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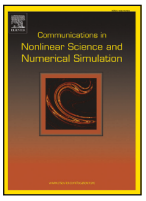

Research paper

Global asymptotic stability for the rigid polynomial differential systems of degree three

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ABSTRACT

A globally asymptotically stable differential system is a differential system having a unique equilibrium point p satisfying that all its orbits, distinct from p, tend in forward time to p. To classify the differential systems that are globally asymptotically stable in the plane is one of the classical problems of the qualitative theory of differential systems. This problem is relevant due to its applications. Here we characterize the global asymptotic stable rigid polynomial differential systems of degree three. A rigid system is a differential system having in polar coordiantes its angular velocity constant. We recall that all the global asymptotic stable polynomial differential systems of degree two already have been classified.

1. Introduction and statement of the main results

In this paper we work with *polynomial differential systems* of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where P and Q are polynomials, and the dot denotes derivative with respect to the time t. The *degree* d of this polynomial differential systems is the maximum of the degrees of the polynomials P and Q.

The differental system (1) is *globally asymptotically stable* if it has an equilibrium point p such that any orbit different from p has its ω-limit at p.

To determine when a differential system (1) is globally asymptotic stable is usually a difficult problem. A usual method for studying the global asymptotic stability of a differential system (1) is to find a Lyapunov's function, but in general is not easy to find a such Lyapunov function.

During the years 1960 and 1997 there was a main interest on the problem studied in this paper about the global asymptotic stability of the differential systems around the Markus-Yamabe conjecture [1]. This conjecture was stated in 1960 and completely solved in 1997. More precisely, the *Markus-Yamabe conjecture* restricted to planar differential systems says: When the trace of the Jacobian matrix of the differential system (1) is negative and its determinant is positive at all the points of the plane, and the differential system has a unique equilibrium the differential system is globally asymptotically stable. This conjecture was proved independently in the papers [2–4], but the conjecture is false in dimension higher than two, see [5,6].

In 2023 the two-dimensional Lotka-Volterra differential systems with a unique equilibrium in the positve quadrant that are globally asymptotically stable in that quadrant were classified in [7]. And in 2025 the polynomial differential systems (1) of degree two that

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are globally asymptotically stable were classified in [8]. But many authors have been interested in the global asymptotic stability of the differential systems during these last three years without being exhaustive we mention the papers [9–17].

The objective of this paper is to classify the cubic polynomial rigid differential systems that are globally asymptotically stable. We recall that a differential system (1) is *rigid* if written in polar coordinates (r,θ), with $x = r\cos\theta$ and $y = r\sin\theta$, it has its angular velocity $\dot\theta$ constant.

It is well known that all the cubic rigid polynomial differential systems can be written as

$$\dot x = -y + x(a + bx + cy + dx^2 + exy + fy^2), \qquad \dot y = x + y(a + bx + cy + dx^2 + exy + fy^2), \tag{2}$$

with $d^2 + e^2 + f^2 \neq 0$, for a proof see for instance [18].

A *bounded* differential system (1) in the plane satisfies that it has no orbits whose ω-limit be at infinity. In order that the rigid polynomial differential systems (2) be globally asymptotically stable need that they be bounded. Our first result is to characterize when the rigid systems (2) are bounded.

Proposition 1. *The cubic rigid polynomial differential systems (2) are bounded if and only if they satisfy one of the following conditions.*

(i) $e^2 - 4df < 0$ *and* $d < 0$.
(ii) $f = e = 0$ *and* $d < 0$.

Proposition 1 is proved in Section 3.

Also in order that the rigid polynomial differential systems (2) be globally asymptotically stable we need that they have a unique equilibrium point. Our second result is to classify when the bounded rigid systems (2) have a unique equilibrium point.

Proposition 2. *Assume that* $f < 0$ *and* $e^2 - 4df < 0$. *Then the cubic rigid polynomial differential systems (2) have the unique equilibrium point* $(0,0)$ *if and only if they satisfy one of the following conditions*

(i) $(be + c(f - d))(e^2 + (d - f)^2) \neq 0$.
(ii) $e^2 + (d - f)^2 = 0$.
(iii) $e^2 + (d - f)^2 \neq 0$ *and* $be + c(f - d) = 0$.

Assume that $f = e = 0$ *and* $d < 0$. *Then the cubic rigid polynomial differential systems (2) have the unique equilibrium point* $(0,0)$.

Moreover, the equilibrium point $(0,0)$ *is stable if and only if* $a \leq 0$.

Proposition 2 is proved in Section 4.

Theorem 1. *The cubic rigid polynomial differential systems (2) with* $e = 0$ *and satisfying* $f \leq 0$ *and* $d < 0$, *have exactly one limit cycle if* $a > 0$, *and have no limit cycles if* $a \leq 0$.

Theorem 1 is proved in Section 5.

Corollary 1. *The cubic rigid polynomial differential systems (2) with* $e = 0$ *are globally asymptotic stable if and only if* $a \leq 0$ *and satisfy the assumptions (i) or (ii) of Proposition 1.*

Corollary 1 is proved in Section 5.

Conjecture *Under the assumptions (i) or (ii) of Proposition 1 the cubic rigid polynomial differential systems (2) have exactly one limit cycle if* $a > 0$, *and have no limit cycles if* $a \leq 0$.

Note that from Theorem 1 this conjecture holds if $e = 0$.

Theorem 2. *If the conjecture holds, then the cubic rigid polynomial differential systems (2) are globally asymptotic stable if and only if* $a \leq 0$ *and satisfy the assumptions (i) or (ii) of Proposition 1* .

Theorem 2 is proved in Section 5.

Note that Theorem 2 extends the result of Corollary 1 when the parameter e is non-zero.

In Section 2 we recall the Poincaré compactification of the polynomial differential systems in the plane $\mathbb{R}^2$ that we shall need for proving the Proposition 1.

2. Preliminaries

In the context of the planar polynomial differential systems the *Poincaré disc* $\mathbb{D}$ is the closed unit disc centered at $(0,0) \in \mathbb{R}^2$ whose interior is identified with the plane $\mathbb{R}^2$ and its boundary, the circle $\mathbb{S}^1$, is identified with the infinity of $\mathbb{R}^2$. A polynomial differential system in $\mathbb{R}^2$ always can be extended analytically to the Poincaré disc, and in this way we can study the orbits of the polynomial differential system in a neighbourhood of the infinity, for details see Chapter 5 of [19].

For working with the differential system extended to the Poincaré disc $\mathbb{D}$ we use the following four local charts (U_i,ϕ_i) and (V_i,ψ_i) for $i = 1,2$, where $U_1 = \{(x,y) \in \mathbb{D} : x > 0\}$, $U_2 = \{(x,y) \in \mathbb{D} : y > 0\}$, $V_1 = \{(x,y) \in \mathbb{D} : x < 0\}$, $V_2 = \{(x,y) \in \mathbb{D} : y < 0\}$, $\phi_i : U_i \to \mathbb{R}^2$ and $\psi_i : V_i \to \mathbb{R}^2$ are defined as $\phi_1(x,y) = \psi_1(x,y) = (1/x, y/x) = (u,v)$ and $\phi_2(x,y) = \psi_2(x,y) = (x/y, 1/y) = (u,v)$ for $i = 1,2$.

Then the polynomial differential system (1) in the local chart (U_1,ϕ_1) becomes

$$\dot u = v^d\Big[-uP\Big(\frac{1}{v},\frac{u}{v}\Big) + Q\Big(\frac{1}{v},\frac{u}{v}\Big)\Big], \qquad \dot v = -v^{d+1}P\Big(\frac{1}{v},\frac{u}{v}\Big). \tag{3}$$

and in the local chart (U_2, ϕ_2) is

$$\dot{u} = v^d\left[P\left(\frac{u}{v}, \frac{1}{v}\right) - uQ\left(\frac{u}{v}, \frac{1}{v}\right)\right], \qquad \dot{v} = -v^{d+1}Q\left(\frac{u}{v}, \frac{1}{v}\right). \tag{4}$$

The expressions in the local charts (V_i, ψ_i) are the same than in the charts (U_i, ϕ_i) but multiplied by $(-1)^{d-1}$, for $i = 1, 2$.

An equilibrium point of the extended differential system in the whole Poincaré disc $\mathbb{D}$ are called *finite equilibrium* if it is in the interior of $\mathbb{D}$, and *infinite equilibrium* if it is in its boundary $\mathbb{S}^1$.

Note that in the four charts the points at infinity have the coordinate $v = 0$, and that if $p = (u, 0) \in \mathbb{S}^1$ is an infinite equilibrium, then $-p = (-u, 0) \in \mathbb{S}^1$ is also an infinite equilibrium. So for studying the infinite equilibria we only need to study the infinite equilibria in the local chart (U_1, ϕ_1) and to check if the origin of the local chart (U_2, ϕ_2) is or not an equilibrium.

3. Proof of Proposition 1

In order to prove Proposition 1 we shall study the local phase portraits of the infinite equilibria. Thus, from (3) the differential system (2) in the local chart (U_1, ϕ_1) writes

$$\dot{u} = (1 + u^2)v^2, \qquad \dot{v} = -v(d + eu + fu^2 + bv + cuv + av^2 - uv^2).$$

Doing a rescaling of the time we eliminate the common factor v between $\dot{u}$ and $\dot{v}$, and we obtain the differential system

$$\dot{u} = (1 + u^2)v, \qquad \dot{v} = -(d + eu + fu^2 + bv + cuv + av^2 - uv^2).$$

At infinity we have that $\dot{v}|_{v=0} = -d - eu - fu^2$. Then, since $d^2 + e^2 + f^2 > 0$ in order that the infinity in the local chart (U_1, ϕ_1) and consequently also in the local chart (V_1, ψ_1) of the rigid system (2) becomes bounded we need that $-d - eu - fu^2 > 0$ for all $u \in \mathbb{R}$. That is, either $e^2 - 4df < 0$ and $d < 0$, or $f = e = 0$ and $d < 0$.

Now it remains to see that no orbits can reach the infinite equilibrium $(0, 0)$ of the local chart (U_2, ϕ_2). From (4) the differential system (2) in the local chart (U_2, ϕ_2) writes

$$\dot{u} = -(1 + u^2)v^2, \qquad \dot{v} = -v(du^2 + cv + buv + av^2 + uv^2).$$

Again rescaling the time we eliminate the common factor v between $\dot{u}$ and $\dot{v}$, and we get the differential system

$$\dot{u} = -(1 + u^2)v, \qquad \dot{v} = -(du^2 + cv + buv + av^2 + uv^2).$$

Now at the origin of the local chart (U_2, ϕ_2) we have that $\dot{v}|_{v=0} = -du^2$. Hence we need that $d < 0$ in order that the rigid system (2) becomes bounded. This completes the proof of Proposition 1.

4. Proof of Proposition 2

We split the proof of Proposition 2 in two cases according with Proposition 1.

Case 1. Assume that $e^2 - 4df < 0$ and $d < 0$. Then, if we compute the finite equilibrium points of the rigid system (2) we obtain the finite equilibrium point $(0, 0)$ and four additional finite equilibrium points, but in the expressions of these four equilibria appears the term

$$\sqrt{-\frac{(be + c(f - d))^2}{\left((d - f)^2 + e^2\right)^2}}.$$

Therefore, if $(be + c(f - d))((d - f)^2 + e^2) \neq 0$ the unique finite equilibrium is the $(0, 0)$ and statement (i) is proved. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0, 0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $(d + f)\pi < 0$. For more information on the Lyapunov constants see for instance the Chapter 4 of [19].

Assume that $(d - f)^2 + e^2 = 0$, i.e. $e = 0$ and $f = d$. Therefore, if we compute the finite equilibrium points of the rigid system (2) we obtain the finite equilibrium point $(0, 0)$ and four additional finite equilibrium points, but in the expressions of these four equilibria appears the term $\sqrt{-(ac + b)^2}$. Hence, if $ac + b \neq 0$ the unique finite equilibrium is the $(0, 0)$ and statement (ii) is proved when $ac + b \neq 0$. When $ac + b = 0$ direct computations show that the unique finite equilibrium is the $(0, 0)$, hence statement (ii) is proved. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0, 0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $2d\pi < 0$.

Assume now that $(d - f)^2 + e^2 \neq 0$ and $be + c(f - d) = 0$. We distinguish the following four subcases:

$b = c = 0$;

$b = 0$, $f = d$ and $c \neq 0$;

$b \neq 0$ and $c = e = 0$;

$be \neq 0$.

Subcase $b = c = 0$ and $d + ae - f \neq 0$. Then $(0, 0)$ is a finite equilibrium and in the other four equilibrium appears the expression $\sqrt{-(ae + d - f)^2}$, hence these other equilibrium are not real. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0, 0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $(d + f)\pi < 0$.

Subcase $b = c = d + ae - f = 0$. Then $f = d + ae$ and $(0,0)$ is a finite equilibrium and in the other four equilibrium are $(\pm 1/\sqrt{e}, (\pm i/\sqrt{e})$, hence these other equilibrium are not real. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0,0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $2d\pi < 0$.
Subcase $b = 0$, $f = d$ and $c \neq 0$. Then $(0,0)$ is a finite equilibrium and in the expressions of the other four equilibrium appears $\pm ci$, hence these other equilibrium are not real. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0,0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $2d\pi < 0$.
Subcase $b \neq 0$ and $e = c = 0$. Then $(0,0)$ is a finite equilibrium and in the expressions of the other four equilibrium appears $\pm(d - f)i \neq 0$, hence these other equilibrium are not real. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0,0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $(d + f)\pi < 0$.
Subcase $be(d + ae - f) \neq 0$. Then $(0,0)$ is a finite equilibrium and in the expressions of the other four equilibrium appears $\sqrt{-e^2(ae + d - f)^2}$, hence these other equilibrium are not real. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0,0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $(d + f)\pi < 0$.
Subcase $be \neq 0$ and $d + ae - f = 0$. Then $b = -ac$ and $(0,0)$ is a finite equilibrium and the other four equilibrium are

$$\left(-\frac{c^2 + \sqrt{c^4 + 4c^2e}}{2ce}, \pm\sqrt{-\frac{c^2 + \sqrt{c^4 + 4c^2e} + 2e}{2e^2}}\right),$$

$$\left(\frac{2c}{c^2 + \sqrt{c^4 + 4c^2e}}, \pm\sqrt{\frac{-c^2 + \sqrt{c^4 + 4c^2e} - 2e}{2e^2}}\right).$$

Since the two inequalities $c^4 + 4c^2e \geq 0$ and $c^2 + 2e + \sqrt{c^4 + 4c^2e} \leq 0$ can not occur at the same time, and the two inequalities $c^4 + 4c^2e \geq 0$ and $-c^2 - 2e + \sqrt{c^4 + 4c^2e} \geq 0$ also can not occur at the same time. The four equilibria distinct from the $(0,0)$ are not real. The eigenvalues of the Jacobian matrix of the differential system (2) at $(0,0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $2d\pi < 0$.
Case 2. Assume that $f = e = 0$ and $d < 0$. Then the rigid system (2) reduces to

$$\dot{x} = -y + x(a + bx + cy + dx^2), \qquad \dot{y} = x + y(a + bx + cy + dx^2). \tag{5}$$

The finite equilibrium points of this differential system are

$$(0,0) \text{ and } \left\{\frac{\sqrt{(b \pm ci)^2 - 4(a \mp i)d} - b \pm ci}{2d}, \frac{\mp i\sqrt{(b \pm ci)^2 - 4(a \mp i)d} \mp bi + c}{2d}\right\}.$$

Hence the unique real finite equilibrium point is the $(0,0)$. The eigenvalues of the Jacobian matrix of the differential system (5) at $(0,0)$ are $a \pm i$, so if $a < 0$ it is a hyperbolic stable focus, and if $a = 0$ it is a stable weak focus because its first Lyapunov constant is $d\pi < 0$. This completes the proof of Proposition 2.

5. On the limit cycles

In this section we assume that we are under the assumptions (i) or (ii) of Proposition 1, so the differential system (2) is bounded. In the Proposition 2 we have proved under the assumptions of Proposition 1 that the unique finite equilibrium is the $(0,0)$. This equiibrium when $a > 0$ is a hyperbolic unstable focus. Then by the Poincaré-Bendixson Theorem (see for instance section 1.7 of [19]) there is at least one limit cycle surrounding the equilibrium $(0,0)$, because it is known that in the region limited by a periodic orbit it must be some equilibria, and the unique equilibria here is the $(0,0)$, for more details see Proposition 6.16 of [19].

We shall prove that one limit cycle bifurcate from the equilibrium $(0,0)$ when $a = 0$ and a increases, i.e. that there is a Hopf bifurcation at $a = 0$. This follows because the equilibrium $(0,0)$ for $a < 0$ is a hyperbolic stable focus, for $a = 0$ is a stable weak focus and for $a > 0$ is a hyperbolic unstable focus. For more details see the proof of Proposition 2.

Proof of Theorem 1. In polar coordinates (r, θ) where $x = r\cos\theta$, $y = r\sin\theta$ the differential system (2) writes

$$\dot{r} = ar + r^2(b\cos\theta + c\sin\theta) + \frac{1}{2}r^3(d + f + (d - f)\cos(2\theta) + e\sin(2\theta)), \quad \dot{\theta} = 1,$$

and taking the variable θ as the new time this differential system becomes the differential equation

$$\frac{dr}{d\theta} = ar + r^2(b\cos\theta + c\sin\theta) + \frac{1}{2}r^3(d(1 + \cos(2\theta)) + f(1 - \cos(2\theta)) + e\sin(2\theta)). \tag{6}$$

Under the assumptions of Theorem 1, i.e. $e = 0$, $d < 0$ and $f \leq 0$ the differential Eq. (6) writes

$$\frac{dr}{d\theta} = ar + r^2(b\cos\theta + c\sin\theta) + \frac{1}{2}r^3(d(1 + \cos(2\theta)) + f(1 - \cos(2\theta))), \tag{7}$$

with the coefficient of r^3, i.e. $d(1 + \cos(2\theta)) + f(1 - \cos(2\theta)) < 0$.

By Theorem 2 of [20] or Theorem A of [21] the differential Eq. (7) on the cylinder $(r, \theta) \in \mathbb{R} \times \mathbb{S}^1$ has at most three limit cycles. One of these three limit cycles is $r = 0$. Since if $r(\theta) > 0$ is a limit cycle of the differential Eq. (7), then $-r(\theta + \pi) < 0$ is another limit

cycle, it follows that the differential system (2) with $e = 0$, $d < 0$ and $f \leq 0$ has at most one limit cycle. From the previous arguments stated just before the proof of this proposition it is clear that the differential system (2) with $e = 0$, $d < 0$ and $f \leq 0$ has a unique limit cycle when $a > 0$. Now we shall prove that it has no limit cycles when $a \leq 0$.

The differential system (2) is a rotated differential system with respect to the parameter f. For more details on the rotated differential systems see Chapter 7 of [19] or [22]. Indeed, since the determinant of the matrix

$$\begin{pmatrix} \dot{x} & \dot{y} \\ \frac{d\dot{x}}{df} & \frac{d\dot{y}}{df} \end{pmatrix}$$

is $-y^2(x^2 + y^2) \leq 0$ it follows that the differential system (2) is a rotated differential system with respect to the parameter f. Here $(\dot{x}, \dot{y})$ is the vector field associated to the differential system (2).

We assume that the differential system (2) with $e = 0$, $d < 0$, $f \leq 0$ and $a \leq 0$ has some limit cycles and we shall arrive to a contradiction. Taking into account that the $(0, 0)$ is the unique finite equilibrium, and that it is stable (see Proposition 2), and that the infinity is bounded, if there is a unique limit cycle this must be semistable, or there are more than one limit cycle. But if there is a unique semistable limit cycle since differential system (2) is rotated with respect to the parameter f, moving conveniently the value of the parameter f this limit cycle splits into one stable and another unstable, see section 7.4 of [19] or [22]. In summary the differential systems (2) with $e = 0$, $d < 0$, $f \leq 0$ and $a \leq 0$ will have at least two limit cycles in contradiction that we know that such differential systems can have at most one limit cycle. The theorem is proved. □

Proof. Proof of Corollary 1 By assumptions in this proof we deal with the cubic rigid polynomial differential systems (2) with $e = 0$, $d < 0$, $f \leq 0$.

If $a \leq 0$ then by Theorem 1 these differential systems have no limit cycles. Moreover, the unique finite equilibrium is the stable $(0, 0)$ (see Proposition 2) and the infinity is bounded (see Proposition 1). Therefore, by the Poincaré-Bendixson Theorem all the orbits, distinct from the equilibrium $(0, 0)$, born at infinity and end at the $(0, 0)$. Hence these differential systems are globally asymptotic stable.

If these differential systems are globally asymptotic stable they can not have limit cycles, and by Theorem 1 it follows that $a \leq 0$. This completes the proof of the corollary. □

Proof. Proof of Theorem 2 By assumptions in this proof we deal with the cubic rigid polynomial differential systems (2) satisfying the conditions (i) or (ii) of Proposition 1.

If $a \leq 0$ then by the conjecture these differential systems have no limit cycles. Moreover, the unique finite equilibrium is the stable $(0, 0)$ (see Proposition 2) and the infinity is bounded (see Proposition 1). Therefore, by the Poincaré-Bendixson Theorem all the orbits, distinct from the equilibrium $(0, 0)$, born at infinity and end at the $(0, 0)$. Hence these differential systems are globally asymptotic stable.

If these differential systems are globally asymptotic stable they can not have limit cycles, and by the conjectue it follows that $a \leq 0$. This completes the proof of the theorem. □

CRediT authorship contribution statement

Jaume Llibre: Writing – original draft, Supervision, Methodology, Investigation, Formal analysis, Conceptualization.

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Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Jaume Llibre reports financial support was provided by State Agency of Research. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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