

Brief review of the background of the proposed investigation

Many authors (Schwartz [1] ⁽¹⁾, Levinson [2], Kahane [3] and others) have studied the properties of the functions $F(s) \in K(\{\lambda_n\}, \Delta)$ in correspondence with those of $\{\lambda_n\}$ and Δ , where $K(\{\lambda_n\}, \Delta)$ denote the closure of the linear combinations of the $\{e^{-\lambda_n s}\}$ in some interval (or domain) Δ .

On the other hand, Mandelbrojt [4], when Δ is a strip, and $\{\lambda_n\}$ is of upper density finite, has proved very much interesting theorems, in which the typical hypothesis is the adherence-one, that may briefly be stated as follows: Let $\Delta(x)$ be the intersection of the strip Δ with the half-plane $\sigma > x$ (which we assume is not empty for any x). Let $d(x)$ be the distance in $\Delta(x)$ from $F(x)$ to the closure of a set of the partial sums of a Dirichlet series, i.e.,

$$d(x) = \inf_{n \geq K} \sup_{s \in \Delta(x)} \left| F(s) - \sum_{n=1}^n d_n e^{-\lambda_n s} \right|,$$

then the adherence hypothesis states that $d(x) \rightarrow 0$ very rapidly when $x \rightarrow \infty$.

I have stated [5] some results in which the typical hypothesis may be stated as follows: Let $\delta(x)$ be the intersection of Δ and the vertical strip $x-b < \sigma < x$. Let $d_1(x)$ be the distance from $F(s)$ to

(1). Numbers in brackets refer to the bibliography at the end.

to $K(\{\lambda_n\}, \delta(x))$, i.e.,

$$d_1(x) = \inf_{\varphi \in \Phi} \sup_{s \in \delta(x)} |F(s) - \varphi(s)|$$

where Φ is the set of the linear combinations of the $\{e^{-\lambda_n s}\}$, then the adherence hypothesis states that $d_1(x) \rightarrow 0$ very rapidly when $x \rightarrow \infty$.

Objectives of the project

The class W of the functions $F(s)$ holomorphic in the strip Δ *where* ~~such that~~ the distance from $F(s)$ to $K(\{\lambda_n\}, \delta(x))$ satisfies an adherence hypothesis, evidently depends on $\{\lambda_n\}$, Δ and on the given adherence hypothesis.

As a ~~To~~ characterize the class W for any $\{\lambda_n\}$ and any Δ ~~seems in~~ *(rule if appears)* general very difficult or practically impossible. But I think that it is likely that if $\{\lambda_n\}$ and Δ have the properties which are mostly involved in the problems of approximations of functions by linear combinations of exponentials, *(be in a position to)* we shall ~~can~~ state interesting results.

Therefore the objectives of the project are to determine the properties of W (i.e. those of the functions $F(s) \in W$) or, if possible, to characterize W , when Δ and $\{\lambda_n\}$ have any of the usual properties. Evidently, *(of every one of this properties)* a different result will follow.

Moreover I think it is likely that if we obtain the aforesaid results they will permit us to prove some theorems that will ~~sta-~~
 reveal a connection between W (for certain adherence hypotheses and certain Δ and $\{\lambda_n\}$) and the class of the almost periodical functions.

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The scientific method to be employed

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In mathematics, before carrying out an investigation, it is very difficult to indicate exactly the method to be used to attain the proposed aims. Generally once the method has been obtained, the problem is already solved. Notwithstanding on the whole I think it is probable that the method will be based on the following ideas:

1º: The results obtained by the authors first indicated [1], [2] and [3]; and

2º: The methods of Mandelbrojt and those I use in the papers about which we have spoken [5].

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Name and background of investigator

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Estimated duration of project and yearly

Budget breakdown

The approximate duration of the investigation and the explanation of the results and demonstrations may be estimated as one year. The expenses are solely composed of the salary of the investigator and perhaps the cost of translation of some Russian papers which may be interesting. *turning out a report in*

An estimate of about 108000 pesetas a year could be sufficient for my salary.

^{I have} There is no data in my possession about the cost of eventually having some Russian papers to be translated. But it will certainly be of very little concern. A little allowance ^{over} by the 108000 pesetas could be sufficient.

Bibliography

- 1.- Schwarts, L. - Etude des sommes d'exponentielles (Actualités scientifiques et industrielles 959, Deuxième édition 1959).
- 2.- Levinson, N. - Gap and density theorems (American Mathematical Society, Colloquium Publications, vol. XXVI 1940).
- 3.- Kahane, J.P. - Sur quelques problèmes d'unicité et de prolongement, relatifs aux fonctions approchables par des sommes d'exponentielles, (Ann. Inst. Fourier, t.V, 1955)

- 4.- Mandelbrojt, S. - *Séries adhérentes. Régularisation des suites Applications* (Paris 1952).
- 5.- Sunyer Balaguer, F. - *Aproximación de funciones por sumas de exponenciales* (Collectanea Math. vol.V, 1952).