

Analytical Modularization of Fully Canonical Filter Networks via Householder Transformations

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Abstract—This letter presents a unified analytical framework for the modularization of fully canonical filter networks based on Householder transformations, along with its particularization to dangling inline structures with a direct impact on the realization of ladder circuit filters. The proposed method extends existing modularization approaches by explicitly addressing the fully canonical case, previously not considered. Compared to conventional extraction- and rotation-based techniques, the proposed formulation exhibits high numerical robustness, particularly for high-order filters. Furthermore, the transversal coupling matrix (CM) is constructed using both coefficient- and root-based formulations, highlighting the limitations of its conventional construction, whereas the latter, combined with the proposed Householder-based formulation, enables the synthesis of high-order extracted-pole filters beyond $N > 60$.

Index Terms—Acoustic wave (AW), coupling matrix (CM), filter synthesis, fully canonical, householder transformations.

I. INTRODUCTION

THE synthesis of RF filter networks has been traditionally addressed through analytical similarity transformation techniques [1], [2], [3] or iterative extraction procedures [4], [5], [6], which, despite recent improvements in numerical stability [7], [8], are often constrained to specific topological classes and exhibit limited scalability for high-order or unconventional structures. In this context, recent works in [9], [10], and [11] have explored algebraic formulations based on Householder transformations to facilitate modular synthesis and systematic manipulation of the network topology. In particular, these approaches enable the splitting of networks into cascaded subnetworks, thereby allowing a structured decomposition of the overall transfer function having a very good fit with extracted-pole networks. Nevertheless, a critical limitation remains in the treatment of fully canonical networks, where the presence of the source-to-load coupling term (M_{SL}) introduces significant challenges. Specifically, this term cannot be modified or eliminated through conventional methodologies without altering the eigen-resonance spectral content of the system, and was therefore not addressed in those frameworks [9], [10], [11]. Moreover, the fully canonical case has a

direct impact on the realization of ladder-type filter structures, since the dangling in-line topology constitutes a fundamental nodal prototype for modeling acoustic wave (AW) ladder networks, which have a huge industrial impact [12]. Existing synthesis methodologies for this class of networks typically rely on iterative extraction of normalized reactance parameters or rotation-based transformations [13], [14], [15]. While effective, these approaches involve multiple operations and are sensitive to numerical conditioning, especially for high-order filters. Although rotation-based techniques can improve stability in some cases [14], their performance depends on the specific transfer function parameters and may vary across design scenarios.

In this work, we extend the methodology introduced in [11] to the fully canonical case, enabling the transformation of coupling matrices (CMs) with the M_{SL} term. The proposed approach is illustrated through the transformation of a fully canonical CM into a dangling in-line topology and its subsequent particularization to AW technology, adopted here due to the authors' interest, while remaining applicable to any fully canonical ladder filter implementation. This leads to an alternative analytical formulation for the synthesis of AW ladder filters based on a purely algebraic matrix framework, employing a compact sequence of Householder transformations and avoiding angle-driven rotations or iterative extraction procedures. Finally, in order to assess the practical performance of the proposed formulation, a comparative study with conventional extracted-pole synthesis techniques is presented, taking as reference the benchmark analysis reported in [14]. In addition, the construction of the transversal CM, which constitutes a critical source of numerical limitations due to the ill-conditioning associated with partial fraction expansion, is reformulated within a root-based framework. This formulation enhances numerical accuracy and enables the synthesis of higher-order filters. This aspect will be further discussed in Section III-A.

II. SYNTHESIS BY HOUSEHOLDER TRANSFORMATIONS

A. Householder Transformations Background

Primary operations (POs) introduced in [11] form the algebraic core of the modularization process based on Householder transformations, which decompose a transversal CM into smaller cascaded subnetworks while preserving the frequency spectrum. The transformation is defined as

$$\mathbf{M}' = \mathbf{H}\mathbf{M}\mathbf{H}, \quad \mathbf{H} = \mathbf{I} - 2\mathbf{u}\mathbf{u}^T \quad (1)$$

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where the vector $\mathbf{u}$ is designed so that the source and load ports remain unchanged. The resulting CM takes the following form:

$$\mathbf{M}' = \begin{bmatrix} 0 & - & (\mathbf{H}\mathbf{m}_1)^T & - & M_{SL} \\ \mathbf{H}\mathbf{m}_1 & - & \mathbf{H}\mathbf{M}_C\mathbf{H} & - & \mathbf{H}\mathbf{m}_N \\ M_{SL} & - & (\mathbf{H}\mathbf{m}_N)^T & - & 0 \end{bmatrix}. \quad (2)$$

PO1 isolates an elementary block without finite transmission zeros Ω_{TZ} s, whereas PO2 extracts a submodule associated with a specific (Ω_{TZ}) , where the vector $\mathbf{u}$ is obtained from (3), so that the

$$\mathbf{h}_1 = \frac{(\mathbf{M}_C + \omega_z \mathbf{I})^{-1} \mathbf{m}_1}{\|(\mathbf{M}_C + \omega_z \mathbf{I})^{-1} \mathbf{m}_1\|}, \quad \mathbf{u} = \frac{\mathbf{e}_1 - \mathbf{h}_1}{\|\mathbf{e}_1 - \mathbf{h}_1\|} \quad (3)$$

transformed column in $\mathbf{H}\mathbf{M}_C\mathbf{H}$ becomes linearly dependent on $\mathbf{H}\mathbf{m}_1$, enabling the separation of the targeted TZ subnetwork. Finally, an auxiliary operation (AO) complements both POs by inserting two empty nodes connected by a unity inverter, allowing the network to be partitioned into two coupled blocks by applying the Schur Complement

$$\begin{bmatrix} \mathbf{M}_A & \mathbf{M}_{AB} \\ \mathbf{M}_{AB}^T & \mathbf{M}_B \end{bmatrix} \xrightleftharpoons[\text{eliminate}]{\text{insert}} \begin{bmatrix} \mathbf{M}_A & \mathbf{k} & \mathbf{0} & \mathbf{0} \\ \mathbf{k}^T & 0 & -1 & \mathbf{0} \\ 0 & -1 & 0 & \mathbf{m}^T \\ 0 & \mathbf{0} & \mathbf{m} & \mathbf{M}_B \end{bmatrix} \quad (4)$$

where $\mathbf{k}$ and $\mathbf{m}$ are vectors, and $\mathbf{k}\mathbf{m}^T = \mathbf{M}_{AB}$. However, such a partition is only feasible when the submatrix $\mathbf{M}_{AB}$ between both partitions has rank one, as discussed in [11]. In the fully canonical case, where all POs are of type II, $\mathbf{M}_{AB}$ exhibits higher rank as a result of the source-to-load coupling (M_{SL}), which remains unaltered during the transformation process (2). Consequently, this condition is not satisfied. To enable the modularization, a rank-1 matrix reformulation is applied [16].

B. Rank-1 Representation of the SubCM Block

For fully canonical networks, the interpartition block $\mathbf{M}_{AB} \in \mathbb{R}^{2 \times n}$ may be formally detected as rank two in finite-precision arithmetic, preventing its direct representation through a single pair of auxiliary nodes in (4). However, the SVD analysis performed at every recursive partition reveals that this second-rank contribution is only a roundoff-level numerical residual. The subblock is expressed as

$$\mathbf{M}_{AB} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sigma_1 \mathbf{u}_1 \mathbf{v}_1^T + \sigma_2 \mathbf{u}_2 \mathbf{v}_2^T \quad (5)$$

where σ_i , $\mathbf{u}_i$, and $\mathbf{v}_i$ denote the singular values and their corresponding singular vectors. For all the recursive partitions and filtering functions considered in this work, the second singular value satisfies $\sigma_2 < 10^{-12}$ and therefore remains at the level of double-precision numerical roundoff. Consequently, $\mathbf{M}_{AB}$ is numerically rank one and can be represented as

$$\mathbf{M}_{AB} \approx \sigma_1 \mathbf{u}_1 \mathbf{v}_1^T = \mathbf{k}\mathbf{m}^T, \quad \begin{cases} \mathbf{k} = \sqrt{\sigma_1} \mathbf{u}_1 \\ \mathbf{m} = \sqrt{\sigma_1} \mathbf{v}_1 \end{cases} \quad (6)$$

By the Eckart–Young–Mirsky theorem [17], the dominant singular component provides the optimal rank-1 approximation of $\mathbf{M}_{AB}$. Since $\sigma_2 < 10^{-12}$ in all the considered partitions, the neglected component remains at the level of numerical precision and the block is treated as numerically rank one. Therefore, (4) can be implemented using a single pair of white nodes within double-precision numerical accuracy.

C. Synthesis of AW Fully Canonical Networks

The synthesis procedure is based on the following steps graphically illustrated in Fig. 1.

- 1) *Step 0*: Partial fraction expansion from the two-port admittance matrix $\mathbf{Y}_N$ to obtain the transversal form [1].
- 2) *Step 1*: Perform PO2 using the selected transmission zero to be embedded in an EP section of the network. Note that, the M_{SL} remains invariant.
- 3) *Step 2*: Perform the subnetwork division which isolates the selected Ω_{TZ} within a trisection (subnetwork A) from the subnetwork B, by performing the SVD decomposition followed by the Schur Complement (see Section II-B).
- 4) *Step 3*: Diagonalize the remaining subnetwork B to reconstruct a transversal network containing $(N - k)$ finite transmission zeros (being k the current iteration number), representing the residual transfer function after removing the contribution of the extracted transmission zero(s).
- 5) *Step 4*: Repeat steps 1–3, $(N - 1)$ times until the network is fully described by a sequence of N in-line tri-sections.
- 6) *Step 5*: Apply the trisection-to-dangling transformation to each of the obtained tri-sections. Each contains a subpart of the overall transfer function associated with the Ω_{TZ} selected according to the chosen extraction order.
- 7) *Step 6*: Perform the reduction of redundant nodes through the multiple-FIR annihilation process (see Section II-D).
- 8) *Step 7*: Diagonal scaling to normalize all main-line inverters to evenly distributed values, enabling the serialization of resonators along the main signal path.

After completing all the steps, the extracted-pole topology is obtained. This network can then be mapped to its equivalent BVD prototype and scaled to the desired frequency range [13].

D. Multiple-FIR Annihilation

During the process, additional auxiliary nodes are generated (see Fig. 1), expanding the network dimensionality to $4N$ nodes. These nodes, which become non-null after the transformations, must then be eliminated to restore the physical $(2N + 2)$ -dimensional representation of the network. This elimination can be achieved through a single operation using the *Kron reduction* [18]. Let the nodal CM be partitioned as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{KK} & \mathbf{M}_{KE} \\ \mathbf{M}_{EK} & \mathbf{M}_{EE} \end{bmatrix}. \quad (7)$$

$\mathbf{M}_{KK}$ contains the retained nodes and $\mathbf{M}_{EE}$ corresponds to the redundant ones. The reduced matrix is then obtained as

$$\mathbf{M}_{\text{red}} = \mathbf{M}_{KK} - \mathbf{M}_{KE} \mathbf{M}_{EE}^{-1} \mathbf{M}_{EK}. \quad (8)$$

In this work, the elimination set is directly defined as

$$\mathbf{E} = \{\mathbf{B}_1^B, \mathbf{B}_1^C, \mathbf{B}_2^B, \mathbf{B}_2^C, \dots, \mathbf{B}_{N-1}^B, \mathbf{B}_{N-1}^C\} \quad (9)$$

corresponding to the auxiliary nodes generated during the process. By reordering $\mathbf{M}$, the Kron reduction annihilates these redundant nodes in a single step, collapsing the expanded $4N$ -node structure back into the $(2N + 2)$ ladder representation while maintaining the eigenvalue spectrum.

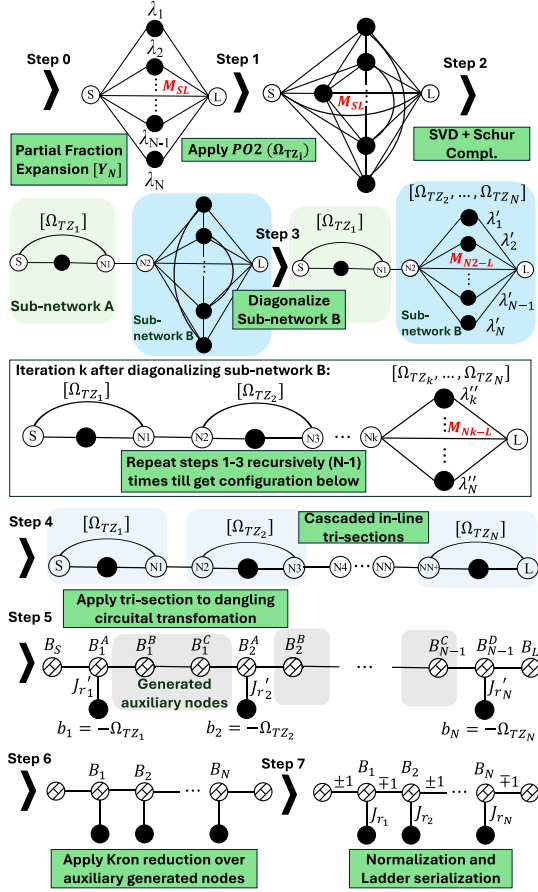


Fig. 1. Workflow for modularizing a fully canonical network into a dangling in-line configuration through sequential householder transformations.

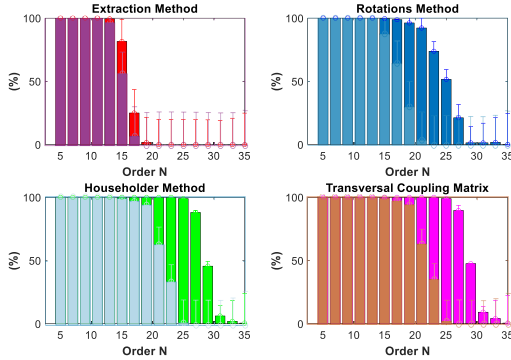


Fig. 2. Statistical comparison of the synthesis success rate as a function of the filter order (N). For each order, 500 independent pseudorandom realizations of Ω_{TZ} were generated within the interval $[\pm 1.1, \pm 3]$, with an RL of 10 and 20 dB (superposed). The bar height represents the (%) of successful synthesis, while the error bars indicate the standard deviation across the ensemble.

III. SYNTHESIS METHODOLOGIES COMPARISON

In order to evaluate the performance of the proposed synthesis framework, a comparative analysis with established analytical synthesis techniques for AW ladder prototypes is presented in this section. All numerical experiments have been carried out using 16-digit precision software.

Following the evaluation procedure reported in [14], the accuracy of each synthesis method is assessed by comparing the reflection response obtained from the polynomial definition, S_{11}^P , with that of the synthesized network, S_{11}^C .

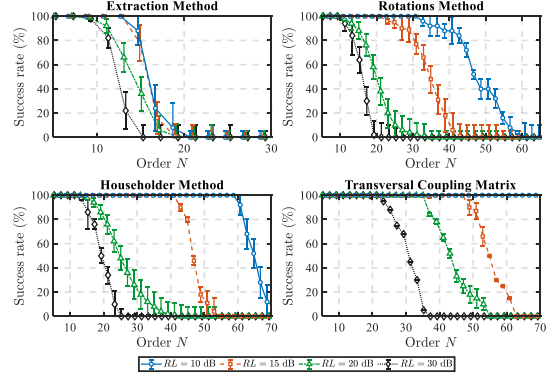


Fig. 3. Statistical evaluation of synthesis success rate versus filter order using $\Omega_{TZ} \in [\pm 1.1, \pm 3]$ for different RLs.

The comparison is performed via mean absolute error (10). A synthesis attempt is considered successful when the MAE remains below a predefined threshold (< 0.25 dB)

$$\text{MAE} = \frac{1}{N+1} \sum_{i=1}^{N+1} |20 \log_{10} |S_{11}^P(j\Omega_i) / S_{11}^C(j\Omega_i)||. \quad (10)$$

The statistical results presented in Fig. 2 reveals a clear differentiation in robustness among the considered synthesis methodologies as the filter order increases. For low-order networks, all approaches exhibit comparable performance. However, as the order increases, significant differences emerge due to the intrinsic numerical and structural limitations of each formulation. This behavior is consistent with the observations reported in [14], where different combinations of return loss levels and transmission-zero distributions lead to markedly different performance across synthesis techniques, indicating a strong dependence on the specific filtering function. In contrast, the proposed Householder-based formulation exhibits an input-agnostic transfer function behavior, consistently maintaining a higher success rate across different distributions of Ω_{TZ} s and RL specifications. Finally, the feasibility of the transversal representation provides a reference baseline to identify the origin of synthesis failures. Its construction in terms of polynomial coefficients becomes increasingly ill-conditioned as the filter order grows, leading to a marked degradation in synthesis success, as observed in Fig. 2. Moreover, for the proposed formulation, synthesis success is directly tied to this feasibility, as a valid initialization consistently leads to a successful transformation. These observations motivate a root-based formulation, which is next addressed.

A. CM Root-Based Formulation

As discussed in [10], constructing the transversal CM using roots significantly improves numerical stability and accuracy compared to coefficient-based formulations, particularly for high-order filtering functions where polynomial ill-conditioning becomes dominant [19]. For this reason, a root-based formulation is adopted in this section, and the same statistical comparison of synthesis methodologies is carried out to isolate the impact of the transformation stage from numerical errors introduced during the initial matrix construction.

Fig. 3 presents the statistical success rate versus filter order for the considered methods and for four return-loss levels,

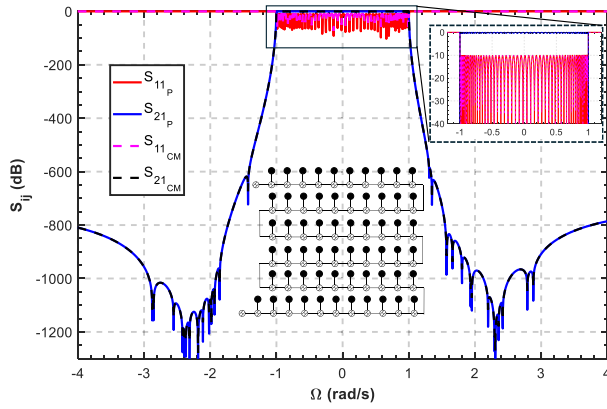


Fig. 4. Comparison between the response obtained from the polynomials and the synthesized extracted-pole dangling in-line topology for $N = 61$.

with the transmission zeros distributed within $|\Omega_{TZ}| \in [1.1, 3]$. A behavior consistent with the coefficient-based formulation is observed, confirming that the proposed Householder-based approach systematically outperforms both extraction and rotation-based techniques across the entire order range. A significant improvement is nevertheless observed for the rotation method when the transversal CM is constructed from the polynomial roots. More importantly, the root-based CM extends the synthesis feasibility to substantially higher orders. In particular, for the least restrictive return-loss case, the Householder formulation maintains a 100% success rate up to approximately $N > 55$ and only begins to exhibit a gradual degradation beyond $N > 60$. As expected from classical filter theory, the synthesis difficulty increases as the return-loss specification becomes more stringent. Thus, for $RL = 15$ dB, the maximum order achievable with a high success rate is reduced to approximately $N \approx 40$, whereas for $RL = 20$ dB it decreases further to approximately $N \approx 30$. Nevertheless, for all four return-loss levels, the Householder-based formulation remains significantly more robust than the alternative methods. Finally, Fig. 4 compares the scattering parameters obtained from the characteristic polynomials and the extracted dangling in-line topology, showing excellent agreement and confirming the spectral consistency of the proposed transformation.

IV. CONCLUSION

This work has presented an analytical modularization framework for fully canonical networks based on a systematic sequence of Householder transformations. Beyond providing an alternative synthesis route for ladder filters, the results demonstrate that the proposed method outperforms conventional extraction- and rotation-based techniques in terms of achievable filter order, while also demonstrating enhanced robustness with respect to different realizations of transmission zero allocation and return loss specifications. Furthermore, the incorporation of a root-based construction of the transversal CM significantly enhances numerical robustness, enabling the extension of the synthesis feasibility toward substantially higher orders.

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