

Fourth quarterly report of the contract

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Almost the same proof of theorem V permits us to obtain the following result:

Theorem IX.- If

$$F(z) = \sum a_n z^n \in S_W$$

there exists a sequence  $\{n_k\}$  of natural numbers such that if  $\varepsilon_n = -1$  when  $n \in \{n_k\}$  and  $\varepsilon_n = 1$  when  $n \notin \{n_k\}$  then

$$F_1(z) = \sum \varepsilon_n a_n z^n \in B_W.$$

From the relations  $B_W \subset S_W \cap B \not\subset B_W$  it follows that the conclusion of this theorem is stronger than the conclusion of theorem V.

In the third report I said that "it would be interesting to know if  $S_W$  is of second category". In this quarter I have been able to prove that, contrarily to what I implicitly supposed probable,  $S_W$  is of first category. This result has theorem VIII as a corollary without interest. Hence the space  $S_W$  is not useful for my objective.

Let  $t(r)$  be a given continuous function such that  $t(r) > 0$  and  $t(r) \rightarrow 0$  as  $r \rightarrow \infty$ . Let  $H_{W,t}$  represent the set of the entire functions  $F(z) = \sum a_n z^n$  such that if  $\mathcal{F}(z) = \sum |a_n| z^n$  then

$$\log_2 M(r, \mathcal{F}) \leq (1 + t(r)) \log W(r)$$

Now if we write

$$S_{w,t} = H_{w,t} \cap S_w$$

$$A_{w,t} = H_{w,t} \cap A_w$$

$$B_{w,t} = H_{w,t} \cap B_w$$

it follows that

$$A_{w,t} \cap B_{w,t} = \emptyset,$$

$$A_{w,t} \cup B_{w,t} = S_{w,t}$$

i.e.,  $B_{w,t}$  is the complement of  $A_{w,t}$  respect  $S_{w,t}$ .

If now we suppose that the topology of  $S_{w,t}$  is the relativization of the topology of  $S_w$  to  $S_{w,t}$  it is known that:

Theorem X.-  $S_w$  and  $S_{w,t}$  are metrizable.

It is also easy to prove:

Theorem XI.-  $S_{w,t}$  is of second category.

This theorem does not contradict the result which affirms that  $S_w$  is of first category since it is known that a subspace of a space of first category can be of second category when it is considered relatively to itself.

Finally we can obtain:

Theorem XII.-  $B_{w,t}$  is a dense  $G_\delta$  set of second category.

Theorem XIII.-  $A_{w,t}$  is a set of first category.

Theorems XII and XIII are not topological consequences from theorems VII and VIII respectively and it is necessary to prove them. The proofs are similar to the proofs of theorems VII and VIII but do

not depend on them.

Theorems XI and XIII permit us to affirm that the functions  $F \in A_{w,t}$  are "exceptional" in  $S_{w,t}$ . On the contrary theorem VIII does not permit us to affirm that the functions  $F \in A_w$  are "exceptional" in  $S_w$  since  $S_w$  is of first category, although I think that the affirmation is true in some sense. I wish to remark that the functions  $F \in A_{w,t}$  are functions which have an exceptional function in the sense of Borel- Lindelöf.

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F. Sunyer Balaguer  
 Angel Guimera 66 pral. 2º  
 Barcelona - 17, Spain