Sixth quarterly report of the contract

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As a first consequence of lemma 1 (fifth report) I shall state a more general form of theorem VII, viz.

Theorem X.- If

1º $\{\lambda_n\}$ is such that $0 \leq \lambda_n < \lambda_{n+1}$ and $\delta < g$(¹)

2º In the half-strip $\Delta' = \{|\sigma| < g,\ t > 0\}$, F(s) is holomorphic and the linear combinations $\varphi(s) \in \Phi'$ (fourth report) represent F(s) in Δ' with the logarithmic b-precision p(t), with $b > 2\pi B$

3º The hypothesis $A_1(g, p(t))$ holds, where A_1 is an adequate hypothesis of adherence

then F(s) is holomorphic in the strip $|\sigma| < g$ and almost periodic in the $|\sigma| < g - \delta$

This theorem is a consequence of lemma 1 and of the properties of the index of condensation δ of $\{\lambda_n\}$.

From lemma 1 it follows also that in theorem V (third report) the condition

$$(1) \qquad B \cos\alpha < \lim g(u) - \bar{D}^*$$

can be replaced by

$$(2) \qquad 2B \cos\alpha < \lim g(u)$$

provided that we suppose

$$(3) \qquad b > 2\pi B \sin\alpha$$

instead of

(¹) According to Bernstein [1,p.26] from $\delta < g < \infty$ it follows $B < \infty$.

(4) $$b > 2\pi \overline{D}^{*}.$$

Evidently the condition (1) is more restrictive than (2) if

(5) $$\cos\alpha < \overline{D}^{*}/B$$

and the (4) is more restrictive than (3) if

(6) $$\sin\alpha < \overline{D}^{*}/B$$

The conditions (5) and (6) can only be satisfied simultaneously if $\overline{D}^{*}/B > 1/\sqrt{2}$, and hence only in this case the lemma 1 can give a result containing theorem V. In the other cases the new result does not contain the theorem V but also in general this theorem does not contain the new result.

In theorem VI we can make the same substitution.

Finally I shall state the result that it follows from lemma 1 when $\alpha = 0$; this result is that which corresponds to theorem IV (second report)

Theorem XI.- If

1º $\{\lambda_n\}$ is such that $0 \leq \lambda_n < \lambda_{n+1}$ and $B < \infty$

2º $\Delta = \{\sigma > \sigma_0,\ |t| < \pi g(\sigma)\}$, where $g(\sigma)$ is a continuous function of bounded variation such that $g(\sigma) > B$, $\lim g(\sigma) > B$ and $g(\sigma) > 2B$ in an open interval $\sigma_1 < \sigma < \sigma_1 + \varepsilon$ with $\varepsilon > 0$ and $\sigma_1 > \sigma_0$

Then $F(s) \in W(\Delta, \{\lambda_n\}, b, A_2)$, with $b > 0$ and where A_2 is an adequate hypothesis of adherence, if, and only if,

(i) $F(s)$ is holomorphic in H_{σ_0} (same notation that in the second report).

(ii) There exists a Dirichlet series $\sum d_n e^{-\lambda_n s}$ and a sequence $\{n_k\}$ of natural numbers such that

$$\lim S_{n_k}(s) = F(s)$$

uniformly in every domain

$$\sigma \geq \sigma_0 + \varepsilon \qquad \left|\frac{t}{\sigma - \sigma_0}\right| \leq C$$

for any $\varepsilon > 0$ and any $C>0$, where

$$S_{n_k}(s) = \sum_{1}^{n_k} d_n e^{-\lambda_n s}$$

and where the sequence $\{n_k\}$ depends only on $\{\lambda_n\}$.

In the next quarter I shall begin to study how the results obtained in this and in previous reports change when $\{\lambda_n\}$ is not formed by real numbers.

REFERENCES

1.- V. Bernstein, Leçons sur les progres récents de la theorie des series de Dirichlet (Paris 1933).

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