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Preservice teachers' knowledge on area measurement

Sofía Caviedes[1], Genaro de Gamboa[1]and Edelmira Badillo[1]

[1]Autonomous University of Barcelona, Faculty of Education, Barcelona, Spain; sofia.caviedes@autonoma.cat; genaro.degamboa@uab.cat; edelmira.badillo@uab.cat

This study aims to characterise elements of specialised knowledge of a group of pre-service teachers using procedures related to the comparison and rearrangement of surfaces when solving area tasks. For this purpose, emphasis is placed on the subdomains of knowledge of topics and knowledge of the structure of mathematics. The written justifications and procedures that pre-service teachers used to solve two area tasks are analysed. The results indicate that when tasks condition the strict use of procedures related to the comparison and rearrangement of surfaces, written justifications supporting the procedures become more rigorous. These rigorous justifications appear, exclusively, when pre-service teachers mobilise specific geometric properties and principles, either implicitly or explicitly.

Keywords: *Area measurement, knowledge of the topics, knowledge of the structure of mathematics.*

Introduction

Area measurement has been identified as a problematic topic to both students and pre-service teachers (PST). The literature shows that students' difficulties related to the limited variety of procedures for solving area tasks are also evident in PST (Baturo & Nason, 1996; Caviedes, De Gamboa & Badillo, 2019; Chamberlin & Candelaria, 2018; Hong & Runnalls, 2020; Murphy, 2012). Hong and Runnalls (2020) show that PSTs have difficulties for accepting the conservation of area in non-prototypical figures, as they do not have numerical values to compare the areas of the triangles with the same base and height but different kinds of shapes, so they prioritise visual estimation without reasoned justification. The same authors emphasised that understanding the ideas behind area conservation would enable a better understanding of formulas for PSTs, helping them develop procedural fluency; based on the acquisition of the initial concepts. Similarly, Caviedes, De Gamboa, and Badillo (2019) point out that PSTs have a limited repertoire of strategies to solve area tasks, prioritising formulas rather than geometric procedures (procedures related to the comparison and rearrangement of surfaces) that may simplify a solving process.

This study assumes the specialised nature of mathematics teachers' knowledge in the sense of Carrillo et al. (2018) since it allows a first approach to characterise aspects of knowledge about specific mathematical concepts. In this context, we pose the following research question: *What specialised knowledge about area measurement do PSTs mobilise when solving tasks that require using geometric procedures?* Thus, we attempt to characterise elements of knowledge of topics (KoT) and knowledge of the structure of mathematics (KSM) in a group of PSTs when solving two tasks that require the use of geometrics procedures.

Theoretical framework

Amongst different possible ways of perceiving teachers` knowledge we consider the Mathematics Teachers` Specialised Knowledge -MTSK- (Carrillo et al., 2018). The MTSK model studies, mainly, the knowledge at stake in teachers' practice, but recent studies consider that it is possible to assume the MTSK model as a reference of the desirable components of a teacher's specialised knowledge, and as a consequence, as a first approximation of what PSTs should know for their future practice (Policastro, Ribeiro & Fiorentini, 2019). The MTSK conceptualisation is conceived as a theoretical and analytical tool to better understand teachers' knowledge specificities and it has been shown that MTSK model could be useful in conceptualising tasks for accessing and developing knowledge of procedures, representations and connections in certain concepts (Policastro, Mellone, Ribeiro & Fiorentini, 2019). Specifically, we are interested in two subdomains of knowledge - Knowledge of Topics (KoT) and Knowledge of the Structure of Mathematics (KSM). KoT includes teachers' knowledge of definitions (i.e., what is area mathematically speaking?); properties and their principles (i.e., the role of each element involved in solving an area task); the phenomenology or contexts of use (i.e., comparing and reproducing shapes; measuring, or sharing fairly); procedures (i.e., knowing how, when and why using certain procedures), and representations (i.e., the geometric, numerical and algebraic representations involved in solving area tasks).

The KSM refers to teachers' knowledge of connections, considering four categories: simplification connections and complexities connections, auxiliary connections, and transversal connections. For instance, knowing that the area of an unknown figure can be calculated by decomposing the area into known figures, such as triangles, rectangles, and-or squares is a simplification connection because figure decomposition can be a precursor to formulae; knowing that the area of a scalene right triangle can be calculated using Heron's formula is a complexity connection, because it involves more advanced mathematical knowledge, also considering the perimeter; knowing that the procedure of iterating units of measurement, lined up in rows and columns, can evoke the measurement procedure involving multiplying number of rows by the number of columns, is an auxiliary connection because it uses one procedure to introduce a different one; and knowing that an area model can be used as a basis for working on fractions and algebraic operations is a transversal connection since the concept of the area can relate different mathematical contents. Due to the scope of our work, we focus only on knowledge of auxiliary connections.

Method

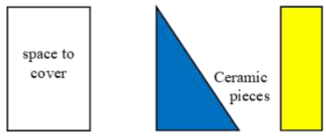
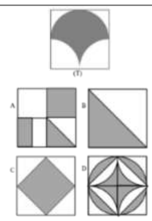
The study is situated in an interpretative paradigm with a qualitative approach (Bassey, 1990) and is part of broader research that seeks to characterise the specialised knowledge about area measurement in a group of PSTs. Data collection was carried out in the first term of the 2020-2021 school year. The participants were non-randomly selected, and they were 70 PSTs studying the third year of the Primary Education Degree at the Autonomous University of Barcelona. The PSTs had previous instruction on different procedures for measuring areas as part of their study program. Content analysis is carried out (Krippendorff, 2004) using two of the subdomains of the MTSK model: The KoT and the KSM. The MTSK defines, for each of these subdomains, specific categories. For the KoT, we consider representations, procedures and justifications, properties and principles, and intra-conceptual connections. For the KSM, the auxiliary connections. In each category there are indicators which have been constructed by the authors based on a previous study (Caviedes, De Gamboa & Badillo, 2020). The MAXQDA software is used to facilitate the process of assigning indicators to the PSTs responses.

Instrument and procedure

A semi-structured open-ended questionnaire (Bailey, 2007) was designed to be completed individually. The PSTs were asked to justify each procedure in writing. To solve the tasks the PSTs could use manipulative material (cut-outs as an annex to the questionnaire), as well as measuring instruments (ruler, square, protractor). The questionnaire consisted of 8 tasks and was structured as follows: three tasks responding to contexts of equal sharing, and comparison and reproduction of shapes (Tasks 1, 2, and 3); two measurement tasks (Tasks 4 and 5); one task to classify statements and one task to define the concept of area (Tasks 6 and 7); finally, one task to analyse students' responses (Task 8). In Tasks 1, 2, and 3 the use of calculations and measuring instruments was prohibited. The questionnaire was administered by the subject teacher in online format due to the COVID-19 health contingency. The official language to administrate and to answer the questionnaire was spanish and the translation was executed by a professional translator. The validation of the instrument considered external research experts, in-service and pre-service primary school teachers.

The PSTs had one week to answer it and send it in pdf or word format. In order to answer our research question, the analysis of the resolutions to Tasks 2 and 3 is presented (Table 1).

Table 1: Tasks proposed to the PST group

Task statement	**Graphical representation of the tasks**
TASK 2: Camila is making a ceramic mosaic with different colours. To finish her mosaic, she needs to cover a rectangular space, but she doesn't have any ceramic pieces with that shape. **What can Camila do to cover the missing space in her mosaic? Justify your answer.**	 (Adapted from Puig and Guillén, 1983)
TASK 3: Using the shaded surface of Figure (T) as a reference: **Which of the following figures has a shaded surface equivalent to Figure (T)? Why? Justify your answer.**	 (Elaborated by the authors)

Methods of analysis

Since we have not found any studies detailing the KoT and KSM indicators of area measurement, these have been constructed based on the results of a previous study that allowed for the construction of an epistemic configuration of area concept (Caviedes, De Gamboa & Badillo, 2021). From this epistemic configuration we define the KoT indicators to focus on the analysis of the PSTs' responses to the tasks. Each indicator was adapted to the categories that the MTSK model proposes for KoT (representations, procedures and justifications, properties and principles, and intra-conceptual connections) and allowed a deductive coding of the PST responses, with the support of MAXQDA software. The indicator corresponding to the KSM (auxiliary connections) emerges from the analysis of PSTs' responses to the questionnaire (Table 2).

Table 2: Categories of specialised knowledge

Categories of KoT and KSM	**Indicators**
Representations (R)	(R1) W*ritten:* use of adjectives such as "equal", "thinner" "wider", "double", "half" "a quarter" related to surfaces. (R2) *Manipulative:* use of physical objects or dynamic geometry software. (R3) *Geometric:* use of convenient decompositions to compare and-or estimate surfaces quantities.. (R4) *Symbolic:* use of the R^+ set to compare two or more surfaces, for counting units or adding up areas.
Procedures (P) and justifications (J)	(P1) Compare two or more surfaces directly by total and-or partial overlapping. (P2) Compare two or more surfaces indirectly by cutting and pasting. (P3) Decompose in a convenient way, graphically or mentally, two or more surfaces. (P4) Carry out movements of rotation, translation and superimposition of figures. (P5) Measure areas as an additive process by counting units and-or sub-units that cover the surface. (J1) The act of comparing two or more surfaces by placing one shape over another is useful for establishing equivalence and/or to include relationships

	(J2) The mental act of cutting the two-dimensional space into parts of equal area as a basis for comparing areas. (J3) The act of changing the shape of a surface does not change the area of the surface, as the figures can be decomposed and reorganised while keeping the same "parts".
Properties (Pp) and principles (Pr)	(Pp1) Use of conservation (Pp2) Use of accumulation and additivity (Pp3) Use of transitivity (Pr1) Use of the fact that a parallelogram that has the same base as a triangle, both placed between the same parallels, has twice the area of the triangle. (Pr2) Use of the fact that the triangles placed on equal bases and between same parallel are equal. (Pr3) Use of the fact that two polygons are congruent if their sides and angles are respectively equal or congruent. (Pr4) Use of the fact that every polygon can be broken down into triangles. (Pr5) Use of the fact that every triangle is equidecomposable from a parallelogram.
Auxiliary connections (Cau)	(Cau1) Use of a procedure or concept to introduce a new procedure or concept.

Results

Figures 1, 2 and 3 show examples of resolutions of PST 2 and PST 39, who mobilise specialised knowledge from the KoT and KSM subdomains. These resolutions are considered representative of PSTs' set that evidence mobilization of these subdomains, as these are the type of resolutions that allows for the emergence of different indicators from PSTs' resolutions. Figure 1 shows indicators of KoT in Task 2.

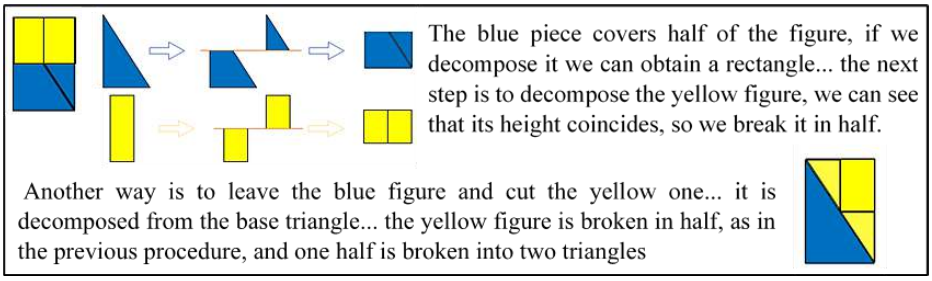


Figure 1: PST 2 resolution for Task 2

PST 2 uses written representations (R1) in her justifications, evidencing use of (J1) and (J2), as she superimposes the surfaces, as well as breaking them, in order to compare them. When breaking surfaces and rearranging their parts, she makes use of (J3). PST 2 uses manipulative representations (R2) to make visible decompositions and reorganizations of figures; and geometric representations (R3) as it compares surfaces indirectly (P2) and decomposes the surface in a convenient way (P3). In addition, she performs rotation and translation movements of the parts (P4), in order to check that both the triangle and the rectangle correspond to half of the piece that needs to be covered. PST 2 also shows an implicit use of the transitivity property (Pp3), while she compares between the surface to be covered and those represented by the pieces. Furthermore, PST2 shows an implicit use of the properties of accumulation and additivity (Pp2) and conservation (Pp1), as PST2 recognizes that figures can be decomposed and recomposed into other figures, while retaining the same "parts". Finally, it is possible to infer that PST 2 recognizes that a triangle is equidecomposable to a parallelogram (Pr5); that is, that a triangle can be decomposed into a finite number of polygons and form a parallelogram (and vice versa), conserving the area. Similarly, she implicitly recognises that a parallelogram with the same base and height as a triangle, both placed between the same parallels, is twice as large as the triangle (Pr1).

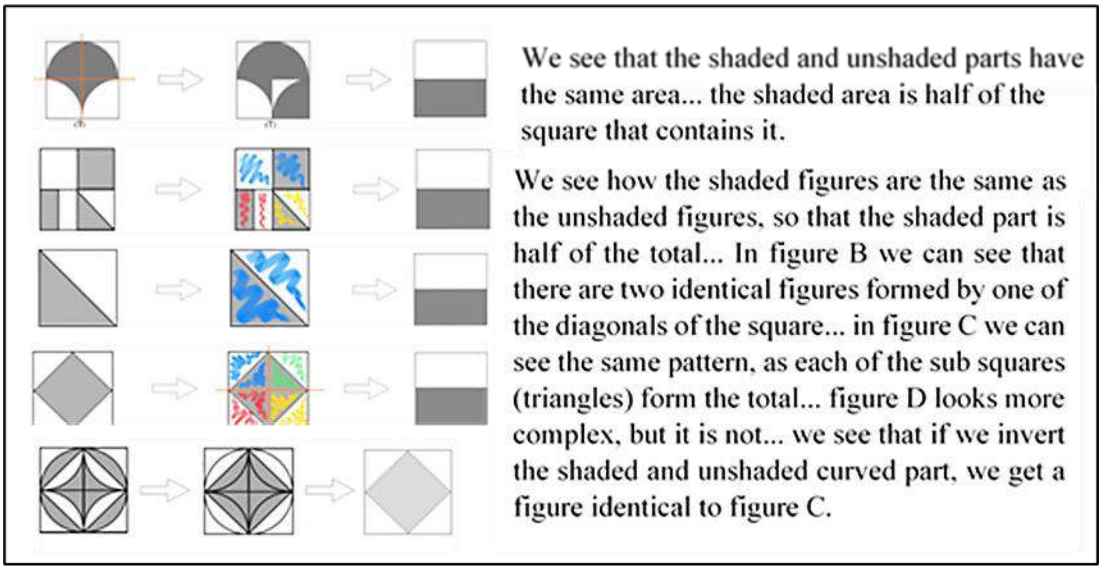


Figure 2: PST 2 resolution for Task 3

Figure 2 shows indicators of KoT knowledge in Task 3. It can be seen that PST 2 uses written (R1), manipulative (R2) and geometric (R3) representations, as she decomposes each of the figures in a convenient way (P3) and performs rotation and translation movements (P4) to verify and illustrate, manipulatively, that each of the figures is equivalent to the model figure. Thus, PST 2 manifests an implicit use of the properties of accumulation and additivity (Pp2) and conservation (Pp1), as she rearranges the shaded surfaces into rectangles that represent half of the square containing them. The property of transitivity (Pp3) is made explicit when PST 2 says that "if Figure D is equal to Figure C, and Figure C is equal to the model, then Figure D is equivalent to the model." PST 2 recognizes that triangles placed on equal bases and between the same parallels are equal (Pr2) and that every polygon can be decomposed into triangles (Pr4).

It is inferred that the PST 2 also makes use of (J2) which indicates that the mental act of cutting two-dimensional space into parts of equal area serves as a basis for comparing areas; and of (J3), since the PST 2 recognizes that changing the shape of a surface does not change the area of the surface.

Figure 3 shows indicators of knowledge of the KoT and KSM of PST 39 in Task 3. The PST 39 uses symbolic representations (R4) in a fractional register; that is, she uses fractions to compare the shaded area of each of the figures, and sets the shaded fraction relative to the total area. PST 39 also shows knowledge of addition of fractions with different denominators and simplification of fractions, showing an auxiliary connection (Cau1) to this concept. By rearranging the parts, PST 39 implements the procedure of surface decomposition (P3) and rotation and translation movements (P4). In this sense, it is inferred that PST 39 recognizes the properties of conservation (Pp1) and accumulation and additivity (Pp2). In turn, the comparisons made between the figures are associated with the property of transitivity using (Pp3). The procedures allow us to infer an implicit use of (J2), since PST 39 recognizes the usefulness of cutting the two-dimensional space.

T= $\frac{1}{2}$ of the figure. We can rearrange the bottom parts and fit them into the upper corners.

A= $\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

B= $\frac{1}{2}$ of the figure

C= $\frac{1}{2}$ of the figure

D= $\frac{1}{2}$ of the figure. D = C

Figure 3: PST 39 resolution for Task 3

Due to the geometric nature of the tasks, the results showed that PTSs solve Tasks 2 and 3 mostly using geometric type representations (R3) that are materialised through convenient surface decomposition procedures (P3), or the use of rotation, translation, and superimposition movements of figures (P4). These procedures, in turn, are supported by some of the properties as the mental act of cutting the two-dimensional space into parts of equal area serves as a basis to compare areas (J2); or that changing the shape of a surface does not produce changes in its area, since the figures can be decomposed and reorganized while retaining the same "parts" (J3).

The resolutions presented in the analysis show the way in which the PSTs mobilise knowledge about the different categories of the KoT and KSM, but do not allow us to identify the tendency of PSTs at the time of solving the task. For this reason, we consider it appropriate to show the frequency of each of the categories mobilised by the PSTs. Table 3 details this frequency and it can be seen that only 11 PSTs establish auxiliary connections (KSM), which are linked to KoT and associated with knowledge about fractions. Thus, knowledge about fractions is made explicit through the use of convenient surface decomposition procedures (P3), or the use of rotation, translation and superposition of figures (P4). Moreover, this type of knowledge is associated with a context that requires establishing equivalence or inclusion relations between different surfaces, linked to the properties of conservation (Pp1), transitivity (Pp3) and accumulation and additivity (Pp2). In this way, the relationship between KoT and the establishment of auxiliary connections becomes clear.

Table 3: Categories of specialised knowledge mobilized by PST (N=70)

Code	**Frequency**	**Code**	**Frequency**
P3	57	R4	13
Pp3	57	P2	11
Pp2	57	Cau 1	11
Pp1	57	R2	6
P4	56	Pr1	3
R3	56	J1	3
J3	56	No response	3
Pr5	56	Pr4	3
J2	53	Pr3	2
Pr2	51	P1	2

Discussion and final remarks

This study focused on the characterisation of PST's knowledge related to the topics and the structures of mathematics, according to the MTSK conceptualization. Results show a relation between descriptive answers that do not justify what is done and why it is done with the lack of use of geometric principles. As it can be seen in the examples presented in the above section (Figures 1, 2 and 3), PST 2 and PST 39 were able to justify and support the procedures they were using based in some of the geometric principles, such as: two polygons are congruent if their sides and angles are respectively equal or congruent (Pr3); every polygon can be broken down into triangles (Pr4); and a parallelogram that has the same base as a triangle, both placed between the same parallels, has twice the area of the triangle (Pr1). However, these principles are not mentioned in most of the PST's resolutions and written justifications, revealing a gap in their KoT and a link between the lack of this kind of knowledge and descriptive answers. Regarding KSM, the auxiliary connections that emerge from the PSTs responses showed a close relationship between knowledge about fractions and the use of procedures, properties and representations of a geometric nature. Although some research highlights the difficulties of PSTs in accepting area conservation (Hong & Runnalls, 2020), in the present study the use of this property is implicit in the justifications and-or procedures used and PSTs do not present major difficulties. This may be due to the time PSTs had to solve the questionnaire, or to the geometric nature of the tasks themselves, since by restricting the use of calculations and measuring instruments, PSTs are forced to use procedures of a geometrical nature.

The indicators proposed for the KoT subdomain serve as a reference of what PSTs should know for their future practice (Policastro, Mellone, Ribeiro & Fiorentini, 2019), as they allow detailing different representations, procedures and justifications, properties and principles underpinning area measurement. This could provide hints for PSTs trainers on how to propose tasks to promote the mobilisation of specialised knowledge, gradually increasing the indicators of knowledge to be developed. The need for further research seems evident, specifically in both at enriching the conceptualisation theoretically and in conceptualising tasks for developing a deeper knowledge on area measurement processes. It can also be seen that there is a need to explore the potential of MTSK model as a tool for promoting and developing an understanding of area measurement in relation to different kind of connections and in relation to other sub-domains of knowledge that have not been considered in this study.

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