

Third quarterly report of the contract
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In this last quarter I have been able to prove the conjecture that I have stated in the last report, i.e.:

Theorem V.- If $F(z) = \sum a_n z^n$ is an entire function corresponding to a $p \in S_1$, there exists a sequence $\{n_k\}$ of natural numbers such that if $\varepsilon_n = -1$ when $n \in \{n_k\}$ and $\varepsilon_n = 1$ when $n \notin \{n_k\}$ then $F_1(z) = \sum \varepsilon_n a_n z^n$ belongs to a class corresponding to a $p \in B$ of S_1 .

Remark.- If in this theorem $p \in B$, it is sufficient not to change the sign of any coefficient or to change the sign of only a finite number of them.

As I said in the second report theorems III and IV follow from known results. In order to obtain more precise and completely new results I think that I ought to consider other spaces and other topologies.

Given a function $W(r)$ which verifies the conditions stated in the second report I consider the space S_W of all entire functions $F(z)$ such that

$$\limsup \frac{\log_2 M(r, F)}{\log W(r)} = 1$$

(Given an entire function $F(z)$ of infinite order there is at least one W such that $F(z) \in S_W$).

Let $\{F_n\}$ be a sequence of functions such that $F_n \in S_W$ for any n . If there exists a $F \in S_W$ such that for any $\varepsilon > 0$ and any $R > 0$ we can find a $n_0 = n_0(\varepsilon, R)$ such that

$$(1) \quad M(R, F - F_n) < \varepsilon \quad \text{for} \quad n > n_0$$

Then we say that the sequence $\{F_n\}$ converges to the limit F . (I remark that if $F \notin S_W$, although $\{F_n\}$ satisfies the conditions (1) I consider that it is not convergent).

Henceforward instead of saying "space S_W with the topology defined by the aforesaid convergent sequences", I shall say briefly "space S_W ". With these definitions it is easy to show:

Theorem VI.- The space S_W is an L' space which satisfies the axioms I, II and III of Kuratowski.

For the definition of L' spaces see [1, p. 83-84] and for axioms I, II and III of Kuratowski see [1, p. 20].

If $F(z) \in S_W$ is such that there exists an entire function $f(z)$, or a constant, which verifies

$$\limsup \frac{\log_2 M(r, f)}{\log W(r)} < 1$$

and

$$\limsup \frac{\log n(r, 1/(F-f))}{\log W(r)} < 1$$

Then I shall say that $F \in A_W$. We shall denote by B_W the complement of

A_w , i.e., $B_w = S_w - A_w$. Then we can state:

Theorem VII.- B_w is a dense G_6 set.

Theorem VIII.- A_w is a set of first category.

It would be interesting to know if S_w , and hence B_w , is of second category. Perhaps in the next quarter I shall try to look into it.

References

1.- Kuratowski, C. - Topologie (vol. 1, Edition troisième, Warszawa 1952)

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F. Sunyer Balaguer
 Angel Guimera 66 pral 2^a
 Barcelona - 17, Spain