Fuzzy Multilevel Graph Embedding for Recognition, Indexing and Retrieval of Graphic Document Images

presented by
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Objectives of thesis

❖ Problematic
  ▪ Lack of efficient computational tools for graph based structural pattern recognition

❖ Proposed solution
  ▪ Transform graphs into numeric feature vectors and exploit computational strengths of state of the art statistical pattern recognition
Introduction

Fuzzy Multilevel Graph Embedding (FMGE)

Automatic indexing of graph repositories for graph retrieval and subgraph spotting

Conclusions and future research challenges
Introduction

- Structural and statistical pattern recognition
- Graph embedding
- State of the art on explicit graph embedding
- Limitations of existing methods

Fuzzy Multilevel Graph Embedding (FMGE)

Automatic indexing of graph repositories for graph retrieval and subgraph spotting

Conclusions and future research challenges
<table>
<thead>
<tr>
<th></th>
<th>Structural</th>
<th>Statistical</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data structure</strong></td>
<td>symbolic data structure</td>
<td>numeric feature vector</td>
</tr>
<tr>
<td><strong>Representational strength</strong></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Fixed dimensionality</strong></td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Sensitivity to noise</strong></td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Efficient computational tools</strong></td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
- Graph matching and graph isomorphism
- Graph edit distance
- Graph embedding
Graph matching and graph isomorphism

[Messmer, 1995] [Sonbaty and Ismail, 1998]

Graph edit distance

Graph embedding

A' ↔ C
B' ↔ B
C' ↔ A
D ↔ ϕ
Graph matching and graph isomorphism
[Messmer, 1995] [Sonbaty and Ismail, 1998]

Graph edit distance
[Bunke and Shearer, 1998] [Neuhaus and Bunke, 2006]

Graph embedding
Graph embedding (GEM)

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Structural PR
Expressive, convenient, powerful but computationally expensive representations

Statistical PR
Mathematically sound, mature, less expensive and computationally efficient models

Graph embedding
### Explicit GEM
- embeds each input graph into a numeric feature vector
- provides more useful methods of GEM for PR
- can be employed in a standard dot product for defining an implicit graph embedding function

### Implicit GEM
- computes scalar product of two graphs in an implicitly existing vector space, by using graph kernels
- does not permit all the operations that could be defined on vector spaces
State of the art on explicit GEM

- Graph probing based methods
- Spectral based graph embedding
- Dissimilarity based graph embedding
- **Graph probing based methods**
  
  [Wiener, 1947] [Papadopoulos et al., 1999] [Gibert et al., 2011] [Sidere, 2012]

- **Spectral based graph embedding**

- **Dissimilarity based graph embedding**

![Graph](image)

number of nodes = 6
number of edges = 5 etc.

v = 6,5, ...
- **Graph probing based methods**
  
  [Wiener, 1947] [Papadopoulos et al., 1999] [Gibert et al., 2011] [Sidere, 2012]

- **Spectral based graph embedding**
  
  [Harchaoui, 2007] [Luo et al., 2003] [Robleskelly and Hancock, 2007]

- **Dissimilarity based graph embedding**

  Spectral graph theory employing the adjacency and Laplacian matrices

  Eigen values and Eigen vectors

  PCA, ICA, MDS
- **Graph probing based methods**
  
  [Wiener, 1947] [Papadopoulos et al., 1999] [Gibert et al., 2011] [Sidere, 2012]

- **Spectral based graph embedding**
  
  [Harchaoui, 2007] [Luo et al., 2003] [Robleskelley and Hancock, 2007]

- **Dissimilarity based graph embedding**
  
  [Pekalska et al., 2005] [Ferrer et al., 2008] [Riesen, 2010] [Bunke et al., 2011]

![Diagram of prototype graphs](image)

 Prototype graphs

 P1
 P2
 P3
 ...

 v = d(g, P1), d(g, P2), ...
Limitations of existing methods

- Not many methods for both directed and undirected attributed graphs
- No method explicitly addresses noise sensitivity of graphs
- Expensive deployment to other application domains
- Time complexity
- Loss of topological information
- Loss of matching between nodes
- No graph embedding based solution to answer high level semantic problems for graphs
Introduction

Fuzzy Multilevel Graph Embedding (FMGE)
- Method
- Experimental evaluation
- Application to symbol recognition
- Discussion

Automatic indexing of graph repositories for graph retrieval and subgraph spotting

Conclusions and future research challenges
- Fuzzy Multilevel Graph Embedding (FMGE)
- Graph probing based explicit graph embedding method

\[ \phi : G \rightarrow \mathbb{R}^n \]
\[ AG \leftrightarrow \phi(AG) = (f_1, f_2, \ldots, f_n) \]
Multilevel analysis of graph

- Graph level information [macro details]
- Structural level information [intermediate details]
- Elementary level information [micro details]

- Graph order
- Graph size
- Node degree
- Homogeneity of subgraphs
- Edge attributes
- Node attributes
Numeric feature vector embeds a graph, encoding:

- Numeric information by *fuzzy* histograms
- Symbolic information by *crisp* histograms
Input : Collection of attributed graphs
Output : Equal-size numeric feature vector for each input graph
Fuzzy Structural Multilevel Feature Vector

**Graph Level Information**
- Graph order
- Graph size
- Fuzzy histogram of node degrees
- Fuzzy histograms of numeric resemblance attributes
- Fuzzy histograms of numeric node attributes

**Structural Level Information**
- Fuzzy histograms of numeric resemblance attributes
- Crisp histograms of symbolic resemblance attributes
- Crisp histograms of symbolic node attributes
- Fuzzy histograms of numeric edge attributes

**Elementary Level Information**
- Crisp histograms of symbolic edge attributes
- Node-resemblance for an edge
- Edge-resemblance for a node
- Node-resemblance for an edge
- Edge-resemblance for a node

\[
\text{numeric resemblance} = \frac{\min(|a_1|, |a_2|)}{\max(|a_1|, |a_2|)}
\]

\[
\text{symbolic resemblance} = \begin{cases} 
1 & \text{if } b_1 = b_2 \\
0 & \text{otherwise}
\end{cases}
\]
- Node-resemblance for an edge
- Edge-resemblance for a node

\[
\begin{align*}
\text{numeric resemblance} &= \frac{\min(|a_1|,|a_2|)}{\max(|a_1|,|a_2|)} \\
\text{symbolic resemblance} &= \begin{cases} 
1 & \text{if } b_1 = b_2 \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]
- Unsupervised learning phase
- Graph embedding phase
Unsupervised learning phase of FMGE

Graph dataset
\{AG_1, AG_2, \ldots, AG_n, \ldots, AG_m\}

Learning

Number of fuzzy intervals
\{c_i\text{, for attribute }i\}

Fuzzy intervals
(for computing the fuzzy histograms in FSMFV)
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Unsupervised learning phase of FMGE

- First fuzzy interval \((-\infty, -\infty, \ldots, \ldots)\)
- Last fuzzy interval \((\ldots, \ldots, \infty, \infty)\)

List of values of attribute in input collection of graphs
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Graph embedding phase of FMGE

Graph
\( AG_e = (V_e, E_e, \mu_{V_e}, \mu_{E_e}) \)

Embedding

Fuzzy overlapping trapezoidal intervals
(s, for attribute \( i \))

Fuzzy Structural Multilevel Feature Vector
(FSMFV_e)

- Numeric information embedded by fuzzy histograms
- Symbolic information embedded by crisp histograms

\[
\alpha(x) = \begin{cases} 
(x-a)/(b-a) & \text{if } a \leq x < b \\
1 & \text{if } b \leq x \leq c \\
(x-d)/(c-d) & \text{if } c < x \leq d \\
0 & \text{otherwise}
\end{cases}
\]
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Example - FMGE

1

2

3

4

RL : 1
Angle: B
r_L: 1
r_NodeDegree: 0.5

2

3

4

RL : 1
Angle: B
r_L: 1
r_NodeDegree: 0.5

3

4

RL : 0.5
Angle: B
r_L: 0.5
r_NodeDegree: 0.5

4

1
Fuzzy Multilevel Graph Embedding

Automatic Indexing of graph repositories

Conclusions and future research challenges

Example - FMGE

FSMFV: 4,3,2,2,1,3,0,0,0,1,1,0,2,1,2,0,0,3,0,2,0,0,2,1

- Node degree: \([-\infty, -\infty, 1,2]\) and \([1,2,\infty,\infty]\)
- Attributes \(\{L, RL\}\): \([-\infty, -\infty, 0.5,1]\), \([0.5,1,1.5,2]\) and \([1.5,2,\infty,\infty]\)
- \(r_\text{Angle}\): \([-\infty, -\infty, 0,1]\) and \([0,1,\infty,\infty]\)
- Resemblance attributes: \([-\infty, -\infty, 0.25,0.5]\), \([0.25,0.5,0.75,1.0]\) and \([0.75,1.0,\infty,\infty]\)
- The symbolic edge attribute Angle has two possible labels
## Experimental evaluation of FMGE

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Size</th>
<th>Classes</th>
<th>Avg</th>
<th>Max</th>
<th>Attributes $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Train</td>
<td>Valid</td>
<td>Test</td>
<td>$</td>
<td>V</td>
</tr>
</tbody>
</table>
| IAM graph database

- Graph classification experimentations
- Graph clustering experimentations

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Size</th>
<th>Classes</th>
<th>Avg</th>
<th>Max</th>
<th>Attributes $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Letter LOW</td>
<td>750</td>
<td>750</td>
<td>750</td>
<td>15</td>
<td>4.7</td>
</tr>
<tr>
<td>Letter MED</td>
<td>750</td>
<td>750</td>
<td>750</td>
<td>15</td>
<td>4.7</td>
</tr>
<tr>
<td>Letter HIGH</td>
<td>750</td>
<td>750</td>
<td>750</td>
<td>15</td>
<td>4.7</td>
</tr>
<tr>
<td>GREC</td>
<td>836</td>
<td>836</td>
<td>1628</td>
<td>22</td>
<td>11.5</td>
</tr>
<tr>
<td>Fingerprint</td>
<td>500</td>
<td>300</td>
<td>2000</td>
<td>4</td>
<td>5.4</td>
</tr>
<tr>
<td>Mutagenicity</td>
<td>500</td>
<td>500</td>
<td>1500</td>
<td>2</td>
<td>30.3</td>
</tr>
</tbody>
</table>

$^a$ Number of attributes is given as a pair “numeric:symbolic”.
<table>
<thead>
<tr>
<th>Dataset</th>
<th>Graph edit distance based reference system</th>
<th>Dissimilarity based embedding</th>
<th>FMGE resemblance:AVG</th>
<th>FMGE resemblance:STD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[k-NN classifier]</td>
<td>Bunke et al. [Bunke and Riesen, 2011b]</td>
<td>[1-NN classifier]</td>
<td>[1-NN classifier]</td>
</tr>
<tr>
<td>Letter LOW</td>
<td>99.3</td>
<td>99.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Letter MED</td>
<td>94.4</td>
<td>94.9</td>
<td>75.7</td>
<td>75.7</td>
</tr>
<tr>
<td>Letter HIGH</td>
<td>89.1</td>
<td>92.9</td>
<td>60.5</td>
<td>60.5</td>
</tr>
<tr>
<td>GREC</td>
<td>82.2</td>
<td>92.4</td>
<td>97.5</td>
<td>97.5</td>
</tr>
<tr>
<td>Fingerprint</td>
<td>79.1</td>
<td></td>
<td>74.9</td>
<td>73.5</td>
</tr>
<tr>
<td>Mutagenicity</td>
<td>66.9</td>
<td></td>
<td>68.6</td>
<td>68.6</td>
</tr>
</tbody>
</table>

- Supervised machine learning framework for experimentation, employing the training, validation and test sets
- 1-NN classifier with Euclidean distance.
- Equal-spaced crisp discretization and the number of fuzzy intervals empirically selected on validation dataset.
Graph clustering experimentations

<table>
<thead>
<tr>
<th>Dataset</th>
<th>FMGE feature vector space</th>
<th>correctly clustered graphs (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Letter LOW</td>
<td></td>
<td>89</td>
</tr>
<tr>
<td>Letter MED</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>Letter HIGH</td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>GREC</td>
<td></td>
<td>82</td>
</tr>
<tr>
<td>Fingerprint</td>
<td></td>
<td>87</td>
</tr>
<tr>
<td>Mutagenicity</td>
<td></td>
<td>82</td>
</tr>
</tbody>
</table>

- Merged training, validation and test sets
- K-means clustering with random non-deterministic initialization
- The measure of quality of K-means clustering w.r.t. the ground truth: ratio of correctly clustered graphs to the graphs in the dataset
- Equal-frequency crisp discretization for automatically selecting the best number of fuzzy intervals
The average Silhouette width ranges between [-1, 1]. The closer it is to 1, the better is the clustering quality.
Unsupervised learning phase is performed off-line and is linear to:

- Number of node and edge attributes
- Size of graphs

Graph embedding phase is performed on-line
- 2D linear model symbols from GREC databases
- Learning on clean symbols and testing against noisy and deformed symbols
- SESYD dataset
- Learning on clean symbols and testing against noisy symbols

<table>
<thead>
<tr>
<th>Floor plans</th>
<th>Noise</th>
<th>Model symbol (classes)</th>
<th>Query symbol (each class)</th>
<th>Recognition rate (match with topmost result)</th>
<th>Recognition rate (a match in top-3 results)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level-1</td>
<td>16</td>
<td>100</td>
<td></td>
<td>84%</td>
<td>95%</td>
</tr>
<tr>
<td>Level-2</td>
<td>16</td>
<td>100</td>
<td></td>
<td>79%</td>
<td>90%</td>
</tr>
<tr>
<td>Level-3</td>
<td>16</td>
<td>100</td>
<td></td>
<td>76%</td>
<td>87%</td>
</tr>
<tr>
<td>Average recognition rate</td>
<td></td>
<td></td>
<td></td>
<td><strong>80%</strong></td>
<td><strong>91%</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electronic diagrams</th>
<th>Noise</th>
<th>Model symbol (classes)</th>
<th>Query symbol (each class)</th>
<th>Recognition rate (match with topmost result)</th>
<th>Recognition rate (a match in top-3 results)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level-1</td>
<td>21</td>
<td>100</td>
<td></td>
<td>69%</td>
<td>89%</td>
</tr>
<tr>
<td>Level-2</td>
<td>21</td>
<td>100</td>
<td></td>
<td>66%</td>
<td>88%</td>
</tr>
<tr>
<td>Level-3</td>
<td>21</td>
<td>100</td>
<td></td>
<td>61%</td>
<td>85%</td>
</tr>
<tr>
<td>Average recognition rate</td>
<td></td>
<td></td>
<td></td>
<td><strong>65%</strong></td>
<td><strong>87%</strong></td>
</tr>
</tbody>
</table>
- Not many methods for both directed and undirected attributed graphs
  - FMGE: Directed and undirected graphs with many numeric as well as symbolic attributes on both nodes and edges
- No method explicitly addresses noise sensitivity of graphs
  - FMGE: Fuzzy overlapping intervals
- Expensive deployment to other application domains
  - FMGE: Unsupervised learning abilities
- Time complexity
  - FMGE: Linear to number of attributes
    - Linear to size of graphs
    - Graph embedding performed on-line
- Loss of topological information
  - FMGE: Multilevel information (global, topological and elementary)
    - Homogeneity of subgraphs in graph
Loss of matching between nodes

No graph embedding based solution to answer high level semantic problems for graphs
Introduction

Fuzzy Multilevel Graph Embedding (FMGE)

Automatic indexing of graph repositories for graph retrieval and subgraph spotting
  - Method
  - Experimental evaluation - application to content spotting in graphic document image repositories
  - Discussion

Conclusions and future research challenges
- Bag of words inspired model for graphs
- Index the graph repository by elementary subgraphs
- Explicit GEM for exploiting computational strengths of state of the art machine learning, classification and clustering tools
- Unsupervised indexing phase
- Graph retrieval and subgraph spotting phase
- Unsupervised indexing phase
- Graph retrieval and subgraph spotting phase

Resemblance attributes → Cliques of order-2 → FSMFVs
- **Unsupervised indexing phase**

- Graph retrieval and subgraph spotting phase

---

FSMFV clusters using an hierarchical clustering technique

Classifier

INDEX
- Unsupervised indexing phase

- Graph retrieval and subgraph spotting phase

Resemblance attributes  Cliques of order-2  FSMFVs
- Unsupervised indexing phase

- **Graph retrieval and subgraph spotting phase**

Adjacency matrix of a result

\[ AG(i, j) = \begin{cases} 
0 & \text{no edge between } i \text{ and } j \\
1 & \text{an edge between } i \text{ and } j \\
2 & \text{clique is in result} 
\end{cases} \]

Where,

- clique for which score is computed
- clique having the same cluster label as one of cliques in query graph
- clique not having the same cluster label as one of the cliques in the query graph
- no edge in graph

\[ \text{score} = \frac{0 \times 4}{9} + \frac{1 \times 2}{9} + \frac{2 \times 3}{9} = 0.89 \]

\[ |z| \text{ is frequency of value } z \text{ in neighborhood} \]

\[ w \text{ is number of connected neighbors looked-up} \]
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Content spotting in document images 50
- SESYD dataset
- Corresponding graph dataset is made publically available
  

<table>
<thead>
<tr>
<th></th>
<th>Image</th>
<th>Attributed graph</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electronic diagrams</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Backgrounds</td>
<td>8</td>
<td>Avg. order</td>
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<tr>
<td>Models</td>
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<td>Avg. size</td>
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<tr>
<td>Symbols</td>
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<td>Node attribs.</td>
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<tr>
<td>Documents</td>
<td>800</td>
<td>Edge attribs.</td>
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<td>Queries</td>
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<td>Graphs</td>
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<td>800</td>
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<td></td>
<td></td>
<td>1000</td>
</tr>
<tr>
<td><strong>Architectural floor plans</strong></td>
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<td>Queries</td>
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<td></td>
<td></td>
<td>1000</td>
</tr>
</tbody>
</table>
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Experimental evaluation 53
Electronic diagrams: (517K 2-node subgraphs) (455 classes) (~17h)
Architectural diagrams: (306K 2-node subgraphs) (211 classes)
- Loss of matching between nodes
  - Score function is a first step forward

- No graph embedding based solution to answer high level semantic problems for graphs
  - FMGE based framework for automatic indexing of graph repositories
Introduction

Fuzzy Multilevel Graph Embedding (FMGE)

Automatic indexing of graph repositories for graph retrieval and subgraph spotting

Conclusions and future research challenges
Conclusions

- Last two decade’s research on structural pattern recognition can access state of the art machine learning tools.

- An impossible operation in original graph space turns into a realizable operation with an acceptable accuracy.

- Application to domains where the use of graphs is mandatory for representing rich structural and topological information and a computational efficient solution is required.

- Feature vector not capable of preserving the matching between nodes of a pair of graphs.
Conclusions

- Unsupervised and automatic indexing of graph repositories
- Domain independent framework
- Incorporating learning abilities to structural representations
- Ease of query by example (QBE)
- Granularity of focused retrieval
Future research challenges

- **Ongoing and short term**
  - Dimensionality reduction
  - Feature selection
  - More topological information

- **Medium term**
  - Detection of outliers for cleaning learning set
  - Multi-resolution index using cliques of higher order (≥3)
Future research challenges

- Long term
  - Surjective mapping of nodes of two graphs
<table>
<thead>
<tr>
<th>Publication Type</th>
<th>Description</th>
<th>Count</th>
</tr>
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<tbody>
<tr>
<td><strong>Journal paper</strong></td>
<td>Pattern Recognition (under review, submitted December 2011)</td>
<td>1</td>
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<tr>
<td><strong>Book chapter</strong></td>
<td>Bayesian Network by InTech publisher</td>
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<tr>
<td><strong>International conference contributions</strong></td>
<td>ICDAR 2011, ICPR 2010, ICDAR 2009</td>
<td>3</td>
</tr>
<tr>
<td><strong>Selected papers for post-workshop LNCS publication</strong></td>
<td>ICPR 2010 contests, GREC 2009</td>
<td>2</td>
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<tr>
<td><strong>International workshop contributions</strong></td>
<td>GREC 2011, GREC 2009</td>
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<td><strong>Francophone conference contributions</strong></td>
<td>CIFED 2012, CIFED 2010</td>
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</table>
Thank you for your attention.
Fuzzy Multilevel Graph Embedding for Recognition, Indexing and Retrieval of Graphic Document Images

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Friday, 2nd of March 2012

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