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Let $n(r, \alpha, \varepsilon, F)$ be the number of zeros of $F(z)$ within the sector

$$|z| < r, \quad |\arg z - \alpha| < \varepsilon$$

Then if $F \in S_{w,t}$ and for any $\varepsilon > 0$ and every finite value of a except at most for one

$$(1) \quad \limsup \frac{\log n(r, \alpha, \varepsilon, F-a)}{\log W(r)} = 1$$

holds we shall say that the direction $\arg z = \alpha$ is a direction of Borel-Valiron. If the equality (1) holds for every finite value of a without ~~exception~~ exception we shall say that $\arg z = \alpha$ is a directions of Borel-Valiron without exceptional value.

Let $C_{w,t}$ be the family containing all the $F \in S_{w,t}$ for which there exist a value of α (which may depend on F) such that $\arg z = \alpha$ is not a direction of Borel-Valiron without exceptional value. Then we shall represent by $D_{w,t}$ the complement of $C_{w,t}$ respect $S_{w,t}$ i.e., a function $F \in D_{w,t}$ if $F \in S_{w,t}$ and any direction $\arg z = \alpha$ is a direction of Borel-Valiron without exceptional value for F .

With these notations and following a result of mine [3] we can prove.

THEOREM XIV.- $D_{w,t}$ is a dense G_δ of second category.

THEOREM XV.- $C_{w,t}$ is a F_σ of first category.

These results can be roughly expressed as follows: Most functions $F \in S_{w,t}$ have every direction for a Borel-Valiron direction without exceptional value.

This results reminds as of result of Polya [2] for entire functions of infinite order and some results of Littlewood and Offord [1] for entire functions of finite order. With our notations and from results of Polya there follows

THEOREM .- If

$$F(z) = \sum a_n z^n \in S_w$$

most functions

$$\sum^+ a_n z^n$$

have every direction for a Julia direction (With or without exceptional value).

Let $F^{(p)}(z)$ be an integral of $F(z)$, the function $F(z)$ or a derivative of $F(z)$ respectively when $p < 0$, $p = 0$ or $p > 0$.

If $F \in S_{w,t}$ for any p

$$\limsup \frac{\log_2 M(r, F^{(p)})}{\log W(r)} = 1$$

holds and we can define the set $C_{w,t}^*$ as follows: $F \in C_{w,t}^*$ if $F \in S_{w,t}$ and at least for one p the function $F^{(p)}(z)$ has a direction that it is not a direction of Borel-Valiron without exceptional value. Moreover I define the set $D_{w,t}^*$ as the complement of $C_{w,t}^*$ respect $S_{w,t}$. Then I have proved

THEOREM XVI.- $D_{w,t}^*$ is a dense G_δ of second category.

THEOREM XVII.- $C_{w,t}^*$ is a F_σ of first category.

Remark.- $D_{w,t}^*$ is the set of the functions $\in S_{w,t}$ such that every direction is a direction of Borel-Valiron without exceptional value for the function and for every integral and every derivative.

REFERENCES

- 1.- Littlewood, J.E. and Offord A.C. - On the distribution of zeros and a -values of a random integral function (II) (Annals of Math. vol.49, 1948).
- 2.- Polya, G. - Untersuchungenuber Lucken und Singularitaten von Potenzreihen. (Math. Zeits. t. 29, p.549).
- 3.- Sunyer Balaguer, F. - Sobre la distribución de los valores de una función entera representada por una serie de Diriclet laguanr (Rev.Acad. Ciencias Zaragoza serie2, t.5, 1950).

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