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I said in the tenth report that the definition of condition $B_0(g,P,\{\lambda_n\})$ (or B_1) supposes that $P(s,R,F)\to 0$ only in $\{\sigma>\sigma_0, t=0\}$ while the condition $A(g(\sigma),p(\sigma),\{\lambda_n\})$ supposes that $P\to 0$ in all the half-strip $\{\sigma>\sigma_0, |t|<\pi g-\pi\overline{D}^*\}$, but B_0 (or B_1) supposes that $P\to 0$ more rapidly than in the condition A. Although if by means of the method used in the case of the condition A we wish to prove a result with the hypothesis that $P\to 0$ only in a half-strip of very little width, then $P\to 0$ more rapidly than in the condition B_0 (or B_1). Hence in general the condition B_0 (or B_1) is weaker than the A.

On the other hand, if we suppose that the sequence $\{\lambda_n\}$ is formed by natural numbers, some theorems in the previous reports have interesting corollaries; for example:

Corollary of theorem IV (second report).- Let $\{n_k\}$ be a sequence of natural numbers of upper mean density $\overline{D}^*$ and of maximum density $B<\infty$. Let Φ_0 be the class of the linear combinations of $\{z^{n_k}\}$. If $f(z)$ is holomorphic in the domain $\{0<|z|<1, |\arg z|$

$<\pi\gamma\}$ where $\gamma > \bar{D}^*+B$ and if

$$\operatorname*{Imf}_{\psi\in\Phi_0}\ \sup_{\substack{\beta r<|z|<r\\ |\arg z|<\pi\gamma}} |f(z) - \psi(z)| < e^{-p(r)}$$

where $\log\beta < -2\pi\bar{D}^*$ and

$$\int_0 p(r)\, r^{\frac{1}{2(\gamma-\bar{D}^*-\varepsilon)}-1}\, dr = \infty$$

where $0<\varepsilon<\gamma-\bar{D}^*-B$, then $f(z)$ is holomorphic in $|z|<1$.

Theorem V has a similar corollary where the domain $\{0<|z|<1,\ |\arg z|<\pi\gamma\}$ is changed by a domain bounded by logarithmic spirals instead of the two parts of rays.

Finally theorem VI for $\alpha=\pi$ has the following corollary:

Corollary of theorem VI (third report).- Let $\{n_k\}$ be a sequence of natural numbers of upper mean density $\bar{D}^*$ and maximum density $B<\infty$. Let Φ_0 be the class of the linear combinations of $\{z^{n_k}\}$. If $f(z)$ is holomorphic in the domain $\{1<|z|,\ |\arg z|<\pi\gamma\}$ where $\gamma>\bar{D}^*+B$ and if

$$\operatorname*{Imf}_{\psi\in\Phi_0}\ \sup_{\substack{r<|z|<\beta r\\ |\arg z|<\pi\gamma}} |f(z) - \psi(z)| < e^{-p(r)}$$

where $\log\beta > 2\pi\bar{D}^{\bullet}$ and

$$\int^{\infty} p(r) r^{\frac{-1}{2(\gamma-\bar{D}^{\bullet}-\varepsilon)}-1} dr = \infty$$

where $0<\varepsilon<\gamma-\bar{D}^{\bullet}-B$. then $f(z)$ is an entire function.

For the other values of α we have similar results but the parts of the two rays of the boundary become two logarithmic spirals.

Evidently these corollaries are closely related with theorems VIII and IX (fourth and fifth reports) and we can state a similar corollary of theorem XI (sixth report).

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