

Seventh quarterly report of the
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(1 January - 31 March 1967)

According to the definition of $S_{w,t}^*$ (sixth report) a function $F(z) = \sum (a_n + ib_n)z^n$ belongs to $S_{w,t}^*$ if $F \in S_w$ (for the definition of S_w see third report of the contract N62558-4161) and if

$$|a_n| \leq M_n$$

$$|b_n| \leq M_n$$

Where

$$M_n = \inf_{r \geq 0} \frac{e^{(w(r))^{1+t(r)}}}{r^n}$$

Putting, in order to simplify, $d_n = a_n + ib_n$, we can prove that for each

$$F(z) = \sum d_n z^n \in S_{w,t}^*$$

the family of all functions

$$\sum d_n z^n$$

form an equivalence class $R[F] \subset S_{w,t}^*$. Now if P is the projection of $S_{w,t}^*$ onto the family $S_{w,t}^*/R$ of equivalence classes, we shall consider the quotient space $S_{w,t}^*/R$, that is, the family of equiva-

lence classes with the quotient topology relative to P .

On the other hand, I shall define the space $s_{w,t}$ as follows:
~~XXXXXX~~ $f \in s_{w,t}$ if and only if for some $F \in S_{w,t}^\circ$ we have

$$F(z) = \sum a_n z^n \quad \text{and} \quad f(z) = \sum a_n^2 z^n.$$

Therefore if F_1 and F_2 belong to the same $\equiv$ equivalence class then the corresponding functions f_1 and f_2 verify

$$f_1 \equiv f_2$$

Hence there exists a one-to-one correspondence between $s_{w,t}$ and $S_{w,t}^\circ/R$.

Moreover if we suppose that the topology of $s_{w,t}$ is the compact open one, then I have proved:

THEOREM XXV.- The spaces $S_{w,t}^\circ/R$ and $s_{w,t}$ are homeomorphic.

It is easy to prove that if $f \in s_{w,t}$ then

$$\limsup_{r \rightarrow \infty} \frac{\log_2 M(r, f)}{\log W(r^{1/2})} = 1$$

and writing $W_0(r) = W(r^{1/2})$, since the properties of $W_0(r)$ are very similar to those of $W(r)$, we can see that there exists a function $t_0(r)$, where $t_0(r) > 0$ and $t_0(r) \rightarrow 0$ as $r \rightarrow \infty$, and such that

$$\log_2 \left(\sum |a_n|^2 r^n \right) \leq (1 + t_0(r)) \log W_0(r).$$

Hence we have

$$s_{w,t} \subset s_{w_0,t_0} \subset s_{w_0,t_0}^*$$

Therefore as in the sixth report we can define a metric and a metric outer measure on $s_{w,t}$, and following theorem XXV also on $s_{w,t}^*/R$.

For my object, as a first step, I think it would be interesting to prove, if it is possible, that $s_{w,t}^*/R$ is not of measure zero.

I consider it useful to state explicitly the following result in spite of the fact that it is an immediate consequence of the theory of normal families:

LEMMA 5.- In the topology of $s_{w,t}^*$ the family of the neighbourhoods

$$\{F: M(1, F-F_0) < \varepsilon, \quad F \in s_{w,t}^*, \quad F_0 \in s_{w,t}^*\}$$

is a base.

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