

Second quarterly report of the contract

N62558-4161 NR 043- 266 (1 October - 31

December 1964)

In order to obtain for the entire functions of infinite order some results like those theorems that I have stated for the finite order [3] it has been necessary for me to improve the orders of King-Lai Hiong [1], i.e. to modify the properties of the function $W(r)$ that I utilized in my previous report.

Now I suppose that the functions $W(r)$ verify the following conditions.

1° $\log W(r)$ is an increasing convex function of $\log r$

$$2^\circ \quad W \left(r \left(1 + \left(\frac{\log r}{\log W(r)} \right)^{\xi(r)} \right) \right) \leq (W(r))^{1+o(1)}$$

3° $\xi(r) > 0$ and $\xi(r) \rightarrow 0$ as $r \rightarrow \infty$

$$4^\circ \quad \left(\frac{\log r}{\log W(r)} \right)^{\xi(r)} \rightarrow 0 \text{ monotonously as } r \rightarrow \infty$$

From these conditions it follows

$$W \left(r \left(1 + \left(\frac{\log r}{\log W(r)} \right)^{\xi(r)} \right)^k \right) \leq (W(r))^{1+o(1)}$$

where k is a fixed natural number.

It is evident that any function $W(r)$ which verifies the fore-said conditions also verifies the conditions of King-Lai Hiong. The-

refere by means of this function $W(r)$ we may define the space S_1 as in the previous report.

On the other hand it can be proved that for any entire function $F(z)$ of infinite order there exists at least one μ function $W(r)$ which satisfies the foresaid conditions and such that

$$0 < \limsup \frac{\log_2 M(r, F)}{\log W(r)} < \infty$$

where $\log_2 X = \log \log X$.

Hence for any entire function $F(z)$ of infinite order there exists at least one space S_1 and a point p of this space such that $F(z)$ belongs to the class corresponding to p .

Given a space S_1 we shall say that $p \in A$ if in the class corresponding to p there is a $F(z)$ such that

$$\limsup \frac{\log n(r, 1/F)}{\log W(r)} < \limsup \frac{\log_2 M(r, F)}{\log W(r)}$$

where, following Nevanlinna, $n(r, 1/F)$ is the number of zeros of $F(z)$ in the circle $|z| < r$.

The set B is the complement of A , i.e., $B = S_1 - A$. Now we can state:

Theorem III.- The set B is an open and dense set in S_1

Theorem IV.- Every point of A is an isolated point of A .

Really the proof of theorem III gives a more precise result of which theorem III is a corollary. This precise result is an improvement of a classic theorem. Besides ^{with} the new properties of the functions $W(r)$, theorem III follows from a theorem of Rauch [2] which does not happen with the former properties.

Hitherto I have been unable to prove the following theorem, which I think is true.

If

$$F(z) = \sum a_n z^n$$

is an entire function corresponding to a $p \in S_1$ there exists a sequence $\{n_k\}$ of natural numbers such that if $\varepsilon_n = -1$ when $n \in \{n_k\}$ and $\varepsilon_n = 1$ when $n \notin \{n_k\}$ then

$$F_1(z) = \sum \varepsilon_n a_n z^n$$

belongs to a class corresponding to a $p \in B$ of S_1

In the next quarter my main objective will be to try to prove this theorem which I think is interesting.

References

- 1.- Hiong, K.L. - Sur les fonctions entières et les fonctions méromorphes d'ordre infini (Jour. Math. 9^e serie, t.14, 1935).
- 2.- Ruch, A. - Extensions de théorèmes relatifs aux directions de Borel des fonctions méromorphes (Jour de Math. 9s., t.12, 1934)
- 3.- Sunyer Balaguer, F. - Propiedades de las funciones enteras representadas por series de Taylor lagunares (Orden finito) (Collectanea Math. vol.II, 1949).

Barcelona 31 December 1964

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