

Statement of the main results obtained
During the period of the Contract N62558-4484

In the first quarterly report I have continued studying the properties of some subsets of the space $S_{w,t}$. In order to state the results that I have obtained it is necessary to give some definitions.

Let $n(r, \alpha, \varepsilon, F)$ be the number of zeros of $F(z)$ within the sector

$$|z| < r, \quad |\arg z - \alpha| < \varepsilon.$$

Then if $F \in S_{w,t}$ and for any $\varepsilon > 0$ and every finite value of a

$$\limsup \frac{\log n(r, \alpha, \varepsilon, F-a)}{\log W(r)} = 1$$

holds, we shall say that the direction $\arg z = \alpha$ is a direction of Borel-Valiron without exceptional value

Now let $C_{w,t}$ be the subset of $S_{w,t}$ such that $F \in C_{w,t}$ if and only if there exists a value of α for which $\arg z = \alpha$ is not a direction of Borel-Valiron without exceptional value of F . We shall represent by $D_{w,t}$ the complement of $C_{w,t}$ with respect to $S_{w,t}$, i.e. when $F \in D_{w,t}$ every direction is a direction of Borel-Valiron without exceptional value of F .

Then I stated:

Theoreme I.- $C_{w,t}$ is a F_σ -set of first category and $D_{w,t}$ is a dense G_δ -set of second category.

Besides if we represent by $F^{(p)}(z)$ an integral of $F(z)$, the function $F(z)$ or a derivative of $F(z)$ respectively when $p < 0$, $p = 0$ or $p > 0$, we can define $C'_{w,t}$ as the subset of $S_{w,t}$ such that $F \in C'_{w,t}$ if and only if

there exists at least one integer p for which $F^{(p)}(z)$ has a direction that is not a direction of Borel - Valiron without exceptional value. Moreover I defined the subset $D_{w,t}^*$ as the complement of $C_{w,t}^*$ with respect to $S_{w,t}^*$. Then I have stated:

Theorem II.- $C_{w,t}^*$ is a F_0 -set of first category and $D_{w,t}^*$ is a dense G_0 -set of second category.

Methods very similar to those that have permitted us ~~results~~ ~~results~~ to prove the previous results also permit us to prove a very different result. I define in the first place a family of functions $f(z)$ holomorphic on $|z| < 1$ and with the topology of uniform convergence on compacts. ~~It~~ ~~there~~ results a topological space E in which we can consider the subset H formed by all those elements of E which can be continued analytically outside $|z| \leq 1$. Let H^* be the complement of H with respect to E . I have stated.

Theorem III.- $H \subset \Lambda$, where Λ is a F_0 -set of first category and $H^* \supset \Lambda^*$; where Λ^* is a dense G_0 -set of second category.

This result belongs to a very numerous class of results obtained by many authors.

Later I have studied the family S of the functions

$$\sum a_n z^n$$

where

$$F(z) = \sum a_n z^n$$

is an entire function such that

$$\limsup \frac{\log_2 M(r, F)}{\log W(r)} = 1$$

where $W(r)$ is a function with the properties that we have defined in the second report of the Contract N62558- 4161.

With the aid of Rademacher's functions we can define a measure on the S and if we suppose that the topology of S is the topology of ~~uniform~~ uniform convergence on compacts we can consider S as a topological space on which we have a measure.

Now I obtain three lemmes interesting in themselves and that permit us to state the three theorems that I shall give (I do not state the lemmes since they would be very long).

Let Φ be the subset of S such that $F \in \Phi$ if and only if there exists an entire function $f(z)$ such that

$$\limsup \frac{\log_2 M(r, f)}{\log W(r)} < 1$$

and

$$\limsup \frac{\log nr, 1/(F-f)}{\log W(r)} < 1$$

and I represent by Ψ the complement of Φ respect to S . Then:

Theorem IV.- Φ is a ~~measurable~~ F_σ -set of measure zero and therefore Ψ is a G_δ -set of measure 1.

If $\Gamma \subset S$ is such that $F \in \Gamma$ if and only if F has at least a direction that is not a direction of Borel-Valiron without exceptional value of F and if Δ is the complement of Γ respect to S , we have:

Theorem V.- Γ is a F_σ -set of measure zero and therefore Δ is a G_δ -set of measure 1.

Besides if $\Gamma' \subset S$ is such that $F \in \Gamma'$ if and only if ^{at} least for one

integer p the $F^{(p)}(z)$ has a direction that is not a direction Borel-Valiron without exceptional value and if Δ' is the complement of Γ' with respect to S , we can state:

Theorem VI.- Γ' is a F_σ -set of measure zero, and therefore Δ' is a G_δ -set of measure 1.

Finally I define a sequence $\{G_n\}$ of bounded domains such that if

$$F(z) = \sum d_n z^n$$

is such that $F \in S_w$, and $d_n \in G_n$ we say that $F \in Z$ and I assign to Z the compact-open topology.

Then I decompose Z in equivalence class $R[F]$ and I define a metric and a metric outer measure in Z/R . On the other hand each equivalence class is similar to space S in which I have defined a metric and a metric outer measure.

Now I am trying to define a metric outer measure in Z . After I shall try to obtain some results on the measure of the subsets of Z formed by entire functions F with determinate properties; for example by the functions which have an exceptional values in the sense of the Picard theorem, or by entire functions which have a direction which is not a Borel-Valiron direction without exceptional value

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