

Third quarterly report of the contract

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The lemma 1 can be improved. Moreover in the statement of this lemma in the second report there is anⁿ errata. Therefore I shall state lemma 1 again:

LEMMA 1.- If $k > W(r)$ and $q < k(\log k)^{-1-\delta}$ then

$$\sum_{n \geq 1} n^q |a_n| r^n \leq (1 + \alpha(r))^{W(r) - k/2} \mu(r),$$

where $R = r(1 + \alpha(r))$ and $\alpha(r) = \left(\frac{\log r}{\log W(r)} \right)^{o(1)}$

On the other hand it is evident that in β (second report) we can define a measure as follow: We shall say that a set $H \subset \beta$ is measurable if and only if the set $H \cap \beta_2$ is measurable in the Lebesgue sense in the closed interval $(0,1)$ and then the measure $|H|_\beta$ of H will be given by $|H|_\beta = |H \cap \beta_2|_L$, where $|X|_L$ is the measure of X in the Lebesgue sense.

Besides the homeomorphism $g: \sigma \rightarrow F(z, \sigma)$ of theorem XIX permits us to define a measure in S by $|\Phi|_S = |H|_\beta$ where $H = g^{-1}(\Phi)$.

Similarly to Littlewood and Offord [1] I write $D = z \frac{d}{dz}$ and therefore

$$(D - p)[\varphi(z)] = z \varphi'(z) - p \varphi(z)$$

and given q numbers

$$P_1, P_2, \dots, P_q, (r).$$

we shall put

$$D_{q,h} = (D - p_1) \dots (D - p_{h-1})(D - p_{h+1}) \dots (D - p_q)D$$

With these notations and those of the second report we can state the following:

LEMMA 2.- Given a number $\varepsilon (0 < \varepsilon < 1)$ there exists a $R_0 = R_0(\varepsilon)$ such that for every z which verifies $|z| = r > R_0$ it is possible to determine a set of q positive integers

with the following properties: If $k = 2 \vee(r(1 + \alpha(r))) + W(r)$, then

- 1° no p exceeds k and $q < (W(r))^\varepsilon$
- 2° there is an exceptional set $\Phi = \Phi(\varepsilon, z)$ of measure $\leq \exp(-(W(r))^{\varepsilon/2})$ and such that if $F(z, \sigma) \notin \Phi$

$$\sup_h |D_{q,h}[F(z, \sigma)]| \geq \exp(-(W(r))^{2\varepsilon}) \mu(r).$$

From this lemma it follows

LEMMA 3.- Given two positive numbers $\varepsilon < 1$ and $\theta < 1$ there is a $R_1 = R_1(\varepsilon, \theta)$ such that for every z_0 with $|z_0| = r > R_1$ there exists a set $\Phi_1 = \Phi_1(\varepsilon, \theta, r)$, depending only on ε, θ and r , and of measure $\leq \exp(-(W(r))^{\varepsilon/3})$, such that if $F(z, \sigma) \notin \Phi_1$ in the circle $|z - z_0| \leq (W(r))^{-\theta}$ there exists a point z' for which

$$|F(z', \sigma)| \geq \exp(-(W(r))^{3\epsilon}) \mu(r).$$

Since ϵ is arbitrary we can suppose $\epsilon < 1/4$ and then improving on a method of one of my papers [2] we can state an interesting theorem, but before we ought to give some new notations:

In the second report I state that the space S of the functions $\{F(z, \sigma)\}$ verifies $S \subset S_{w,t}$, where $S_{w,t}$ is a space of the class of those defined in the fourth report of the contract N62558-4161 and in this latter report I have defined the subsets $A_{w,t}$ and $B_{w,t}$ of $S_{w,t}$. Then if we write

$$\Phi = S \cap A_{w,t}$$

$$\Psi = S \cap B_{w,t}$$

by definition $F(z, \sigma) \in \Phi$ if and only if there exists an entire function $f(z)$ such that

$$\limsup \frac{\log_2 M(r, f)}{\log W(r)} < 1$$

and

$$\limsup \frac{\log n(r, 1/(F-f))}{\log W(r)} < 1$$

and also by definition Ψ is the complement of Φ ^{with} ~~write~~ respect to S .

Now we can state:

THEOREM XX .- Φ is an F_σ of measure zero, and therefore Ψ is an G_δ of measure 1.

From some results of one of my previous papers [2] it follows that if the Taylor series which represent $F(z, \sigma)$ are gap series we have $\Phi = \emptyset$, but the theorem XX is interesting when those series are not gap series and $\Phi \neq \emptyset$.

On the other hand, it is evident that theorem IX (fourth report, contract N62558-4161) is a corollary of theorem XX.

References

- 1.- Littlewood, J.E. and Offord, A.C. - On the distribution of zeros and α -values of a random integral function (II) Annals of Math. vol.49, 1948.
- 2.- Sunyer Balaguer, F. - Sur la substitution d'une valeur exceptionnelle par une propriété lacunaire (Acta Math. t. 87, 1952).

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