

A Direct Bandpass Numerical Coupling-Matrix Reconfiguration Framework for the Synthesis of Unconventional Acoustic-Wave Wideband Filter Topologies

Joel Mesas¹, Graduate Student Member, IEEE, Santi Cano¹, Graduate Student Member, IEEE, Mario Faura¹, Graduate Student Member, IEEE, Laia Garcia¹, Graduate Student Member, IEEE, Sergio López de Pablo¹, Graduate Student Member, IEEE, Jordi Verdú¹, Senior Member, IEEE, and Pedro de Paco¹, Senior Member, IEEE

Abstract—A unified direct bandpass numerical coupling-matrix (CM) reconfiguration framework is proposed for the synthesis of unconventional wideband acoustic-wave (AW) filter topologies. Unlike classical analytical approaches, whose applicability is largely restricted to predefined configurations, the proposed method performs numerical synthesis directly in the bandpass domain through a nonorthogonal transformation of the CM. This avoids intermediate low-pass-to-bandpass transformations, eliminates the need for deriving explicit analytical network solutions through laborious procedures, thereby facilitating a rapid and systematic exploration of alternative filter topologies and enabling the direct treatment of frequency-variant reactive terms required for broadband network modeling. From a prescribed filtering function, the proposed method directly derives the target nodal topology and its associated reactive network, preserving the spectral characteristics of the original transfer function. Furthermore, it enables a seamless mapping to a hybrid AW implementation through the systematic identification of Butterworth–Van Dyke (BVD) resonator cores and complementary auxiliary lumped elements, thereby establishing a direct link between the synthesized circuit model and its physical realization in AW technologies. The approach is validated through the synthesis of multiple unconventional network structures spanning diverse classes of filtering responses, thereby demonstrating its generality and versatility within a unified formulation. Experimental verification is finally provided by a fabricated bulk AW solid-mounted resonator (BAW-SMR) bandstop filter operating in the 2.95–4.45-GHz frequency range. The prototype achieves a 41.4% fractional bandwidth (FBW), with measured attenuation levels ranging from 15 to 27 dB. The implemented topology comprises two AW resonators with an electromechanical coupling factor of $k_t^2 \approx 12.5\%$, complemented by auxiliary inductive elements.

Index Terms—Acoustic-wave (AW), bandpass filter, coupling matrix (CM), direct synthesis, frequency-variant coupling (FVC), reconfiguration.

Received 11 May 2026; revised 9 July 2026 and 15 September 2026; accepted 19 September 2026. This work was supported in part by the Agencia Estatal de Investigación (AEI)—Ministerio de Ciencia e Innovación under Project PID2024-159605OB-I00 and in part by the Departament d'Universitats i Recerca. (Corresponding author: Joel Mesas.)

The authors are with the Department of Telecommunications and Systems Engineering, Universitat Autònoma de Barcelona, 08193 Barcelona, Spain (e-mail: joel.mesas@uab.cat).

Digital Object Identifier 10.1109/TMTT.2026.3736730

I. INTRODUCTION

THE continuous evolution of wireless communication systems is imposing increasingly stringent requirements on radio frequency (RF) filters in terms of selectivity, fractional bandwidth (FBW), and architectural flexibility. In this context, acoustic-wave (AW) filters have emerged as a key enabling technology for RF front-end modules due to their low insertion loss, high quality factor, compact footprint, and compatibility with high-volume manufacturing [1], [2]. However, the growing diversity of spectral allocations in modern standards such as 5G, Wi-Fi 6E, and emerging multiband systems is pushing filter design beyond the limits traditionally addressed by classical synthesis approaches.

On one hand, these classical analytical methods, which are based on predefined rotation sequences, circuitual transformations, or iterative sequential extractions, have proven their practical usability in the design process, benefiting from their high precision and computational efficiency in obtaining valid solutions [3], [4], [5], [6], [7]. Among these approaches, those grounded in the coupling-matrix (CM) framework [8], [9] have established themselves as the most systematic and versatile, serving as a cornerstone for the synthesis and reconfiguration of microwave filters. The CM formulation provides a compact algebraic representation of resonator interactions, thereby enabling structured topological transformations and the systematic derivation of a wide range of realizable network configurations.

Nevertheless, their applicability remains largely limited to a reduced set of well-established configurations. When more intricate or unconventional topologies are considered, the synthesis procedure often becomes undefined, and designers must rely on iterative trial-and-error processes or subsequent optimization stages to enforce structural constraints, achieve the desired response, or to fulfill with a given technology specification. This lack of a systematic synthesis framework for complex topologies hinders the exploration of novel configurations that could potentially offer enhanced performance or better address the demands of emerging applications.

TABLE I
RELATIONSHIP BETWEEN CIRCUIT AND NODAL REPRESENTATION

	Shunt Capacitor	Series Capacitor	Shunt Inductor	Series Inductor
Nodal Representation				
Circuit Representation				

In recent years, considerable efforts have been made to overcome the inherent limitations of analytical synthesis techniques as a means to decouple the desired electrical response from a priori assumptions on network topology. By formulating filter synthesis as a numerical problem, it becomes possible to directly operate in the topology space and to explore novel network configurations that would be difficult and time-consuming to obtain using analytical recipes alone. These approaches, grounded in CM theory, aim to transform canonical topologies into prescribed target configurations through advanced mathematical techniques [10], [11], [12], [13], [14], [15], [16], [17]. They are typically formulated in the low-pass prototype domain and mapped to the bandpass regime via frequency transformations [18]. However, this mapping introduces denormalization errors and fails to capture the behavior of the network far from the passband, leading to inaccuracies in wideband scenarios. This issue was discussed in [19] and [20], where it is shown that working directly in the bandpass domain is essential to accurately capture the asymptotic behavior of the transfer function at zero frequency and at infinity to ensure the validity of the synthesized models across the entire frequency range.

On the other hand, a recent work presented in [21] has established CM reconfiguration methodologies that operate directly in the bandpass domain for microwave filters. By embedding frequency-variant reactances into the numerical framework and adopting the CM interpretation shown in Table I, the approach enables a more accurate representation of wideband structures. In particular, Mesas et al. [21] employed nonorthogonal congruence transformations to jointly reconfigure the frequency-independent and frequency-dependent matrices. However, its fixed source and load basis vectors prevent the reconfiguration of boundary frequency-variant terms, while the method remains computationally demanding, sensitive to the initial seed, and technology-agnostic with respect to AW constraints. The present work extends this framework through a generalized transformation matrix with additional boundary degrees of freedom, enabling source-load frequency-variant couplings (FVCs). Since the port loading may then change, the termination matrix is explicitly transformed, and its preservation is enforced within the numerical formulation.

From this standpoint, direct bandpass numerical synthesis should not be interpreted as a substitute for established analytical methodologies, but rather as an expanded design framework that complements and allows the exploration of

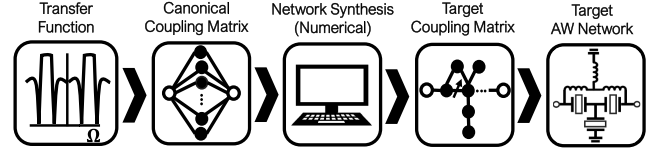


Fig. 1. High-level workflow of the proposed topology-driven numerical direct bandpass synthesis. Starting from a target transfer function in the normalized frequency domain Ω to the circuit realization in the real frequency domain f .

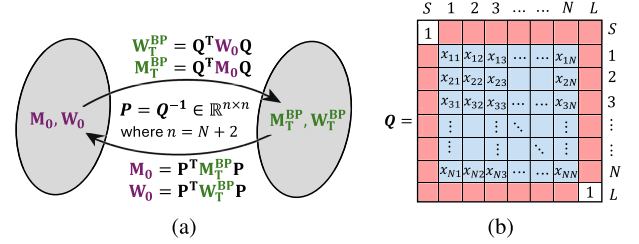


Fig. 2. Mapping between canonical and target CM. (a) Bijective function representation of the transformation matrix. (b) Transformation matrix shape.

filter topologies beyond those typically accessible through closed-form solutions, thereby providing a broader and more flexible design space.

To the best of the authors' knowledge, a comprehensive numerical synthesis framework that operates directly in the bandpass domain and is specifically tailored to the design of unconventional AW filter topologies has not yet been developed. This work aims to fill this gap by introducing a structured numerical synthesis methodology that enables the systematic exploration and realization of nonconventional AW filter configurations within a unified solver-based environment. Within this context, the proposed methodology adheres to the systematic workflow illustrated in Fig. 1, where the central block named *numerical network synthesis* can be interpreted as a closed-box numerical engine. Its inputs consist of the transfer function characteristics, together with an initial CM representation, while its outputs provide the targeted filter topology, along with its frequency response.

The validity of the methodology is demonstrated through multiple AW network examples, highlighting its intrinsic scalability and its adaptability to diverse filtering responses. In particular, the framework is shown to be directly applicable to low-pass, bandpass, bandstop, and high-pass configurations within the same unified formulation, underscoring its flexibility as a generalized synthesis platform. The transition between these design paradigms requires only minimal modifications to the underlying formulation, as detailed in Section IV.

Finally, as an experimental validation, an unconventional bulk AW solid-mounted resonator (BAW-SMR) bandstop filter topology obtained through the proposed synthesis process is fabricated and measured, providing tangible evidence of the robustness and practical relevance of the approach.

II. MATHEMATICAL BACKGROUND

The synthesis reconfiguration problem in the direct bandpass regime can be formulated as a congruent transformation (see Fig. 2) that maps an arbitrary canonical transversal

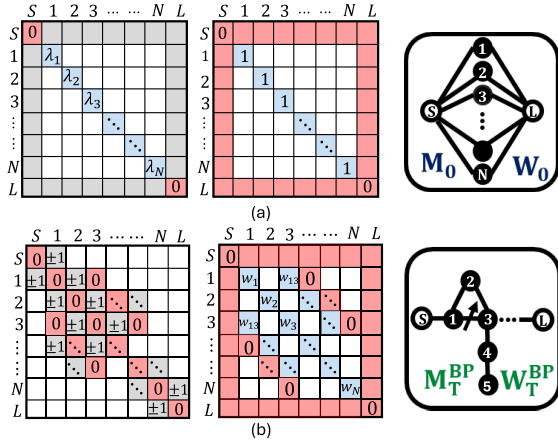


Fig. 3. General structural correspondence between the canonical and target matrices in the direct-bandpass reconfiguration framework and their associated nodal representations. (a) Canonical transversal coupling and frequency matrices M_0 and W_0 . (b) Corresponding target matrices M_T^{BP} and W_T^{BP} . The rows and columns follow the same node ordering as the nodal diagrams.

CM [see Fig. 3(a)] into an arbitrary target frequency-reactive topology [see Fig. 3(b)] by applying the following equation:

$$\begin{cases} M_T^{BP} = Q^T M_0 Q \\ W_T^{BP} = Q^T W_0 Q \end{cases} \quad \text{with } Q^T Q \neq I. \quad (1)$$

In (1), $M_0 \in \mathbb{R}^{(N+2) \times (N+2)}$ denotes the initial transversal CM, whereas $W_0 \in \mathbb{R}^{(N+2) \times (N+2)}$ denotes the corresponding frequency-dependent matrix. M_T^{BP} and W_T^{BP} denote the target bandpass-domain frequency-independent CM and frequency-dependent reactive matrix, respectively, where the superscript BP indicates that the formulation is carried out directly in the bandpass domain. The matrix $Q \in \mathbb{R}^{(N+2) \times (N+2)}$ is a real nonsingular transformation matrix, with dimension $N+2$. The condition $Q^T Q \neq I$ emphasizes that, unlike classical CM rotations [9], the proposed transformation is not orthogonal.

Following the direct-bandpass nodal interpretation described in [20], the couplings in M_T^{BP} are normalized as admittance inverters, allowing W_T^{BP} to directly identify the frequency-dependent reactive elements to be extracted. As summarized in Table I, shunt capacitors are represented by single reactive nodes, series capacitors by π -type FVCs [22], shunt inductors by dangling reactive nodes, and series inductors by reactive nodes placed between admittance inverters of opposite sign.

Accordingly, for the target considered in this work, each nodal position is assigned either to a frequency-independent inverter in M_T^{BP} or to a frequency-dependent reactive contribution in W_T^{BP} , but not to both simultaneously. Hence, the elementwise Hadamard product between the target frequency-dependent and frequency-independent matrices must satisfy (2). As illustrated in Fig. 3(b), the entries highlighted in red correspond to the positions that are structurally constrained to zero. These null locations are complementary

$$M_T^{BP} \circ W_T^{BP} = \mathbf{0}_{(N+2) \times (N+2)}. \quad (2)$$

As shown in Fig. 2(a), the transformation matrix Q adopts the block structure depicted in Fig. 2(b), with unit entries fixed at the source (S) and load (L) positions, and an internal block

$X \in \mathbb{R}^{N \times N}$ containing the free variables x_{ij} that manage the reconfiguration of the resonator subnetwork.

This construction establishes a bijective mapping between the canonical matrices and their bandpass target counterparts, provided that Q is nonsingular. Due to its block-diagonal structure, its determinant factorizes as follows:

$$Q = \text{diag}(1, X, 1) \rightarrow \det(Q) = \det(X). \quad (3)$$

Therefore, the transformation is invertible if and only if

$$\det(X) \neq 0, \quad \text{equivalently} \quad \text{rank}(X) = N. \quad (4)$$

Under this full-rank condition, the inverse transformation exists and can be expressed as follows:

$$Q = \text{diag}(1, X, 1) \rightarrow P = Q^{-1} = \text{diag}(1, Y, 1). \quad (5)$$

Consequently, starting from the target topology, it is always possible to recover the canonical form through the inverse reconfiguration mapping, provided that the internal block X remains nonsingular. Then, by examining the frequency matrices, it is observed that they are structured around an internal block of dimensions $N \times N$. For convenience, let us denote this internal block in the target bandpass domain as W_T^A and its transversal counterpart as W_0^A . The full matrices can therefore be expressed as follows:

$$W_0 = \text{diag}(0, W_0^A, 0) \quad (6)$$

$$W_T^{BP} = \text{diag}(0, W_T^A, 0)$$

$$= \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & w_{11}^T & w_{12}^T & \cdots & w_{1N}^T & 0 \\ 0 & w_{21}^T & w_{22}^T & \cdots & w_{2N}^T & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & w_{N1}^T & w_{N2}^T & \cdots & w_{NN}^T & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad (7)$$

where W_0^A is an identity matrix, $w_{ij}|_{i \neq j}$ are the FVC terms, and $w_{ij}|_{i=j}$ are the frequency-variant self-reactive nodes, according to the nodal representation in Table I. Note that all entries of the frequency-independent target matrix M_T^{BP} are constrained to unity (± 1), ensuring that they operate as ideal admittance inverters. This allows the frequency matrix W_T^{BP} to directly identify all frequency-dependent elements that will be extracted as lumped components in the final circuit realization. Then, by substituting, the target frequency-dependent block matrix can be written as follows, which shows that the bandpass capacitance inner block is fully determined by the free parameters in X :

$$W_T^A = X^T W_0^A X = X^T I_N X = X^T X. \quad (8)$$

By construction, the matrix product $X^T X$ is symmetric and, under the invertibility assumption on X , positive definite. In general, for any real matrix $X \in \mathbb{R}^{N \times N}$, the matrix $X^T X$ is positive semidefinite since

$$\mathbf{v}^T X^T X \mathbf{v} = \|X\mathbf{v}\|^2 \geq 0 \quad \text{for all } \mathbf{v} \in \mathbb{R}^N. \quad (9)$$

Since the congruent transformation is required to be bijective, X must be full rank ($\text{rank}(X) = N$) and therefore invertible [23]. Under this condition, the inequality is strict for all nonzero $\mathbf{v}$, and consequently $X^T X$ is positive definite.

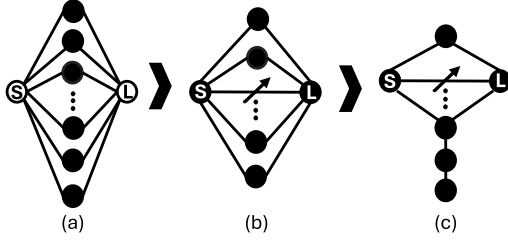


Fig. 4. (a) Initial structure without source–load FVC. (b) Intermediate configuration including the source–load coupling W_{SL} . (c) Final arbitrary target inner topology preserving W_{SL} .

In spectral terms, this implies that

$$\lambda_i(\mathbf{W}_T^A) > 0, \quad i = 1, \dots, N. \quad (10)$$

Therefore, the admissible target frequency-dependent blocks are restricted to the set of symmetric positive-definite matrices. Negative eigenvalues cannot arise from any real parametrization $\mathbf{X}$, which imposes an intrinsic structural constraint on the reconfiguration problem.

On the other hand, the inverse relation follows analogously, which leads to the reconstruction of the original transversal capacitance matrix:

$$\mathbf{W}_0 = \text{diag}(0, \mathbf{Y}^T \mathbf{W}_T^A \mathbf{Y}, 0) = \text{diag}(0, \mathbf{Y}^T \mathbf{X}^T \mathbf{X} \mathbf{Y}, 0). \quad (11)$$

This explicit block formulation highlights the bijective nature of the transformation: the internal bandpass block $\mathbf{W}_T^A$ can be obtained from the canonical identity block through $\mathbf{X}$, and conversely, the canonical form can be reconstructed by $\mathbf{Y}$.

A. Special Case: Source–Load Frequency Dependent Coupling and Inner Reconfiguration

In certain practical configurations, it is convenient to operate on an inner coupling representation [20], [22] that includes a nonzero frequency-dependent source–load interaction term, denoted as W_{SL} in (12), as illustrated in Fig. 4. Such a term typically arises when the admittance numerators $y_{11_n}(s)$, $y_{21_n}(s)$, and $y_{22_n}(s)$ exhibit a higher polynomial order than the common denominator $y_d(s)$, which renders the constant term of the partial-fraction expansion frequency-dependent. Consequently, additional entries appear in the frequency matrix $\mathbf{W}$, including the source–load term W_{SL} as well as the diagonal contributions W_{SS} and W_{LL} in (12), which are no longer null as in the configuration shown in Fig. 4(b) and (c).

Under these conditions, the classical structural form of the transformation matrix defined in (5) is no longer sufficient to guarantee full structural reconfigurability

$$\begin{aligned} W_S &= \left. \frac{y_{11_n}(s)}{y_d(s)} \right|_{s=j\infty}, & W_L &= \left. \frac{y_{22_n}(s)}{y_d(s)} \right|_{s=j\infty} \\ W_{SL} &= \left. \frac{y_{21_n}(s)}{y_d(s)} \right|_{s=j\infty}. \end{aligned} \quad (12)$$

By construction, the block-diagonal structure of $\mathbf{Q}$ preserves the source and load entries, since the first and last rows and columns remain fixed with null entries. Consequently, any congruent transformation of the form (3) cannot alter the

existing terms W_{SL} , W_S , and W_L ; the mapping induced by the classical transformation becomes structurally infeasible.

A similar limitation was previously analyzed in [12] and [15], where, in the context of fully canonical low-pass prototype CMs, it was demonstrated that relaxing the strict internal-block structure of the classical transformation matrix, specifically by allowing the outermost rows and columns to include free variables, enables the elimination of the frequency-independent term M_{SL} and allows controlled adjustments of the source and load frequency-invariant reactances (FIRs) located at the first and last diagonal entries of the matrix $\mathbf{M}$

$$\mathbf{Q}_g = \begin{bmatrix} q_{SS} & \mathbf{q}_S^T & q_{SL} \\ \mathbf{q}_L & \mathbf{X} & \mathbf{q}_R \\ q_{LS} & \mathbf{q}_E^T & q_{LL} \end{bmatrix}. \quad (13)$$

Extending this reasoning to the direct bandpass formulation, whenever a source–load interaction must be reconfigured, the transformation matrix cannot remain a strictly diagonal embedding and must instead allow controlled nonzero entries in the boundary rows and columns. This generalized structure preserves the invertibility of the transformation while enlarging the feasible solution space, thereby making direct source–load and frequency-dependent source–load interactions mathematically admissible within the synthesis framework.

To overcome this limitation, the classical block-diagonal transformation (3) is replaced by a generalized invertible matrix in which the outermost rows and columns are no longer null as (13). Note that, as reported in [12], the presence of nonzero entries in the first and last rows and columns of $\mathbf{Q}_g$ can potentially modify the termination matrix $\mathbf{R}$. For this reason, it is included in the problem formulation detailed in Section III as an additional constraint to ensure the preservation of the source and load terminations. This effect will also be illustrated later through a filter design example in Section V. The transformed target matrices are then obtained through the congruent relations in the following equation:

$$\begin{aligned} \mathbf{M}_T^{\text{BP}} &= \mathbf{Q}_g^T \mathbf{M}_0 \mathbf{Q}_g, & \mathbf{W}_T^{\text{BP}} &= \mathbf{Q}_g^T \mathbf{W}_0 \mathbf{Q}_g \\ \mathbf{R}_T^{\text{BP}} &= \mathbf{Q}_g^T \mathbf{R}_0 \mathbf{Q}_g. \end{aligned} \quad (14)$$

III. NONORTHOGONAL OPTIMIZATION MODEL FOR CM RECONFIGURATION IN THE BANDPASS DOMAIN

In this section, a novel nonorthogonal constrained optimization model is proposed for the reconfiguration of coupling matrices in the direct bandpass domain.

Over the last few years, different optimization models have been developed for the reconfiguration of coupling matrices, with the aim of finding solutions that meet specific structural or response constraints. Recently, in [24], the reconfiguration problem was expressed as (15), which is restricted to strictly orthogonal transformations. This formulation excludes the possibility of handling FVCs, which are an essential requirement when operating directly in the bandpass domain

$$\begin{aligned} \min_{\mathbf{X}} f(\mathbf{X}) &= \left\| \mathbf{Q}(\mathbf{X})^T \mathbf{M}_0 \mathbf{Q}(\mathbf{X}) - \mathbf{S}_M \circ (\mathbf{Q}(\mathbf{X})^T \mathbf{M}_0 \mathbf{Q}(\mathbf{X})) \right\|_F^2 \\ \text{s.t.} \quad & \mathbf{X}^T \mathbf{X} = \mathbf{I}_N. \end{aligned} \quad (15)$$

In contrast and based on the derivation in Section II, the following nonorthogonal constrained optimization model is proposed:

$$\begin{aligned} \min_{\mathbf{X}} f(\mathbf{X}) &= \frac{1}{2} \left\| \mathbf{Q}(\mathbf{X})^\top \mathbf{M}_0 \mathbf{Q}(\mathbf{X}) - \mathbf{S}_M \circ (\mathbf{Q}(\mathbf{X})^\top \mathbf{M}_0 \mathbf{Q}(\mathbf{X})) \right\|_F^2 \\ &+ \frac{1}{2} \left\| \mathbf{Q}(\mathbf{X})^\top \mathbf{W}_0 \mathbf{Q}(\mathbf{X}) - \mathbf{S}_W \circ (\mathbf{Q}(\mathbf{X})^\top \mathbf{W}_0 \mathbf{Q}(\mathbf{X})) \right\|_F^2 \\ \text{s.t. } \mathbf{S}_M \circ (\mathbf{M}(\mathbf{X}) - \mathbf{S}_M^\pm) &= \mathbf{0} \end{aligned} \quad (16)$$

where $\mathbf{X}$ is restricted to be a real nonsingular matrix, so that the inverse transformation introduced in Section II is well defined. Furthermore, the matrix $\mathbf{S}_M^\pm$ is introduced as the signed version of the binary mask $\mathbf{S}_M$, where each admissible frequency-independent coupling is assigned its prescribed unit value $+1$ or -1 . Hence, $\mathbf{S}_M = |\mathbf{S}_M^\pm|$

$$\mathbf{S}_M \circ \mathbf{S}_W = \mathbf{0}_{(N+2) \times (N+2)}. \quad (17)$$

These matrices define the structural information of the desired network: $\mathbf{S}_M$ prescribes the locations of the frequency-invariant couplings (inverters) to be preserved in $\mathbf{M}_T^{\text{BP}}$, whereas $\mathbf{S}_W$ identifies the positions of the frequency-dependent reactive terms in $\mathbf{W}_T^{\text{BP}}$. Both masks are symmetric and complementary, satisfying (17) as well.

However, when the generalized transformation matrix introduced in Section II-A is employed, the outer rows and columns of $\mathbf{Q}$ are no longer fixed. As a consequence, the termination matrix $\mathbf{R}$ may also be modified under the congruent transformation. Since $\mathbf{R}$ encodes the source and load terminations of the network, preserving this information is essential to maintain the correct admittance description of the synthesized filter. Therefore, the termination matrix must be explicitly included in the transformation model. So (16) is complemented with the additional term in (18) that ensures the preservation of the source and load terminations under the transformation

$$\begin{aligned} &\frac{1}{2} \left\| \mathbf{Q}(\mathbf{X})^\top \mathbf{R}_0 \mathbf{Q}(\mathbf{X}) - \mathbf{S}_R \circ (\mathbf{Q}(\mathbf{X})^\top \mathbf{R}_0 \mathbf{Q}(\mathbf{X})) \right\|_F^2 \\ \text{s.t. } \mathbf{Q}(\mathbf{X})^\top \mathbf{R}_0 \mathbf{Q}(\mathbf{X}) &= \mathbf{R}_0 = \mathbf{R}_T^{\text{BP}} \end{aligned} \quad (18)$$

where $\mathbf{Q}(\mathbf{X})$ corresponds to $\mathbf{Q}_g$ in (13) and $\mathbf{S}_R$ is a symmetric binary mask that identifies the positions of the source and load terminations in the matrix $\mathbf{R}$. By including this additional term, the optimization model ensures that the synthesized filter maintains the correct source and load admittance characteristics, even when employing a generalized transformation matrix that allows for modifications in the outer rows and columns.

IV. DESIGN WORKFLOW

A. Design Inputs: Transfer Function Definition

As illustrated in Fig. 1, the overall synthesis workflow begins by specifying a complete set of design parameters that uniquely define the desired frequency response [return losses, transmission zeros (TZs), and reflection zeros (RZs)]. From these specifications, a rational filtering function is constructed in the normalized low-pass domain and expressed in terms of the characteristic polynomials $P(s)$, $F(s)$, and $E(s)$, which define the scattering parameters of a lossless two-port network

as given in (19). This formulation enables the extraction of the canonical CM [9], which serves as the starting point for the subsequent numerical topology synthesis stage

$$S_{11}(s) = \frac{F(s)/\varepsilon_r}{E(s)}, \quad S_{21}(s) = \frac{P(s)/\varepsilon}{E(s)}. \quad (19)$$

Depending on the type of filter response to be realized, alternative formulations of the filtering function are considered.

On the one hand, for the synthesis of low-pass and high-pass responses, the filtering function is defined following the classical polynomial framework [8]. In this formulation, the filtering function belongs to the generalized Chebyshev (GC) class and characterizes the passband within the normalized frequency interval $\Omega \in [-1, 1]$. The associated polynomials in (19) are then swapped to obtain high-pass responses.

Alternatively, for bandpass and bandstop responses, the filtering function is defined as a Hermitian function that accounts for the broadband impedance behavior of the filter network [12] and is obtained using the Remez-like algorithm [25]. In this formulation, the filtering function corresponds to a dual-band response defined within the normalized frequency intervals $\Omega \in [-1, -f_{c1}/f_{c2}]$ and $\Omega \in [f_{c1}/f_{c2}, 1]$, where f_{c1} and f_{c2} denote the lower and upper corner frequencies of the band, ensuring that the same information is preserved across the positive and negative frequency planes. The response is described through a generalized rational filtering function that preserves the prescribed equiripple behavior in the passband. Depending on the imposed transmission and RZ distribution, this formulation may accommodate complex RZs; in such cases, the filtering function reduces to particular subclasses such as the reduced Chebyshev (RC) function. Likewise, for the bandstop case, the corresponding polynomials in (19) are interchanged.

B. Numerical Solver Execution

At this stage, the methodology enters the numerical network synthesis phase. The constrained optimization problem in (16) is solved using the sequential quadratic programming (SQP) algorithm [26]. The free entries of the transformation matrix are collected into a decision vector $\mathbf{x}$, so that, at each iteration, the matrix $\mathbf{Q}(\mathbf{X})$ is reconstructed and the congruent mapping in (1) is evaluated.

The objective function accounts for the structural conditions imposed on the transformed matrices through the prescribed masks $\mathbf{S}_M$ and $\mathbf{S}_W$, whereas the nonlinear equality conditions are supplied directly to the solver as constraints.

The initial seed is generated by assigning the free entries of the decision vector according to a uniform random distribution, $X_{ij}^{(0)} \sim \mathcal{U}(-1, 1)$. A sensitivity analysis of this initialization strategy, including different sampling ranges, the three target topologies, and the associated convergence behavior and computational cost, is reported later in Fig. 5. All numerical tests reported in the manuscript were performed on an Intel Core i7-10700 CPU at 2.90 GHz with 16 GB of RAM.

Note that, in the case explained in Section II-A, the generalized transformation matrix $\mathbf{Q}_g$ in (13) is employed, and the optimization model is complemented with (18).

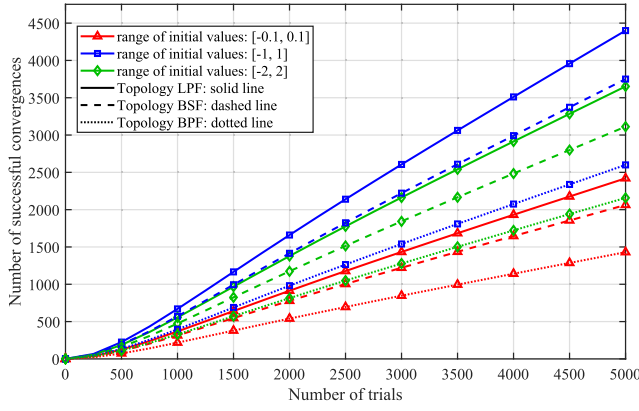


Fig. 5. Initialization sensitivity and computational-cost assessment of the proposed numerical reconfiguration method.

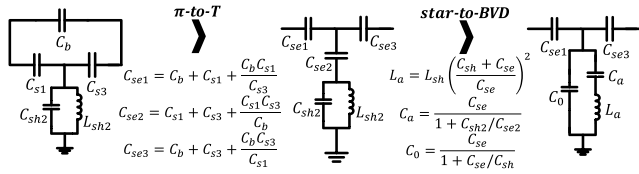


Fig. 6. Circuit transformations to obtain the BVD elements.

It is also worth mentioning that, when the order and structural complexity of the network increase, the dimensionality of the optimization problem also increases. Consequently, a larger computational effort is required, which is a natural limitation shared by numerical optimization-based synthesis techniques. In practice, this burden can be partially alleviated through parallel computing, since different admissible initial seeds can be evaluated independently within a multistart optimization strategy without modifying the mathematical formulation of the proposed method.

C. Synthesized Outputs

Once $\mathbf{Q}$ has been determined, (1) is applied to obtain the corresponding target nodal network. From this reconfigured CM, the normalized reactive elements are systematically extracted and then translated to the desired operating frequency by a low-pass-to-low-pass transformation [18], yielding a fully characterized network composed of lumped inductive and capacitive elements. Subsequently, the synthesized reactive network is reorganized by identifying and transforming only the subcircuits associated with acoustic resonators into their equivalent Butterworth–Van Dyke (BVD) representations [27]. The remaining reactive elements, which do not belong to the resonator cores, are preserved in their original lumped form and act as auxiliary components within the overall network. As a result, the final realization corresponds to a hybrid topology that combines BVD-modeled acoustic resonators with additional lumped reactive elements. Fig. 6 illustrates a circuit transformations used to obtain the BVD model. This procedure will be further illustrated in Section V, where the nodal representation of the resulting topology is explicitly shown.

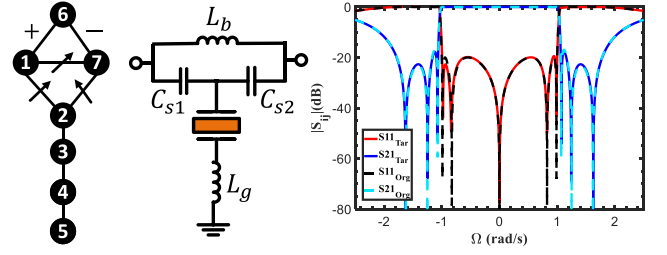


Fig. 7. Low-pass filter design example: nodal target network, AW circuit implementation and corresponding prototype domain S-parameter response of canonical (original) and reconfigured CM (target).

$$\begin{aligned}
 & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & -1.0000 & 0 & 0 & 0 & 0 \\ 0 & -1.0000 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 1.0000 & 0 & 0 \\ 1.0000 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.0000 & 0 \end{bmatrix} \\
 & \quad (a) \\
 & \begin{bmatrix} 1.5562 & -0.6108 & 0 & 0 & 0 & 0 & -0.9544 \\ -0.6108 & 1.2397 & 0 & 0 & 0 & 0 & -0.6108 \\ 0 & 0 & 0.2235 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 14.2615 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0612 & 0 & 0 \\ -0.9544 & -0.6108 & 0 & 0 & 0 & 0.7001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.5562 \end{bmatrix} \\
 & \quad (b)
 \end{aligned}$$

Fig. 8. Bandpass CM of the LPF target topology. (a) $\mathbf{M}_T^{\text{BP}}$. (b) $\mathbf{W}_T^{\text{BP}}$.

V. DESIGN OF UNCONVENTIONAL HYBRID AW FILTER TOPOLOGIES

In this section, the methodology developed in Sections II and III is applied to the synthesis of several AW filters with unconventional topologies, including low-pass, bandpass, and bandstop configurations. The versatility of the proposed approach is demonstrated through its ability to accommodate different design paradigms without modifications to the underlying formulation, apart from the corresponding definition of the filtering function (see Section IV-A). From a technological perspective, the introduced hybrid AW topologies enable wideband filtering responses ($\text{FBW} > 30\%$) while requiring moderate coupling levels.

A. Low-Pass Filter Design Example

A fifth-order GC filtering function is considered, with prescribed TZs located at $\Omega_{TZ} = [\pm 1.250, \pm 1.621, \pm 1.071]$, and designed to achieve a return loss of $RL = 20$ dB. From these specifications, the characteristic polynomials $P(s)$, $F(s)$, and $E(s)$ are computed, and the corresponding canonical CM is obtained, as shown in (20) and (21), at the bottom of the next page, together with their associated structural masks $\mathbf{S}_M$ and $\mathbf{S}_W$, and $\mathbf{S}_R$ in (22), as shown at the bottom of the next page, for the reconfiguration process in (16). Note that, in this case, the formulation in (18) is included to preserve the source and load terminations, since the generalized transformation matrix in (13) is employed to allow the modification of the source and load frequency-dependent terms as explained in Section II-A.

The corresponding nodal network and its AW circuit implementation are illustrated in Fig. 7. The transformation matrix $\mathbf{Q}_g$ obtained from the optimization problem in Section III, and reported in (22), is applied through the congruent mapping in (14), yielding the target bandpass coupling matrices $\mathbf{M}_T^{\text{BP}}$

[illegible]

TABLE II
EXTRACTED ELEMENTS FOR THE LPF

Element	L_a (nH)	C_a (pF)	C_0 (pF)
Resonator 1	10.201	0.143	1.543
C_{s1}, C_{s2}	2.051 pF	2.051 pF	
L_g, L_b	0.207 nH	1.315 nH	

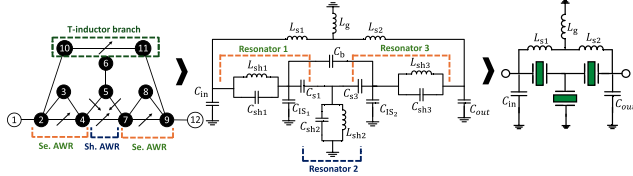


Fig. 10. Hybrid AW filter topology: target nodal topology and final circuit implementation combining BVDs with lumped reactive elements.

in the frequency range of 3.0–4.2 GHz, corresponding to a FBW of 33.8%. The transfer function is expressed in its dual-polynomial form with a specified return loss of 14, dB, TZs located at $\Omega_{TZ} = [0, \pm 1.32, \pm 1.029, \pm 0.71, \pm 0.62]$, and RZs located at $\Omega_{RZ} = [\pm 1 \pm 0.5j]$.

These specifications yield the canonical transversal CM $\mathbf{M}_{0BP}$ and the corresponding frequency-dependent CM $\mathbf{W}_{0BP}$ shown in (23), at the bottom of the page, and (24), as shown at the bottom of the next page, respectively.

The structural masks $\mathbf{S}_{MBP}$ and $\mathbf{S}_{WBP}$ are also included in the same equations.

$$\begin{bmatrix} 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 & -1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.0000 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & -1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0000 & 0 & 0 & -1.0000 & -1.0000 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -1.6991 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.0680 & 0 & -1.6991 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4839 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.6991 & 0 & 2.4885 & -0.3833 & 0 & -0.4061 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3833 & 12.4668 & 0 & -0.4235 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1527 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.4061 & -0.4235 & 0 & 3.2750 & 0 & -2.4455 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3546 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2.4455 & 0 & 2.6929 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0303 & -0.2436 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2436 & 1.0942 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)

Fig. 11. Bandpass CM of BPF target topology. (a) $\mathbf{M}_T^{BP}$. (b) $\mathbf{W}_T^{BP}$.

It should be noted that, in order to enable the subsequent association with the AW circuitual model through the grouping of frequency-dependent reactive elements (see Fig. 10), the interstage capacitances of the network must cancel out with the distributed contributions of the resonator nodes.

To achieve this, the model described in Section III must be complemented with additional structural constraints. The proposed methodology inherently allows the exploration of alternative solutions that enforce such cancellations. In the present case, this requires imposing that the combination of the FVCs and the self-capacitances of nodes 4 and 7 vanish, which is mathematically expressed as follows:

$$\begin{aligned} C_{IS_1} &= W(4, 4) + W(2, 4) + W(4, 7) + W(4, 5) = 0 \\ C_{IS_2} &= W(7, 7) + W(4, 7) + W(7, 9) + W(5, 7) = 0. \end{aligned} \quad (26)$$

$$\begin{aligned} \mathbf{M}_{0BP} &= \begin{bmatrix} 0 & 0.421 & 0.421 & -0.539 & -0.539 & -0.292 & -0.292 & 0.096 & 0.096 & 0.223 & 0.223 & 0 \\ 0.421 & 1.305 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.426 \\ 0.421 & 0 & -1.305 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.426 \\ -0.539 & 0 & 0 & -1.229 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.530 \\ -0.539 & 0 & 0 & 0 & 1.229 & 0 & 0 & 0 & 0 & 0 & 0 & 0.530 \\ -0.292 & 0 & 0 & 0 & 0 & 0.925 & 0 & 0 & 0 & 0 & 0 & 0.290 \\ -0.292 & 0 & 0 & 0 & 0 & 0 & -0.925 & 0 & 0 & 0 & 0 & 0.290 \\ 0.096 & 0 & 0 & 0 & 0 & 0 & 0 & 0.706 & 0 & 0 & 0 & 0.230 \\ 0.096 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.706 & 0 & 0 & 0.230 \\ 0.223 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.704 & 0 & 0.049 \\ 0.223 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.704 & 0.049 \\ 0 & 0.426 & 0.426 & 0.530 & 0.530 & 0.290 & 0.290 & 0.230 & 0.230 & 0.049 & 0.049 & 0 \end{bmatrix} \\ \mathbf{S}_{MBP} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{aligned} \quad (23)$$

With these definitions in place, the optimization procedure is carried out, yielding the transformation matrix $\mathbf{Q}$ in (25), as shown at the bottom of the page, which generates the target matrices shown in Fig. 11.

The equivalent lumped elements are derived from the entries of the bandpass frequency matrix $\mathbf{W}_T^{\text{BP}}$ and subsequently scaled in frequency with $f_c = 4.2$ GHz and $Z_0 = 50 \Omega$. Once the reactive elements are obtained, they are mapped onto their equivalent AW representation through the transformation illustrated in Fig. 6.

The extracted BVD parameters are summarized in Table III, while the corresponding S -parameter responses of both the reconfigured prototype and the final bandpass realization are depicted in Fig. 12(a).

TABLE III

EXTRACTED AW PARAMETERS OF THE BVD EQUIVALENT BPF MODEL

Element	L_a (nH)	C_a (pF)	C_0 (pF)
Resonator 1	5.587	0.355	0.513
Resonator 2	32.898	0.086	0.829
Resonator 3	5.702	0.333	0.636
$C_{\text{in}}, C_{\text{out}}$	0.28 pF	0.188 pF	
L_{s1}, L_{s2}, L_g	1.95 nH	2.07 nH	0.462 nH

In addition, to evaluate the impact of the lumped elements on the bandpass implementation, an additional circuit simulation was performed by assigning finite quality factors to the lumped components, $Q_L = 30$ for the inductors and $Q_C = 100$ for the capacitors. The resulting lossy response is also included

$$\begin{aligned}
 \mathbf{W}_{0_{BP}} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{S}_{W_{BP}} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned} \tag{24}$$

$$\begin{aligned}
 \mathbf{Q}_{BP} &= \begin{bmatrix} 1.000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.705 & 0.248 & -0.393 & -0.002 & -0.038 & 0.433 & 0.184 & -0.755 & 0.323 & 0.327 & 0 \\ 0 & 0.705 & -0.248 & -0.393 & -0.002 & 0.038 & 0.433 & -0.184 & -0.755 & -0.323 & -0.327 & 0 \\ 0 & -0.633 & 0.208 & 0.350 & -0.014 & 0.230 & 0.461 & -0.184 & -0.745 & 0.439 & -0.431 & 0 \\ 0 & -0.633 & -0.208 & 0.350 & -0.014 & -0.230 & 0.461 & 0.184 & -0.745 & -0.439 & 0.431 & 0 \\ 0 & 0.273 & 0.242 & -0.542 & -0.071 & -0.890 & -0.763 & -0.230 & 0.488 & -0.316 & 0.314 & 0 \\ 0 & 0.273 & -0.242 & -0.542 & -0.071 & 0.890 & -0.763 & 0.230 & 0.488 & 0.316 & -0.314 & 0 \\ 0 & -0.153 & -0.157 & 0.460 & 0.166 & 1.594 & -0.826 & -0.190 & 0.295 & 0.135 & 0.326 & 0 \\ 0 & -0.153 & 0.157 & 0.460 & 0.166 & -1.594 & -0.826 & 0.190 & 0.295 & -0.135 & -0.326 & 0 \\ 0 & -0.195 & 0.232 & 0.680 & -0.193 & 1.845 & -0.348 & 0.080 & 0.137 & -0.317 & -0.070 & 0 \\ 0 & -0.195 & -0.232 & 0.680 & -0.193 & -1.845 & -0.348 & -0.080 & 0.137 & 0.317 & 0.070 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.000 \end{bmatrix}
 \end{aligned} \tag{25}$$

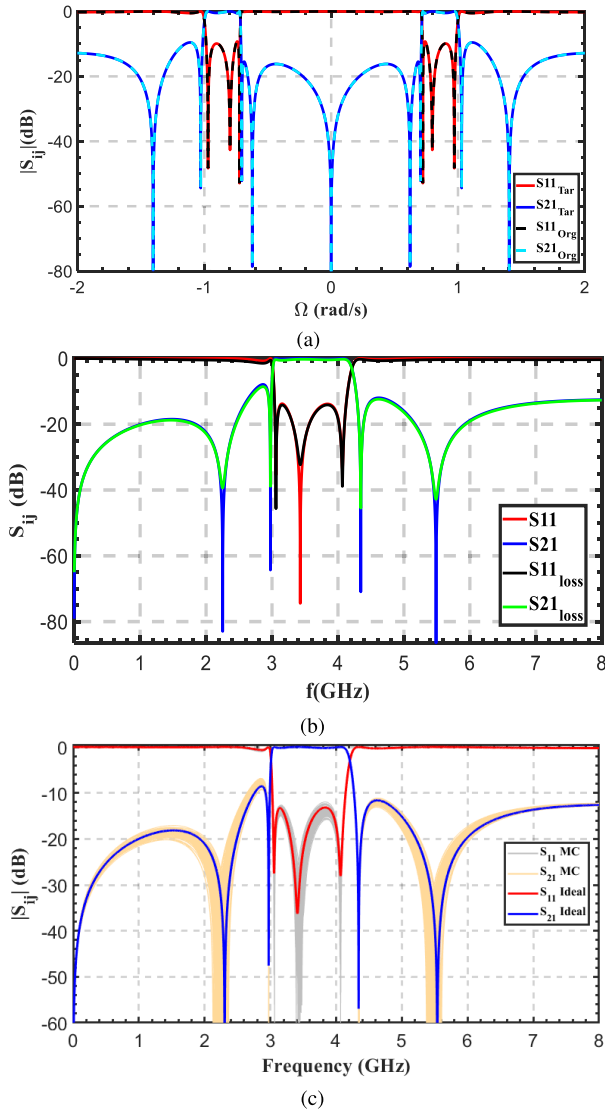


Fig. 12. S-parameter response of the bandpass filter design example. (a) Comparison between the synthesized canonical transversal CM and the reconfigured target CM. (b) Bandpass frequency response of the AW topology under ideal conditions and considering finite quality factors for the lumped elements. (c) Monte Carlo analysis of the lumped-element tolerances, showing the sensitivity of the proposed hybrid AW ladder.

in Fig. 12(b), where $S_{11,loss}$ and $S_{21,loss}$ are compared with the ideal response. As expected, the finite quality factor of the lumped elements mainly increases the in-band insertion loss, while the overall filtering behavior and the prescribed transmission-zero locations are preserved.

Furthermore, a Monte Carlo analysis is carried out to assess the sensitivity of the proposed bandpass topology to practical tolerances in the auxiliary lumped elements. In particular, the elements L_{s1} , L_{s2} , and L_g shown in Fig. 10 were randomly perturbed assuming a Gaussian distribution with a maximum variation of $\pm 5\%$ around their nominal values. The resulting response dispersion is shown in Fig. 12(c).

C. Bandstop Filter Design Example

The bandstop filter case design example will also serve as the experimental validation benchmark. For this reason, the

$$\begin{aligned}
 & \begin{bmatrix} 0 & -1.0000 & 0 & -1.0000 & 0 & 0 & 0 & 0 & 0 \\ -1.0000 & 0 & -1.0000 & 0 & 0 & 0 & 0 & 0 & 1.0000 \\ 0 & -1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.0000 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 1.0000 & 0 & -1.0000 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.0000 & 0 & 0 & 0 & 1.0000 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix} \\
 & \quad (a) \\
 & \begin{bmatrix} 1.4269 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.4269 \\ 0 & 6.5679 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1660 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.4766 & 0 & 0 & 0 & -0.9006 & 0 \\ 0 & 0 & 0 & 0 & 1.0526 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 19.4834 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1201 & 0 & 0 \\ 0 & 0 & 0 & -0.9006 & 0 & 0 & 0 & 1.4766 & 0 \\ -1.4269 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.4269 \end{bmatrix} \\
 & \quad (b)
 \end{aligned}$$

Fig. 13. Bandpass CM of BSF target topology. (a) $\mathbf{M}_T^{\text{BP}}$. (b) $\mathbf{W}_T^{\text{BP}}$.

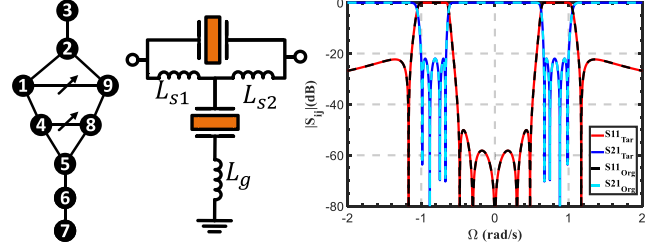


Fig. 14. Bandstop filter design example: nodal target network, AW circuit implementation, and corresponding prototype domain S-parameter response of canonical and reconfigured CM (target).

fabricated bandstop filter and its corresponding measurements are presented and discussed in Section VI.

VI. EXPERIMENTAL VALIDATION AND MEASUREMENTS

Following the same synthesis procedure, let us define an eighth-order dual-band transfer function symmetric around the origin with the set of normalized TZs $\Omega_{TZ} = [\pm 0.300, \pm 0.477, \pm 1.171, 0]$ rad/s and a prescribed return loss of $RL = 22$ dB. The corner passbands are located at $f_{c1} = 2.95$ GHz and $f_{c2} = 4.45$ GHz, resulting in a FBW of approximately $FBW = 41\%$. The characteristic polynomials are computed using the Remez-like algorithm described in [25], and their normalization constants ϵ and ϵ_r are interchanged. The resulting canonical matrices $\mathbf{M}_{0BSF}$ and $\mathbf{W}_{0BSF}$, are reported in (27) and (28), as shown at the bottom of the next page. In this case, the inner transversal configuration is adopted, and therefore the generalized transformation matrix $\mathbf{Q}_g$ introduced in (13) is employed together with the extended cost function including the $\mathbf{R}$ term defined in (18). The resulting transformation matrix $\mathbf{Q}_{BS}$ is reported in (29), as shown at the bottom of page 12. Applying the congruent mapping in (1) yields the target bandpass coupling matrices $\mathbf{M}_T^{\text{BP}}$ and $\mathbf{W}_T^{\text{BP}}$ shown in Fig. 13, which define the nodal representation of the synthesized BS network.

It is worth noting that, although the inner transversal configuration is adopted in this case, the terms W_{sL} , W_s , and W_L remain fixed from their definition in the canonical matrix. Consequently, as shown in $\mathbf{Q}_{BS}$ in (29), the structural form of the transformation matrix still follows the diagonal structure defined in (3). This property is independent of the particular formulation of the cost function and therefore remains valid regardless of the specific optimization framework employed.

[illegible]

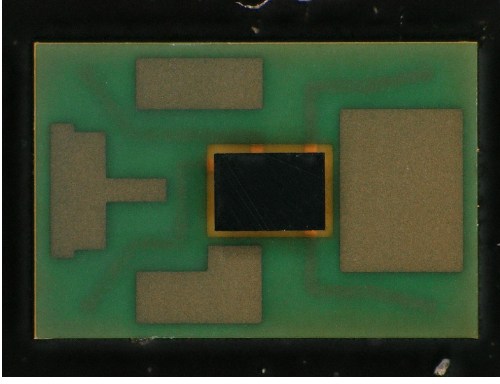


Fig. 16. Top view of the BAW-assisted filter, with the flipped die mounted on the laminate.

branches introduces an additional zero that is governed by the filtering function rather than the individual resonances. The fourth TZ arises from the combined effect of the grounding inductor L_g placed beneath the shunt resonator.

The fabricated prototype is implemented using BAW-SMR technology based on scandium-doped AlN thin films, providing an effective electromechanical coupling factor of $k_t^2 \approx 12.5\%$. The two acoustic resonators are co-simulated and integrated within a die of dimensions $750 \times 550 \mu\text{m}$.

The auxiliary inductive elements L_{s1} , L_{s2} , and L_g are realized externally on the laminate, enabling the hybrid implementation of the synthesized topology. The physical realization of the device is depicted in Fig. 16, which shows a top-view image of the assembled structure after the die has been flipped and mounted onto the laminate. The overall footprint of the prototype, including the supporting laminate, is $3 \times 2 \text{ mm}$. It should be noted that this area slightly exceeds the intrinsic size of the proposed filter topology, as the same laminate platform was shared with additional structures.

Fig. 17 compares the (S)-parameter responses obtained from circuit-level synthesis, electromagnetic (EM) simulation, and

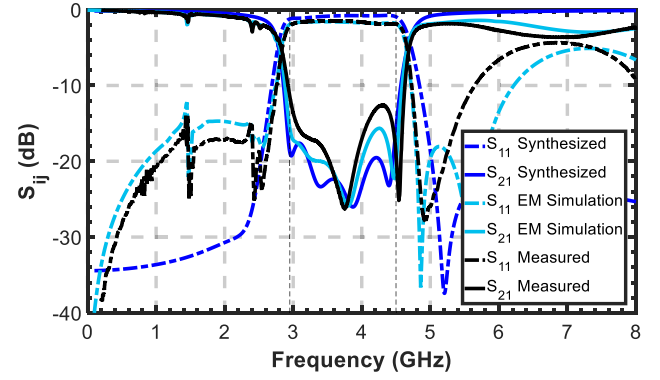


Fig. 17. S-parameter response of the fabricated bandstop filter.

experimental measurements of the fabricated BAW-assisted bandstop filter. In the circuit-level model, losses are introduced through a modified BVD (mBVD) representation, where finite quality factors are assigned to both the acoustic resonators and the lumped elements. The good agreement among the three responses validates the proposed synthesis and reconfiguration methodology, confirming the consistency between the synthesized circuit model and its EM and fabricated implementations.

From a performance perspective, the fabricated prototype achieves rejection levels ranging from approximately 15–26 dB across the stopband region. The observed differences in insertion loss between the synthesized response and the EM and measured results are mainly attributed to additional losses introduced by the evaluation board and the physical implementation, which are not captured in the ideal circuit model. Furthermore, spurious effects associated with the acoustic stack, such as parasitic modes introduced by the Bragg reflector, contribute to a degradation of the insertion loss, particularly at lower frequencies. Finally, it is important to emphasize that the proposed hybrid AW/lumped-element implementation is not intended to replace conventional ladder-type AW filters when standard ladder configurations are

$$\begin{aligned}
 \mathbf{Q}_{\text{BS}} &= \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.8122 & -0.2881 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.8122 & 0.2881 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.6967 & 0.6930 & -1.3357 & 0.0725 & 0.6967 & 0 \\ 0 & 0 & 0 & -0.6967 & -0.6930 & -1.3357 & -0.0725 & 0.6967 & 0 \\ 0 & 0 & 0 & -0.3299 & 0.2147 & 2.8209 & -0.2341 & 0.3299 & 0 \\ 0 & 0 & 0 & -0.3299 & -0.2147 & 2.8209 & 0.2341 & 0.3299 & 0 \\ 0 & 0 & 0 & 0.5366 & 0 & 0 & 0 & 0.5366 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
 \mathbf{S}_{\text{RBS}} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \tag{29}$$

sufficient. Rather, its main advantage lies in enlarging the available synthesis space toward unconventional topologies that can overcome some of the intrinsic limitations of purely ladder-based networks. Hybrid topologies provide additional degrees of freedom; for example, the ability of decoupling the transmission-zero placement from the resonator coupling coefficient, thereby enabling wideband responses with moderate (k_l^2) acoustic resonators.

This benefit comes at the cost of introducing auxiliary lumped elements, whose finite quality factors and tolerances may degrade the response. Moreover, such hybrid topologies can be more sensitive than ladder networks because the stored energy is not as evenly distributed among the different elements. Therefore, the proposed framework should be understood as a topology-exploration and synthesis tool that allows this tradeoff to be systematically evaluated, providing access to hybrid AW topologies with enhanced bandwidth and transmission-zero control while preserving compatibility with BVD-based acoustic resonator implementations.

VII. CONCLUSION

A direct bandpass numerical CM reconfiguration methodology for the synthesis of unconventional wideband AW filter topologies has been presented. By formulating the synthesis problem directly in the bandpass domain through a congruent transformation acting on the canonical coupling and frequency matrices, the proposed approach enables the systematic exploration and synthesis of nonclassical filtering structures without the need to delve into analytical synthesis steps. The method provides a unified framework capable of synthesizing low-pass, high-pass, bandstop, and bandpass AW topologies starting from a prescribed filtering function, while remaining compatible with hybrid implementations combining BVD resonators and auxiliary lumped elements.

The practical validity of the proposed approach has been demonstrated through the design, fabrication, and measurement of an unconventional BAW-SMR bandstop filter operating between 2.95 and 4.45 GHz, corresponding to a FBW of approximately 41%. The measured results exhibit rejection levels ranging from 15 to 26 dB, with good agreement with circuit and EM simulation, confirming the capability of the proposed methodology to accurately translate the synthesized response into a physical implementation. These results highlight the potential of the proposed framework for the realization of wideband AW filters with enhanced flexibility in topology exploration. Future work will focus on the extension of the methodology to higher order networks and multiport AW filtering structures.

REFERENCES

- [1] R. Ruby, "A snapshot in time: The future in filters for cell phones," *IEEE Microw. Mag.*, vol. 16, no. 7, pp. 46–59, Aug. 2015.
- [2] A. Hagelauer et al., "From microwave acoustic filters to millimeter-wave operation and new applications," *IEEE J. Microw.*, vol. 3, no. 1, pp. 484–508, Jan. 2023.
- [3] S. Tamiazzo and G. Macchiarella, "Synthesis of cross-coupled filters with frequency-dependent couplings," *IEEE Trans. Microw. Theory Techn.*, vol. 65, no. 3, pp. 775–782, Mar. 2017.
- [4] L. Acosta, E. Guerrero, P. Silveira, J. Verdú, and P. De Paco, "Synthesis methodology of aw filters for RF applications based on matrix rotations to overcome round-off errors," in *Proc. IEEE Int. Ultrason. Symp. (IUS)*, Jul. 2021, pp. 1–4.
- [5] A. Giménez, J. Verdú, and P. De Paco Sánchez, "General synthesis methodology for the design of acoustic wave ladder filters and duplexers," *IEEE Access*, vol. 6, pp. 47969–47979, 2018.
- [6] S. Amari and G. Macchiarella, "Synthesis of inline filters with arbitrarily placed attenuation poles by using nonresonating nodes," *IEEE Trans. Microw. Theory Techn.*, vol. 53, no. 10, pp. 3075–3081, Oct. 2005.
- [7] S. Tamiazzo and G. Macchiarella, "Synthesis of cross-coupled prototype filters including resonant and non-resonant nodes," *IEEE Trans. Microw. Theory Techn.*, vol. 63, no. 10, pp. 3408–3415, Oct. 2015.
- [8] R. J. Cameron, C. M. Kudsia, and R. R. Mansour, *Microwave Filters for Communication Systems: Fundamentals, Design, and Applications*. Hoboken, NJ, USA: Wiley, 2018.
- [9] R. J. Cameron, "General coupling matrix synthesis methods for Chebyshev filtering functions," *IEEE Trans. Microw. Theory Techn.*, vol. 47, no. 4, pp. 433–442, Apr. 1999.
- [10] G. Cao, Y. Zeng, H. Huang, and M. Yu, "A synthesis framework for microwave filter topologies utilizing predetermined reconfiguration path," *IEEE Trans. Microw. Theory Techn.*, vol. 73, no. 9, pp. 1–12, Sep. 2025.
- [11] Y. Zeng, H. Huang, Y. Wu, Y. Sun, Y. Xiang Xu, and M. Yu, "Coupling matrix reconfiguration based on matrix completion," *IEEE Trans. Microw. Theory Techn.*, vol. 72, no. 11, pp. 6563–6573, Nov. 2024.
- [12] J. Mesas, J. Verdú, and P. de Paco, "Synthesis of in-line fully canonical filters by solving a matrix completion problem under AW ladder constraints," *IEEE Microw. Wireless Technol. Lett.*, vol. 35, no. 9, pp. 1–4, Sep. 2025.
- [13] Y. Wu, Y. Zeng, and M. Yu, "On the reconfiguration of coupling matrix," *IEEE Trans. Microw. Theory Techn.*, vol. 72, no. 1, pp. 308–320, Jan. 2024.
- [14] Q. Wu, R. Ye, B. Liu, Y. Yang, and M. Yu, "Unified framework for coupling matrix reconfiguration through sequential householder transformation," *IEEE Trans. Microw. Theory Techn.*, vol. 73, no. 2, pp. 1051–1061, Feb. 2025.
- [15] J. Mesas et al., "A non-deterministic direct bandpass methodology for the synthesis of acoustic-wave topologies," in *Proc. IEEE Int. Microw. Filter Workshop (IMFW)*, Feb. 2026, pp. 1–3.
- [16] P. Zhang, Y. Zeng, Z. Zhang, H. Huang, and M. Yu, "Searching multisolutions in synthesis of high-order filters and multiplexers aided with multiobjective optimization," *IEEE Trans. Microw. Theory Techn.*, vol. 73, no. 8, pp. 5123–5132, Aug. 2025.
- [17] J. Liu, Y. Chen, W. Wei, Y. Dou, and K.-L. Wu, "Exhaustive synthesis framework of coupled resonator microwave bandpass filters with NRNs," *IEEE Trans. Microw. Theory Techn.*, vol. 74, no. 3, pp. 2238–2247, Mar. 2026.
- [18] D. M. Pozar, *Microwave Engineering*, 4th ed., Hoboken, NJ, USA: Wiley, 2012.
- [19] S. Cano, C. Caballero, O. Barrera, R. Lu, J. Verdú, and P. de Paco, "General synthesis methodology for acoustic wave ladder filters in the bandpass domain," *IEEE Trans. Microw. Theory Techn.*, vol. 73, no. 10, pp. 7055–7068, Oct. 2025.
- [20] S. Cano, J. Verdú, and P. de Paco, "Coupling matrix decomposition of acoustic wave ladder filters in the bandpass domain," *IEEE Trans. Microw. Theory Techn.*, vol. 74, no. 1, pp. 569–580, Jan. 2026.
- [21] J. Mesas, M. Faura, J. Verdú, and P. De Paco, "Broadband modeling using numerical coupling matrix reconfiguration methodologies for microwave filters," *IEEE Microw. Wireless Technol. Lett.*, vol. 36, no. 1, pp. 1–4, Jan. 2026.
- [22] P. Zhao and K. Wu, "Cascading fundamental building blocks with frequency-dependent couplings in microwave filters," *IEEE Trans. Microw. Theory Techn.*, vol. 67, no. 4, pp. 1432–1440, Apr. 2019.
- [23] R. A. Horn and C. R. Johnson, *Matrix Analysis*, 2nd ed., Cambridge, U.K.: Cambridge Univ. Press, 2012.
- [24] H. Qian and Y. Xu, "Robust large-scale coupling matrix reconfiguration via approximate Cayley transform," *IEEE Microw. Wireless Technol. Lett.*, vol. 35, no. 7, pp. 941–944, Jul. 2025.
- [25] G. Macchiarella, "Equi-ripple' synthesis of multiband prototype filters using a Remez-Like algorithm," *IEEE Microw. Wireless Compon. Lett.*, vol. 23, no. 5, pp. 231–233, 2013.
- [26] K. Schittkowski, "NLPQL: A fortran subroutine solving constrained nonlinear programming problems," *Ann. Oper. Res.*, vol. 5, no. 2, pp. 485–500, Jun. 1986.

- [27] S. V. Krishnaswamy, J. Rosenbaum, S. Horwitz, C. Vale, and R. A. Moore, "Film bulk acoustic wave resonator technology," in *Proc. IEEE Symp. Ultrason.*, Dec. 1990, pp. 529–536.



Joel Mesas (Graduate Student Member, IEEE) was born in Barcelona, Spain, in 2001. He received the B.S. and M.S. degrees in telecommunications engineering from the Universitat Autònoma de Barcelona (UAB), Barcelona, in 2022 and 2024, respectively, where he is currently pursuing the Ph.D. degree at the WavesLab Group.

His main research interests include advanced synthesis methodologies for microwave and acoustic-wave filters for wireless applications, with a particular emphasis on the systematic exploration of unconventional filter topologies through numerical reconfiguration techniques.



Santi Cano (Graduate Student Member, IEEE) was born in Sabadell, Spain, in 2000. He received the B.S. and M.S. degrees in telecommunications engineering from the Universitat Autònoma de Barcelona (UAB), Barcelona, Spain, in 2021 and 2023, respectively, where he is currently pursuing the Ph.D. degree at the WavesLab Group.

His main research interests include the synthesis of microwave filters and the synthesis and design of microwave acoustic filters for wireless communications applications.



Mario Faura (Graduate Student Member, IEEE) was born in Barcelona, Spain, in 2000. He received the B.S. (Hons.) and M.S. degrees in telecommunications engineering from the Universitat Autònoma de Barcelona (UAB), Barcelona, in 2021 and 2023, respectively, where he is currently pursuing the Ph.D. degree at the WavesLab Group.

His main research interests include the synthesis of microwave filters and the synthesis and design of microwave acoustic filters and multiplexers.



Laia Garcia (Graduate Student Member, IEEE) was born in Granollers, Spain, in 2000. She received the B.S. and M.S. degrees in telecommunications engineering from the Universitat Autònoma de Barcelona (UAB), Barcelona, Spain, in 2021 and 2023, respectively, where she is currently pursuing the Ph.D. degree at the WavesLab Group.

Her main research interests include the identification of design deviations in microwave acoustic filters, as well as their synthesis and design for wireless communication applications.



Sergio López de Pablo (Graduate Student Member, IEEE) was born in Spain, in 1997. He received the B.S. degree in telecommunications systems engineering and the M.S. degree in telecommunication engineering from the Universitat Autònoma de Barcelona (UAB), Barcelona, Spain, in 2022 and 2024, respectively, where he is currently pursuing the Ph.D. degree.

His research focuses on the design of GaN power amplifiers for microwave and millimeter-wave applications. His research interests include wideband matching networks, power-combining techniques, and RF power amplifier design for wireless and satellite communications.



Jordi Verdú (Senior Member, IEEE) was born in Sabadell, Spain. He received the M.S. degree in telecommunication engineering and the Ph.D. degree (cum laude) from the Universitat Autònoma de Barcelona (UAB), Barcelona, Spain, in 2006 and 2010, respectively.

From 2006 to 2010, he was a member of the antenna and microwave systems (AMSS). In 2010, he joined the RF System Group, European Spallation Source (ESS), Bilbao, Spain, as a Group Coordinator. From 2011 to 2012, he was with the École Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland, through a Marie Curie Grant. In 2013, he joined the Theory Signal Communication Group, Universitat Politècnica de Catalunya (UPC), Barcelona. He is currently an Associate Professor with the Universitat Autònoma de Barcelona, where he teaches microwave engineering courses and a member of the WavesLab Group. He has been extensively working on linear modeling of acoustic-wave devices, and new synthesis techniques and advanced configurations for acoustic-wave filters. His current research interests include the design of microwave devices and linear modeling, but also the design of active devices for space applications.

Dr. Verdú is the Vice-Chair of the MTT-6 MEMS and Microwave Acoustics Technical Committee.



Pedro de Paco (Senior Member, IEEE) was born in Badalona, Spain. He received the M.S. degree in telecommunication engineering and the Ph.D. (cum laude) degree from the Universitat Politècnica de Catalunya (UPC), Barcelona, in 1997 and 2003, respectively.

In 1998, he joined the Electromagnetic and Photonics Engineering Group (EEF), Signal Theory and Communications Department (TSC), UPC, with a grant from the Institut d'Estudis Espacials de Catalunya (IEEC), Castelldefels, Spain in a joint activity related with European scientific space mission Planck. He was a member of the LFI-Radiometer working group and the Planck Consortium. During his Ph.D. degree at UPC, he participated in several national and international research projects mostly related to microwave and millimeter-wave circuits and systems applied to the design and testing of remote sensing instruments and front-end point-to-multipoint broadband communication system. Since 2004, he has been an Associate Professor with the Universitat Autònoma de Barcelona, Barcelona, Spain, where he teaches applied electromagnetics and microwave engineering courses. He has advised eight Ph.D. students. From 2010 to 2013, he was the Deputy Director and a member of the Executive Board at the Telecommunication and System Engineering Department, UAB. From 2018 to 2019, he was a Visiting Researcher with the Microwave and RF Research Group, University of Colorado Boulder, Boulder, CA, USA. His main research interests include microwave filter synthesis and microwave acoustics filter synthesis and design, as well as microwave and radar systems and devices.

Dr. de Paco is a member of the MTT-6 RF MEMS and Microwave Acoustics Committee, and a reviewer of IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES and IEEE MICROWAVE AND WIRELESS COMPONENTS LETTERS. He serves as a member of the Technical Review Board for European Microwave Conference. He has been appointed as a Technical Expert for telecommunications equipment by the National Agency of Accreditation. He was awarded three times with merits in research and recognized with merits in university training by l'Agència de Qualitat, and awarded with Advanced Research by Generalitat de Catalunya.