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In order to obtain the results which interest us it is necessary to state a more precise form of lemma 3 (third report). The same proof gives us:

LEMMA 3'. - Given two positive numbers $\varepsilon < 1$ and $\theta < 1$ there is a $R_1 = R_1(\varepsilon, \theta)$ such that for every $r > R_1$ there exists a set $\Phi_1 = \Phi_1(\varepsilon, \theta, r)$, depending only on ε, θ and r , and of measure $\leq \exp(-(W(r))^{\varepsilon/3})$, such that if $F(\cdot, \sigma) \notin \Phi_1$ and if C_1 represents a circle $|z - z_0| \leq (W(r))^{-\theta}$ with $|z_0| = r$ then

$$\sup_{z \in C_1} |F(z, \sigma) - a| > \exp(-(W(r))^{3\varepsilon}) \mu(r)$$

whatever the complex finite number a

If for every α the direction $\arg z = \alpha$ is a direction of Borel-Valiron without exceptional value of $F(z, \sigma)$ (first report) we shall say that $F(\cdot, \sigma) \in \Delta$. The complement of Δ with respect to S will be represented by Γ . i. e. $F(\cdot, \sigma) \in \Gamma$ if and only if at least for one value of α the direction $\arg z = \alpha$ is not a direction of Borel-Valiron without exceptional value of $F(z, \sigma)$.

With these notations we can state

THEOREM XXI. - Γ is a F_σ -set of measure zero and therefore Δ is a G_δ -set of measure 1.

The proof is much more elaborated than the proof of theorem

XX. In the proof of theorem ~~XXXX~~ XXI I use among other results a transformation that I used in a previous paper [1, p. 55]

On the other hand it is evident that theorem XXI contains as a particular case the theorem of Polya that I have quoted in the first report.

Let $F^{(p)}(z, \sigma)$ be an integral of $F(z, \sigma)$, the function $F(z, \sigma)$ or a derivative of $F(z, \sigma)$ respectively when $p < 0$, $p = 0$ or $p > 0$.

As in the first report we can prove that from

$$\limsup \frac{\log_2 M(r, F)}{\log W(r)} = 1$$

it follows

$$\limsup \frac{\log_2 M(r, F^{(p)})}{\log W(r)} = 1.$$

Then we can define the set Γ^* as follows: $F(, \sigma) \in \Gamma^*$ if and only if at least for one p the function $F^{(p)}(z, \sigma)$ has a direction that is not a direction of Borel-Valiron without exceptional value. Moreover I define the set Δ^* as the complement of Γ^* respect to $\mathbb{C} \setminus S$. Then we can prove:

THEOREM XXII.- Γ^* is a ~~measurable~~ F_σ -set of measure zero and therefore Δ^* is a G_δ -set of measure 1.

The proof of this theorem is more elaborated than the proof of theorem XXI.

Evidently $\Gamma \subset \Gamma^*$ and therefore $\Delta^* \subset \Delta$ and hence from the fact that Γ^* is of measure zero and Δ^* is of measure 1 it follows

that Γ is of measure zero and Δ is of measure 1. But from $\Gamma \subset \Gamma^*$ and the fact that Γ^* is a F_σ -set it does not necessarily follow that Γ is a F_σ -set. Similarly from $\Delta^* \subset \Delta$ and the fact that Δ^* is a G_δ -set it does not necessarily follow that Δ is a G_δ -set.

Therefore theorem XXII contains partially theorem XXI but not totally.

REMARK.- In the second report I have defined a correspondence between β and S and I have affirmed that it is a one-to-one correspondence. This affirmation is only verified if the function

$$F(z, \sigma) = \sum_{n=0}^{\infty} r_n(\sigma) a_n z^n$$

is such that $a_n \neq 0$ for every n . If on the contrary $a_n \neq 0$ only for $n \in \{n_k\}$ we write

$$F(z, \sigma) = \sum_{k=0}^{\infty} r_k(\sigma) a_{n_k} z^{n_k}$$

and theorems XIX, XX, XXI and XXII are also satisfied

REFERENCE

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- 1.- Sunyer Balaguer, F. - Sobre la distribución de los valores de una función entera representada por una serie de Dirichlet lagunar (Rev. de la Acad. Ci. Zaragoza Serie 2ª t. V, 1950).

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