doi.org/10.1016/j.jmathb.2026.101348).

Data availability

Data will be made available on request.

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