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Combining Total Variation and Nonlocal Means Regularization for Edge Preserving Image Deconvolution

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Abstract

We propose a new edge preserving image deconvolution model by combining total variation and nonlocal means regularization. Natural images exhibit an high degree of redundancy. Using this redundancy, the nonlocal means regularization strategy is a good technique for detail preserving image restoration. In order to further improve the visual quality of the nonlocal means based algorithm, total variation is introduced to the model to better preserve edges. Then an efficient alternating minimization procedure is used to solve the model. Numerical experiments illustrate the effectiveness of the proposed algorithm.

Key Words: Total Variation Regularization, Nonlocal Means Filter, Image Deconvolution.

1 Introduction

One of the important tasks in signal/image processing is to recover signals/images from noisy and blurry observations. Image deconvolution can be formulated as an inverse problem. Many linear inverse problems are ill-posed, i.e., either the linear operator does not admit inverse or it is near singular, yielding highly noise sensitive solutions.

The object is to find the unknown true image $u \in \Omega \subset R^2$ from an observed image $f \in \Omega$ defined by

$$f = Au + \eta \tag{1}$$

where η is a white Gaussian noise with variance σ^2, A is a linear operator, typically a convolution operator. One of the techniques to avoid this problem is the regularization framework which solves the following minimization problem:

$$u^* = \arg\min_u \left\{ J(u) + \frac{\lambda}{2} F(f, Au) \right\} \tag{2}$$

where $F(f, Au)$ is the fidelity term which measures the difference between f and Au, $J(u)$ is the regularization term which characterizes some features of the desired solution u and $\lambda > 0$ is a regularization parameter tuning the weight between the regularization term and the fidelity term. A well-known regularized inverse problems

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is the Tikhonov regularization model [1], which can be formulated as a one-step filter via Fourier transform for image deconvolution. An alternative regularization [2] uses H^1 semi-norm of u as the regularization term called Wiener filter. These linear methods are very simple and fast to implement. However, in order to compensate for the noise, a large regularization parameter is necessary, which tends to smear out the edges.

A successful edge preserving image restoration regularization model is the well-known ROF model [3], the total variation (TV) functional with $J_{TV}(u) = |u|_{BV}$ is used as the regularization term because images are assumed to have bounded variation, which has the following form:

$$u^* = \arg\min_u \left\{ |u|_{BV} + \frac{\lambda}{2}\|f - Au\|_{L^2}^2 \right\}. \tag{3}$$

Due to its virtue of preserving edges, it is widely used in many applications of image processing, such as image deconvolution [4, 5, 6], inpainting [7] and super resolution [8]. There are many methods to minimize the TV regularization models, such as FTVd method [6], dual method [9, 10], iterative Bregman algorithm [11], etc.

In order to better respect small details and structures in images, Buades $et\ al.$ introduced in [12] an efficient denoising model called nonlocal means (NL-means) which is a generalization of the Yaroslavsky filter [13]. This model consists in denoising a pixel value by averaging the nearby pixel values with similar structures, which is defined by the simple formula:

$$NL_f(u)(x) = \frac{1}{C(x)} \int_\Omega \omega_f(x,y)u(y)dy \tag{4}$$

where $x \in \Omega$ and $C(x)$ is a normalizing constant with

$$C(x) = \int_\Omega \omega_f(x,y)dy, \quad \omega_f(x,y) = \exp\left(-\frac{(G_a * |f(x+\cdot) - f(y+\cdot)|^2)(0)}{h^2} \right). \tag{5}$$

G_a is a Gaussian kernel with a the standard deviation of the Gaussian kernel and h acts as a filtering parameter. In the variational point of view, Gilboa and Osher in [14] defined a framework based on nonlocal operators which are inspired by the graph Laplacian [15]. Then this framework is successfully applied to nonlocal total variation functionals [16, 17]. Under the Bayesian framework, a nonlocal regularization strategy was proposed in [18] for image deconvolution. However, experimental results show that there visually exist edge oscillations along sharp edges.

In this paper, we combine total variation regularization and nonlocal means strategy for edge preserving image deconvolution. Total variation regularization can suppress the edge oscillations which are caused by the nonlocal means strategy. Comparisons with other classical image deconvolution algorithms are given to illustrate that the proposed algorithm is an effective algorithm for edge preserving image deconvolution.

2 Proposed Edge Preserving Image Deconvolution

In [18], the NL-means is used to derive an efficient image prior distribution, which expresses that the reconstructed images are similar to the solutions exhibiting similar neighborhood configurations. Finally, the regularization term is denoted as

$$J_{NL}(u) = \|u - NL_f(u)\|_p^p \tag{6}$$

where $p > 1$ is a shape factor. For p=2, the regularization term in Eq.(6) has already been proposed by Buades et al [19]. The regularization term (6) is interpreted as nonlocal since pixels belonging to the whole image are used for the estimation of each new pixel. Because natural images have enough redundancy to be restored by NL-means, the small details are well preserved.

In (6), for the computation of nonlocal means step, the nonlocal weight ω_f is computed from the observed image f and the nonlocal averaging is over the reconstructed result. This notation simplifies the solution of the functional (6) when the steepest descent procedure is used. However, there exists a certain deviation along sharp edges for the experimental results. In order to suppress the edge oscillations, the total variation functional is incorporated into the regularization term, and the proposed edge preserving regularization term has the following form

$$J_E(u) = \theta|u|_{BV} + \|u - NL_f(u)\|_p^p. \tag{7}$$

θ is a parameter that tunes the weight between the total variation and nonlocal regularization strategy. For the image deconvolution problem, we solve the following minimization functional:

$$u^* = \arg\min_u \left\{ \underbrace{\theta|u|_{BV} + \|u - NL_f(u)\|_p^p}_{J_E(u)=\theta J_{TV}(u)+J_{NL}(u)} + \frac{\lambda}{2} \underbrace{\|f - Au\|_{L^2}^2}_{F(f,Au)} \right\}. \tag{8}$$

The regularization term $J_E(u)$ composes of the edge-preserving total variation functional and the detail-preserving nonlocal regularization strategy term. The value of the parameter θ tunes the weight between the total variation regularization term and the nonlocal regularization strategy term which should not be too large. For the large value of θ, the total variation term plays an important role so that the restored images are piecewise constant with small blurred details.

For the minimization functional (8), two regularization terms are adopted: one is the TV regularization term J_{TV}, another is the NL-regularization strategy term J_{NL}. However, the nonlocal operators for image recovery introduced in [16, 17] should be the framework called nonlocal framework based on the nonlocal gradient $\nabla_\omega u : \Omega \to \Omega \times \Omega$

$$(\nabla_\omega u)(x, y) := (u(y) - u(x))\sqrt{\omega(x, y)} \tag{9}$$

and nonlocal divergence $div_\omega \overrightarrow{v} : \Omega \times \Omega \to \Omega$

$$(div_\omega \overrightarrow{v})(x) := \int_\Omega (v(x, y) - v(y, x))\sqrt{\omega(x, y)}dy. \tag{10}$$

Based on the nonlocal operators, the NL/TV regularization functional is designed as follows:

$$J_{NL/TV} = \int |\nabla_\omega u| = \int_\Omega \sqrt{(u(x) - u(y))^2 \omega(x, y)}dydx. \tag{11}$$

which is different from the Eq.(8). In [16], the gradient descent of the nonlocal total variation (NLTV) is used to minimize the above functional, and the Bregmanized operator splitting technique is adopted to improve the minimization algorithm of the NLTV in [17].

For numerical implementations, inspired from [6], we use alternating minimization algorithms for solving the above image restoration problems (8). First, a new auxiliary variable v is introduced, and the minimization problem can be rewritten as the following constrained minimization problem:

$$\begin{aligned} \min_{u,v} \quad & \theta|v|_{BV} + \|u - NL_f(u)\|_p^p + \frac{\lambda}{2}\|f - Au\|_{L^2}^2 \\ \text{s.t.} \quad & u = v. \end{aligned} \tag{12}$$

We consider the L^2 norm square penalty formulation

$$\min_{u,v} \theta|v|_{BV} + \|u - NL_f(u)\|_p^p + \frac{\lambda}{2}\|f - Au\|_{L^2}^2 + \frac{\beta}{2}\|u - v\|_{L^2}^2. \tag{13}$$

In order to solve the unconstrained problem (13), an alternating minimization algorithm is proposed [6]. We need to solve two types of subproblems which solve the v problem with u fixed and vice versa. We initialize $u^0 = v^0 = f$.

Step 1. u is fixed to solve the v-subproblem:

$$v^{n+1} = \arg\min_{v} \left\{ \theta |\nabla v|_{BV} + \frac{\lambda}{2} \|u^n - v\|_{L^2}^2 \right\}. \tag{14}$$

This step corresponds to the ROF denoising algorithm [3].

Step 2. v is fixed to solve the u-subproblem:

$$u^{n+1} = \arg\min_{u} \left\{ \|u - NL_f(u)\|_p^p + \frac{\lambda}{2} \|f - Au\|_2^2 + \frac{\beta}{2} \|u - v^n\|_2^2 \right\}. \tag{15}$$

We use the steepest descent procedure similar as the approach proposed in [18] to search the solution of the minimization problem (15)

$$\begin{aligned} u^{n+1} &= u^n + \alpha^n \cdot \lambda \cdot (A^*(f - Au^n) \\ &\quad -\partial_u J_{NL}(u^n) - \beta(u^n - v^{n+1})). \end{aligned} \tag{16}$$

The superscript denotes the iteration number, α^n is the gradient step size changing adaptively [18] according to the following:

$$\alpha^n = \frac{\|q^n\|^2}{\|Aq^n\|^2} \quad \text{with } q^n = A^*(f - Au^n) \tag{17}$$

and $\partial_u J_{NL}(u^n)$ has the following expressions:

$$\partial_u J_{NL}(u^n) = p \cdot \left(1 - \frac{1}{C}\right) \cdot sign(u^n - NL_f(u^n)) \|u^n - NL_f(u^n)\|^{p-1}. \tag{18}$$

C is the normalizing constant computed from NL-means filter. For $C \gg 1$, we therefore simplify the above equation as:

$$\partial_u J_{NL}(u^n) \approx p \cdot sign(u^n - NL_f(u^n)) \|u^n - NL_f(u^n)\|^{p-1}. \tag{19}$$

3 Numerical Experiments

We now present numerical experimental results and comparisons to illustrate the valid performance of the proposed algorithm. For all the algorithms based on the NL-means, the size of the search window is set to 11×11, the decaying parameter h is set to $h = 10\sigma$ where σ is the standard deviation of the Gaussian white noise, the parameter λ and β are manually set to $\lambda = 0.3, \beta = 1$ respectively. The selection of the optimal value of the parameters is not in the scope of this work. As in [18], the shape factor is $p = 1.5$. We use "lena" and "cameraman" image of size $N = 256 * 256$ and use the "rectangular" image of size $N = 128 * 128$ to illustrate the effectiveness of our proposed algorithm, respectively.

Example 1. Figure 1 shows the original clean "lena" image and the corrupted blurry and noisy version. We set the blurring window size equal to 7 and the standard deviation equal to 2 with Gaussian white noise BSNR=40 ($BSNR = 10log\left(\frac{var(blurred\ \ image)}{var(noise)}\right)$). Figure 2 presents the evolution of ISNR with different parameter θ (ISNR = PSNR$\|f - u\|^2/(N \times 255^2)$).

The value of θ plays an important role, large θ yields a piecewise constant image with small blurred details and a small θ corresponds to the method similar to the nonlocal regularization strategy. From Figure 2, we can choose $\theta = 0.02$ for the proposed method. Figure 3 displays the results with FTVd method [6], nonlocal total variation method (NLTV) [17], nonlocal regularization strategy (NL-Re) [18] and the proposed edge preserving method. We can see that the proposed algorithm can suppress edge oscillations and can preserve small details of the image with a higher ISNR.

Example 2. Figure 4 shows the original clean "cameraman" image and its corrupted version. The corrupted "cameraman" image is obtained by the same method as the corrupted "lena" shown in the left of figure 1. The

Figure 1: original clean ”lena” and the blurry noisy version

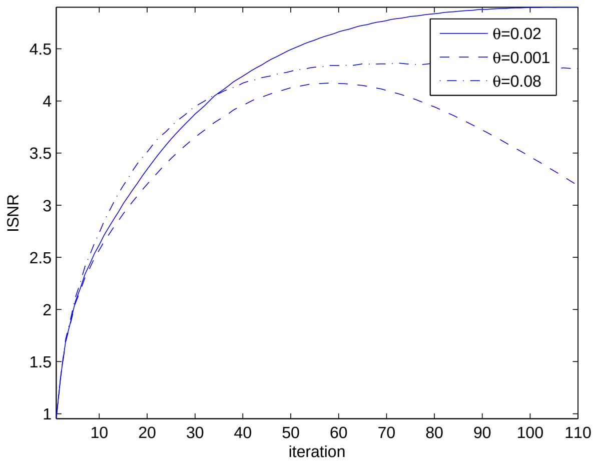

Figure 2: evolution of ISNR along iterations for different parameter θ

Figure 3: deconvolution results with different methods. Up left, FTVd [6] ISNR=3.2999, up right, NLTV [17] ISNR=4.0980, down left, NL-Re [18] ISNR=4.0635 and down right, the proposed algorithm ISNR=4.8987.

Figure 4: original clean ”cameraman” and the blurry noisy version

Figure 5: deconvolution results with different methods. Up left, FTVd [6] ISNR=2.4282, up right, NLTV [17] ISNR=4.3318, down left, NL-Re [18] ISNR=3.8777 and down right, the proposed algorithm ISNR=4.5260.

methods used and the value of θ we choose are the same as those used in example 1. The deconvolution results are shown in figure 5. This is another example to illustrate the effectiveness of the proposed algorithm.

Example 3. Figure 6 shows the original "rectangular" image and the corresponding blurry noisy image with Gaussian blurry and Gaussian white noise. The blurry window size equals to 7 and its standard deviation equals to 2, the Gaussian white noise BSNR=40. It is easily to see from Figure 7 that the proposed method can preserve sharp edges with little edge oscillations.

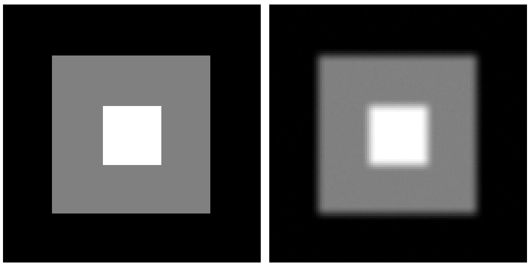

Figure 6: original clean "rectangular" and the blurry noisy version

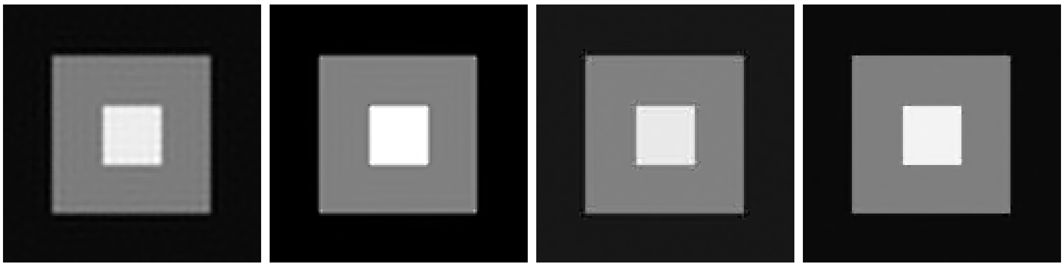

Figure 7: deconvolution results with different methods. From left to right: FTVd [6], NLTV [17], NL-Re [18] and the proposed algorithm.

Example 4. Figure 8 shows the deconvolution results with different methods for a slice of "cameraman" image. From left to right, the original motion blurry (the angle is set to 30 and the distance 10) noisy (the Gaussian white noise BSNR=40) image, FTVd [6], NL-Re [18] and the proposed algorithm. Figure 9 shows the results for a slice of average blurry (average size equals to 7) noisy (BSNR=40) "cameraman" image with the same methods as the ones used in figure 8. It is easily to see that the proposed method can preserve sharp edges with little edge oscillations. From Figure 10, we can see that with our proposed method the root mean square error ($RMSE = \|ture\ image - estimated\ image\|_2$) is smaller than the one obtained with the NL-Re[18].

Figure 8: deconvolution results with different methods. From left to right, the original motion blurry noisy image, FTVd [6], NL-Re [18] and the proposed algorithm

Table 1 shows the ISNR comparison with different methods used in example 1. We test three types of blurring effects: Gauss, averaging and motion blur. We use the functions "fspecial" and "imfilter" from the

Figure 9: deconvolution results with different methods. From left to right, the original average blurry noisy image, FTVd [6], NL-Re [18] and the proposed algorithm

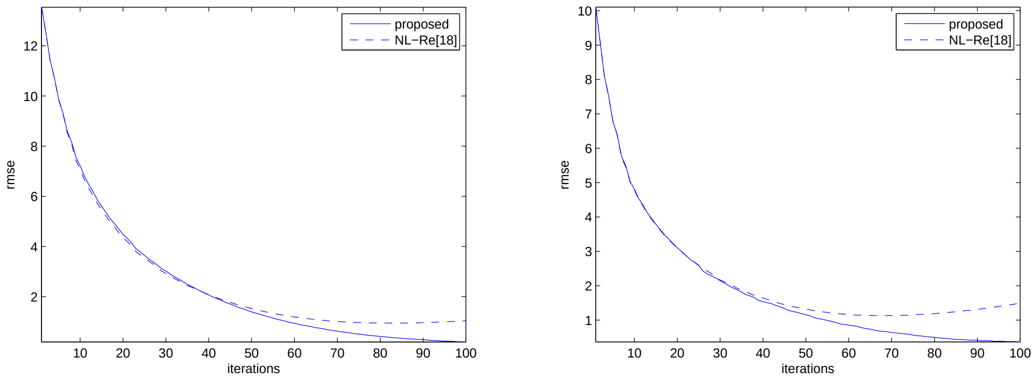

Figure 10: evolution of **rmse** along iterations for a slice of motion blurry noisy "cameraman" image with different methods.

Matlab Image Processing Toolbox with the types "Gauss","ave" and "motion". For the Gaussian blurring we set the blurring window size equal to 7 and the standard deviation equal to 2, for the average blurring we set the size to 5, and for the motion blurring we set the angle to 30 and the distance to 10. For all the experiments, we add Gaussian white noise with BSNR=40. Compared to the existing algorithms, the obtained ISNRs with different methods and different BSNRs for the Gauss blurring are shown in table 2. From the comparison of the two tables, we can see that the proposed algorithm performs effectively with higher ISNRs.

4 Conclusion

We have proposed an image deconvolution model by combining total variation (TV) and nonlocal (NL) regularization to preserve edges and small details. The NL regularization strategy is used as a regularization term to preserve small details, and the total variation functional is incorporated into the NL regularization term. Therefore, incorporating the TV into the NL regularization strategy, edge preserving restored images are obtained while simultaneously the restored images are detail preserved well for the NL regularization. Numerical results have shown that the proposed algorithm is quite effective.

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Table 1: ISNR comparison with different methods for ”lena” and ”cameraman”

	lena				cameraman			
methods	FTVd	NLTV	NL-Re	Proposed	FTVd	NLTV	NL-Re	Proposed
Gauss	3.2999	4.0980	4.0635	4.8987	2.4282	4.3318	3.8777	4.5260
uniform	5.3847	6.4648	6.7106	7.6724	5.4368	7.3876	7.5395	8.3985
motion	7.5675	9.8172	9.6025	11.0688	6.6289	10.2992	9.6401	10.7413

Table 2: ISNR comparison with different methods for Gauss blurring

	lena				cameraman			
BSNRs	FTVd	NLTV	NL-Re	Proposed	FTVd	NLTV	NL-Re	Proposed
40	3.2999	4.0980	4.0635	4.8987	2.4282	4.3318	3.8777	4.5260
30	2.9148	3.3969	3.2174	3.4080	2.4697	2.5570	2.7247	2.8181
25	2.9034	3.1334	3.0635	3.1768	2.2406	2.2472	2.2943	2.3158

References

[1] A.Tikhonov and V.Arsenin, Solution of Ill-Posed Problems. New York: Wiley, 1977.

[2] H.C.Andrews and B.R.Hunt, Digital Image Restoration. Prentice Hall, Englewood Cliffs, NJ, 1977.

[3] L.I.Rudin, S.Osher, and E.Fatemi, ”Nonlinear total variation based noise removal algorithms”, *Physica D* 60:259-268, 1992.

[4] T.F.Chan and C.K.Wong, ”Total variation blind deconvolution”, *IEEE Transactions on Image Processing* 7:370-375, 1998.

[5] L.He, A.Marquina, and S.Osher, ”Blind deconvolution using TV regularization and bregman iteration”, *International Journal of Imaging Systems and Technology* 15(1):74-83, 2005.

[6] Y.Wang, J.Yang, W.Yin, and Y.Zhang, ”A New Alternating Minimization Algorithm for Total Variation Image Reconstruction”, *SIAM J. Imaging Science*, 1(3):248-272, 2008.

[7] T.Chan and J.Shen, ”Mathematical models for local non-texture inpainting”, *SIAM J. Appl. Math.* 62(3):1019-1043, 2001.

[8] F.Malgouyres and F.Guichard, ”Edge direction preserving image zooming: a mathematical and numerical analysis”, *SIAM Journal on Numer. An.* 39:1-37, 2001.

[9] A.Chambolle, ”An algorithm for total variation minimization and applications”, *Journal of Mathematical Imaging Vision* 20:89-97, 2004.

[10] D.Krishnan, Q.Pham and A.Yip, ”A primal-dual active set algorithm for bilaterally constrained total variation deblurring and piecewise constant Mumford-Shah segmentation problems”, *Adv. Comput. Math.* 31:237-266, 2009.

[11] J.Cai, S.Osher and Z.Shen, ”Linear Bregman iteration for compressed sensing”, *Math. Comput.* 78:1515-1536, 2009.

[12] A.Buades, B.Coll and J.M.Morel, ”A nonlocal algorithm for image denoising”, CVPR 2005, *IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, San Diego, CA, USA, 2:60-65, 2005.

[13] L.P.Yaroslavsky, Digital Picture Processing, an Introduction, Springer Verlag, Berlin, 1985.

[14] G.Gilboa and S.Osher, ”Nonlocal operators with applications to image processing”, *Multiscale Model. Simul.* 7(3):1005-1028, 2008.

[15] F.R.K.Chung, Spectral Graph Theory, American Mathematical Society, 1997.

[16] Y.Lou, X.Zhang, S.Osher and A.Bertozzi, ”Image recovery via nonlocal operators”, *Journal of Scientific Computing*, 42(2):185-197, 2010.

[17] X.Zhang, M.Burger, X.Bresson and S.Osher, ”Bregmanized nonlocal regularization for deconvolution and sparse reconstruction”, *SIAM Journal on Imaging Sciences*, 3(3):253-276, 2010.

[18] M. Mignotte, ”A non-local regularization strategy for image deconvolution”, *Pattern Recognition Letters* 29:2206-2212, 2008.

[19] A.Buades, B.Coll, J.Morel, ”Image enhancement by non-local reverse heat equation ”, CMLA, Tech. Rep. 22, 2006.