15 October 1967

PROGRESS REPORT NO.1

PROPERTIES OF A SPACE OF ENTIRE FUNCTIONS OF A INFINITE ORDER

1 JULY 1967 - 30 SEPTEMBER 1967

F. SUNYER BALAGUER

UNIVERSITY OF BARCELONA

THIS RESEARCH HAS BEEN SPONSORED BY THE MATHEMATICS BRANCH, OFFICE OF NAVAL RESEARCH, WASHINGTON D.C. UNDER CONTRACT F61052 67 C 0106. (ONR Task 043-266)

Let $\{n_k\}$ be the sequence of natural numbers corresponding to F(z) such that

$$d_n = 0 \quad \text{for} \quad n \notin \{n_k\}$$
$$d_n \neq 0 \quad \text{for} \quad n \in \{n_k\}$$

threfore we shall have

$$F(z) = \sum d_{n_k} z^{n_k}$$

and if we put (see eighth report of the contract N 62558-4484)

$$\sum r_k(\sigma) d_{n_k} z^{n_k}$$

the correspondence between β and R[F] would be one-to-one, but since the sequence $\{n_k\}$ depends on the equivalence class R[F] I think it is not possible to prove that the correspondence between Z and $\beta X(Z/R)$ is a homeomorphism with the topology which we assign to Z (compact open topology) and with the product topology assigned to $\beta X(Z/R)$. And I think it would be very difficul (perhaps impossible) to utilize the theorem of Fubini.

In order to define a metric outer measure in Z I am trying to follow another method.

In the first place I consider in every equivalence class P = R[F] the member such that

$$-\frac{\pi}{2} < \arg d_n \leq \frac{\pi}{2}$$

and then I write

$$F(z, \sigma) = \sum r_n(\sigma) d_n z^n.$$

Then to each $u \in \beta X(Z/R)$ corresponds a $F \in Z$. This correspondence $\varphi(u) = F$ is not one-to-one because to distinct points u can correspond the same F. However this correspondence has properties which are interesting for my object.

It is evident that the functions which correspond to two or more points u are the F which have some $d_n = 0$. Therefore it is interesting to know the properties of the set K of all equivalence class corresponding to functions which have some $d_n = 0$. I have stated the following theorem which give an interesting property of K:

THeorem XXVII.- According to the metric outer measure defined in Z/R (eighth report contract N62558- 4484 NR 043-266) K is of measure zero.

On the other hand if we represent by K_1 the set formed by all equivalence class corresponding to functions which have some d_n such that

$$|\arg d_n| = \frac{\pi}{2}.$$

Then I have proved

THEOREM XXVIII.- According to the metric outer measure defined in Z/R the set K_1 is of measure zero.

Finally if we consider a set $A \subset Z$ and such that every $F \in A$ verifies

$$R[F] \notin K_1 \cup K$$

we shall say that A has the property p_1. Then I have stated

THEOREM XXIX.- If the set $A \subset Z$ is closed and has the property p_1 then

$$\varphi^{-1}(A) \text{ is closed.}$$

I think that the theorems XXVII, XXVIII and XXIX will permit me to define a metric outer measure addequate to my object but I have not yet a complete proof and I think that it will be very elaborate.

In the next quarter I shall try to complete the proof (if it is possible) and from the definition deduce the measure of some interesting subsets of Z.

F. Sunyer Balaguer
Angel Guimera 66 pral. 2º
Barcelona - 17, Spain