

Eighth quarterly report of the contract
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1967)

Putting

$$M_n = \inf_{r>0} \frac{e^{W(r)}}{r^n}$$

we can define in the complex plane $d=a+ib$ the domain

$$H_n^{(0)} = \{d: a^2 - b^2 \leq M_n^2\}$$

bounded by a hyperbola. Moreover if we suppose that $H_n^{(m)}$ is defined, we define $H_n^{(m+1)}$ by

$$d \in H_n^{(m+1)} \iff d e^{-i\pi/4} \in H_n^{(m)}$$

Finally we shall write

$$G_n = H_n^{(0)} \cap H_n^{(1)} \cap H_n^{(2)} \cap H_n^{(3)}$$

and it is very easy to prove that G_n is a bounded domain

Let

$$F(z) = \sum d_n z^n$$

be an entire function. If $F \in S_w$ (see third report of the contract N62558-4161) and if

$$d_n \in G_n \quad \text{for } n = 0, 1, 2, \dots$$

we shall say that $F \in Z$. According to the definition of the space $S_{w,t}$ (fourth report of the contract N62558-4161) from the properties of $W(r)$ it follows

LEMMA 6.- There exists a function $t(r)$ ($t(r) > 0$ and $t(r) \rightarrow 0$ as $r \rightarrow \infty$) which depends only on $W(r)$ such that $Z \subset S_{w,t}$

On the other hand let Q_n be the square of the complex plane $d = a + ib$ such that $d \in Q_n$ if and only if

$$|a| \leq M_n^2 \quad |b| \leq M_n^2$$

Therefore from the definition of G_n it follows that

$$d_n \in G_n \iff d_n^2 \in Q_n.$$

Hence if we write $W_0(r) = 2W(r^{1/2})$,

$$F(z) = \sum d_n z^n$$

and

$$f(z) = \sum d_n^2 z^n$$

there exists a $t_0(r)$ ($t_0(r) \geq 0$ and $t_0(r) \rightarrow 0$ as $r \rightarrow \infty$) such that

$$F \in Z \iff f \in S_{w_0, t_0}^*$$

(for the definition of $S_{w, t}^*$ see sixth report).

In a similar form as in the seventh report we can affirm that if $F \in Z$ the family of all functions

$$\sum d_n z^n$$

form an equivalence class $R[F] \subset Z$ and with the same hypothesis and using a proof similar to that of the theorem XXV, we can prove.

THEOREM XXVI.- The spaces Z/R and S_{w_0, t_0}^* ~~XXXXX~~ are homeomorphic.

In the sixth report I have defined in S_{w_0, t_0}^* ^ametric outer measure such that the measure of S_{w_0, t_0}^* is = 1. Hence following

Theorem XXVI I have defined in Z/R a metric outer measure such that the measure of Z/R is $= 1$.

As in the second report we shall represent by $\{r_n(\sigma)\}$ the Rademacher's functions and we know that if $\sigma \in \beta$ the function

$$\sum r_n(\sigma) a_n z^n$$

belongs to $R[F]$. If the correspondence between β and $R[F]$ were always one-to-one we should be able to use Fubini's theorem in order to obtain a measure on Z , but when F has some $a_n = 0$ the correspondence between β and $R[F]$ is not one-to-one. Perhaps in the next quarter I shall try to remove this difficulty.

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