On the Relation between Natural Philosophy, Mathematics and Logic in the Investigation on Atomism in the Middle Ages

Lídia Queiroz
Universidade do Porto
Centro de Filosofia das Ciências da Universidade de Lisboa (CFCUL)
liqueiroz@sapo.pt

Abstract

In the first half of the fourteenth century, a great polemic surrounding atomistic conceptions arose at the University of Oxford and the University of Paris. This paper will focus on the use of two tools of investigation on atomism, to wit, "thought experiments" and the "theory of supposition", which reveal the prominence of the *a priori* in late medieval debates on atomism. The paper intends to show the heuristic role of those two tools in the investigation of unobservable phenomena. The imaginative scenarios express the relation between natural philosophy, mathematics and logic, illustrating the medieval conception of science.

Keywords: a priori; thought experiments; theory of supposition; Thomas Bradwardine

Resumen. *Sobre la relación entre la filosofía natural, la matemática y la lógica en la investigación del atomismo en la Edad Media*

En la primera mitad del siglo XIV, se ha levantado en la Universidad de Oxford y en la Universidad de París una gran polémica en torno a las concepciones atomistas. Este trabajo se centrará en el uso de dos herramientas de investigación en el atomismo, a saber, los «experimentos de pensamiento» y la «teoría de la suposición», que revelan la prominencia de lo *a priori* en los debates medievales tardíos sobre el atomismo. El artículo pretende mostrar el papel heurístico de estas dos herramientas en la investigación de fenómenos no observables. Los escenarios imaginativos expresan la relación entre la filosofía natural, la matemática y la lógica, ilustrando la concepción medieval de la ciencia.

Palabras clave: a priori; experimentos de pensamiento; teoría de la suposición; Thomas Bradwardine

Summary

ISBN 978-84-490-8043-2

1. Introduction

In the early fourteenth century, a great polemic around atomistic conceptions arose at the University of Oxford and the University of Paris. At Oxford, the philosophers Henry of Harclay (in *Questiones*) and Walter Chatton (in *Reportatio super Sententias*) have defended atomism, while Thomas Bradwardine (in *Tractatus de continuo*), William of Alnwick (in *Determinationes*) and Adam Wodeham (in *Tractatus de indivisibilibus*) have combated it. A little later, the same scenario of controversy erupted at Paris, with the atomists Gerard of Odon (in *Tractatus de continuo*) and Nicholas Bonetus (in *Praedicamenta*), among others, and their opponents, as John the Canon (in *Questiones super octo libros physicorum)*. The fact that all this controversy, both in England and France, being formed and developed in a short time (between approximately 1315 and 1335) and being illustrated by a significant number of texts written by supporters of opposing doctrines seems to show the importance given by medieval authors to the question.

What led to the emergence of atomistic conceptions may have only been the recognition that the arguments in book VI of *Physics* (where Aristotle develops a deep treatment of the continuum) were no longer fully convincing. One cannot also assure if it was the treatment of theological questions that led to arrive at an annulment of the traditional anti-indivisibilist perspective or if the theological questions where, to a large extent, we see appearing the discussion around the indivisibles only served as pretexts to the defense of their own atomistic conceptions (for instance, we can see the Aristotelian thesis of the impossibility of composition of continua by points or indivisibles appearing in a *quaestio* about the mouvement of angels). Anyway, what we do know is that in the late Middle Ages emerges a theory that had been adopted (and discussed) in Antiquity: atomism. And, in most authors, this medieval atomism is of mathematical nature: the *indivisibilia* are seen as geometric points, that is, absolutely unextended.

2. The Oxford Calculators and the thought experiments

Let's take Bradwardine's *Tractatus de continuo* as a reference text for this paper on atomism: this treatise is considered by John Murdoch as being perhaps "the brightest" (1974: 18) of all medieval works written in the context of the controversy around the atomistic composition of the continuum. This text also provides a clear image of the character essentially "analytical and critical" which characterizes the fourteenth century[1]. Bradwardine is known to be one of the so-called *Calculatores* of Oxford, that is, a group of logicians, mathematicians and physicists of the first half of the fourteenth century, who sought to apply the logico-mathematical calculation in solving problems of natural philosophy. Because of that methodology, they were called "Calculators".

1. *Vide* Murdoch, 1978; 1975a: 280-289; 1975b. This work is supported by a research grant from the Fundação para a Ciência e a Tecnologia (Portugal).

Logic plays a prominent role in the resolution of natural philosophy problems in the late Middle Ages and takes a leading role in the construction of arguments in the aforementioned *Treatise on the continuum*. The analytic character of late medieval learning can be seen as something like "natural philosophy without nature":

> what one finds as a dominant feature of fourteenth-century natural philosophy — especially at Oxford, but so too in the dissemination of the ideas and methods of *Anglicana* at Paris — is the extension of the application of logic and logico-mathematical techniques in not just resolving, but even in creating and then resolving, problems in natural philosophy (Murdoch, 1982b: 174-175).

The *philosophia naturalis* or *physica* that the "Calculators" of Oxford study relied on reasonings *secundum imaginationem* ("by imagination"), building up thought experiments (hypothetical or imaginary) based on objects expressed by the use of letters to designate the variables that are idealized in order to serve as the "thought experiments" of the problems in question (for instance, calculations on the movement)[2]. The Oxford philosophers developed then a new methodology for analyzing various issues of physical or theological nature to which they have called *procedere secundum solam imaginationem* (to proceed only according to imagination). Thus, the expression *secundum imaginationem* often begins to appear in the texts, to designate the speculative exercise of formulation and development of hypothetical theses for the analysis of issues that could never be considered if it depended on the direct observation of the listed cases.

The ambitioned goal is the refutation or approval of a particular position (real or imaginary) based on reason (and not on empirical verification). The technique of the imaginary or thought experiments allows to distinguish between natural and logical impossibilities: cases that could not be physically observed are, however, *imaginabiles absque contradictione* (imaginable without contradiction). There are things that are *impossibile per se*, that is, logically impossible. It can be done (*esse factibilis*) only that which does not imply contradiction (*quod non implicat contradictionem*) and — it is known — there are two possible senses of *implicare contradictionem*: (1) in the strict sense, what *includit contradictionem*, that is, a formal *contradictio* between two realities that cannot occur simultaneously (two contradictories); (2) in a broad sense, that which *includit repugnantiam*, that is, a reality that in no way can the mind or *intellectus* accept or conceive.

The reasoning *secundum imaginationem* supported on the application of logical subtleties of mathematical abstraction. One do not see in Oxford masters a clear phenomenon of valuing experience and observation in view of the "experimentation" of their theories. One can say that

2. For thought experiments in late medieval debates on atomism, *vide* Grellard, 2011.

> the question upon which we ourselves place so much emphasis, namely empirical applicability, was never seriously considered. [...] [Philosophers of the Merton College were] concerned with consistency rather than truth. [...] In the almost total absence of the requisite sorts of measuring apparatus it is hard to see how matters could have been otherwise (North, 1992a: 88).

Alternatively, those philosophers test their theories "mentally"[3], considering the remarkable advances in logic of Oxonian tradition. The reasoning *secundum imaginationem* must obey a requirement of plausibility: if something was consistently imagined, it was then demonstrated its possibility.

As said before, in *Tractatus de continuo*, Bradwardine denounces the absurdities that the defense of *compositio continui ex indivisibilibus* engenders, and he reaches this goal approaching the question *de compositione continui* from various angles. It is in the "Conclusions" of the treatise that the philosopher seeks to refute the atomistic theses, highlighting the impossibility of being true because they collide with basic principles of all domains of knowledge. In a treatise conducted according to the axiomatic method, the "Conclusions" begin by being demonstrated by logical deduction from a set of undemonstrated propositions but admitted as true (a series of "Definitions" and "Assumptions" presented at the beginning of the work) and, then also, from propositions already proven. Let's turn our attention to an example of a thought experiment present in Bradwardine's *Tractatus de continuo*, more specifically in the porism of Conclusion 19:

> Vas concavum resupinum positum equedistantis orizonti supra locum elementi fluxibilis plus istius elementi continere in loco humo quam alto. Tali vero vase pleno elementi huius ascendente affluere quasdam partes: descendente vero contentum fluidum congregari, et maxima vasis latera vacua derelinqui, atque liquidi summitatem ultra vasis dyametrum continue elevari. Rursum tale vas talis elementi semiplenum ascendens fieri aliquotiens magis plenum, aliquotiens vero plenum et superius cumulatum: et aliquando in tantum quod affluent quedam partes descendens effici minus plenum. Si vero tale vas ponatur simpliciter infra locum huius elementi, per totum contraria prioribus evenire (Bradwardine, 2013: 110 and 112).

This porism is extracted from a Conclusion of mathematical nature, in which Bradwardine intends to demonstrate the infinite divisibility of a finite straight line. Demonstrating that a geometric line can be infinitely divided into parts (one can always take a part out of what has already been taken, without ever reaching a minimum from which one can no longer advance in division, that is, an indivisible), what Bradwardine tries to prove is that every magnitude, of whatever extension, can be infinitely divided. We will approach this subject again further on (in another section of this paper); for now, let us focus on the porism, which is related to these two phenomena of nature:

3. About the role of the procedure *secundum imaginationem* and the "natural confirmation" of a thesis, *vide* Murdoch, 1982b.

condensation and rarefaction. Consider "horizon" as what we call the "horizon line", that is, the line of apparent contact between Heaven and Earth. And from here we must consider that such a vessel would not remain immobile on this line all the time; on the contrary, it would be subject to both upward and downward movements, of different vessel positions: to above that line and sometimes further down. And follow the "natural" results: the surface of a liquid in a container will rise progressively the more the container is taken to the center of the earth, and, conversely, it will decrease at its peak the further the container is raised to the heavens (or in other words, the same container will have more water when lower than when higher). The natural corollary is demonstrated as follows:

> A centro mundi ducantur linee recte ad singula puncta in superficie suprema talis particule, que omnes, si sunt equales, habetur intentum. Si non sunt equales, igitur illud grave fluxibile exeuns in termino linee longioris et per consequens superius descendet inferius ad latus, et sic donec omnes partes sperice adequantur […] (Bradwardine, 2013: 114).

As we have just seen, this imaginative scenario expresses the relation between natural philosophy and mathematics. And, as we can easily understand, such experience of natural science could only be a thought experiment: given the structure of the experiment, it wouldn't be possible to perform it. Thought experiments are very useful when particular physical experiments are impossible to conduct — and so they are made in imagination, becoming theoretical experiments. In Peter King's words, "thought experiments, in their mediæval use, support theories which have no check or control, no way to test their correctness or incorrectness, as opposed to the modern experimental method" (1991: 56). A thought experiment is a device with which one performs an intentional, structured process of speculation within a problem domain, where the emphasis is on the conceptual, rather than on the experimental part.

According to James Weisheipl,

> the more valuable contribution to problems of physics was made by Thomas Bradwardine and the other Mertonian 'calculators' who tried to examine physical problems in terms of mathematical principles. All such problems were argued *longe et late* in what might be called a 'letter-calculus', originally suggested by Aristotle himself in *Physics* but carried to extremes by the Oxford calculators […]. This letter-calculus was simply the use of letters of the alphabet to signify terms or concepts, and the practice of arguing logically with these letters, much as Euclid had done in his *Elements* (Weisheipl, 1984: 626).

In fact, in the approach of the thematic of continuity, it had become common practice, in the late Middle Ages, the use of two new methods: the "logical" and the "geometrical". Intellectuals of the fourteenth century developed them. And among twelve areas of knowledge used by Bradwardine in his refutation of atomism, one can easily perceive the prominence of two powerful weapons: mathematics and logic.

Taken as a rigorous paradigm, Euclid's *Elements* is of crucial importance in the aforesaid medieval treatise, where a unique role is recognized to mathematics in the discovery of the truth about problems of natural philosophy, as it can be appreciated in Conclusion 56 of the *De continuo*:

> Nullus enim physico certamine se speret gavisurum triumpho nisi mathematica utatur consilio et auxilio confortetur. Ipsa est enim revelatrix omnis veritatis sincere et novit omne secretum absconditum at omnium litterarum subtilium clavem gerit. Quicunque igitur ipsa neglecta physicari presumpserit, sapientie ianuam se nunquam ingressurum agnoscat (Bradwardine, 2013: 158).

Bradwardine has not neglected mathematics at all, showing himself absolutely aware that the gateway to physics has to be opened by it. More contemporaneously, it started to be discussed whether it is appropriate to speak of the embryonic development of a new physics at Oxford, in the fourteenth century, the beginning of a mathematized physics[4]. Meanwhile, among medievalists it has become common to disseminate the idea that it was developed a "new physics" or "mathematical physics" in Oxford school and that the *new physics* of the fourteenth century was transmited from England to the continent.

3. The theory of supposition

Examining Bradwardine's *Tractatus de continuo*, we can also ask ourselves whether we are facing a logicized physics[5]. Another aspect that deserves our attention when addressing the question of atomistic theses being "mentally" tested is the "theory of supposition" within logic. The theory of supposition, of the terminist logic developed in the twelfth and thirteenth centuries, stimulated great debates and scientific developments in medieval logic of the fourteenth century, affecting other areas of knowledge, namely natural philosophy. In William Courtenay's *Schools & Scholars in Fourteenth-Century England* one reads that:

> the thrust of fourteenth-century terminist logic was to simplify a more elaborate procedure by reducing the necessary steps for arriving at solutions and by using symbols to represent longer phrases or procedures. Often this was done by substituting a letter for a phrase or proposition. The more extensive use of Latin abbreviations also served this purpose […]. Syncategorematic terms were usually more highly abbreviated than categorematic terms. Again, the

4. Among many other reading possibilities, *vide*, *v. g.*, Sylla, 1997. The thesis that modern science began in the Middle Ages is controversial. The reading of Grant, 1996: xi-xiv, is quite inspiring for a reflection.
5. On the prominent role of Logic in solving problems of natural philosophy, in the late Middle Ages, *vide* Murdoch, 1989.

> use of symbols and the shortening of steps in problem solving were aspects that logic shared with mathematics (Courtenay, 1987: 241)[6].

Logic became the methodological basis of all disciplines and the Oxonian masters took into extreme the Aristotelian idea that logic serves as *organon/instrumentum* for sciences.

The theory of supposition was created in the medieval period and it corresponds to what today we would call a "theory of quantification" (Murdoch, 1974: 25)[7]. Medieval philosophers saw in the theory of supposition an extraordinary resource to show the errors in which the atomists incurred, drawing attention to the grammatical position where the syncategoremes appear in a proposition affecting the extension of categorematic terms, and allowing to perceive big differences between apparently equal propositions.

The *logica modernorum* (distinct from *logica antiquorum*, that is, Aristotelian and Boecian logic) is a *logica terminorum* (logic of terms): the logic centered on proposition (the *logica antiqua*) is now conjugated with that of the syntax and semantics of terms (articulated in the distinction between *significatio* and *suppositio*). The scholastic logicians conduct a study *de proprietatibus terminorum*, giving the prominent place not so much to the signification (*significatio*) of a term (given by its definition) but to its supposition (*suppositio*).

The supposition of a term is discovered by an intrapropositional analysis, by observing the syntactic grammatical function (and sometimes semantics) of the terms in a proposition. The terms are either categoremes or syncategoremes: the firsts, signify something by themselves, by conventional representation (nouns, adjectives, pronouns and verbs); the seconds, the remaining, nothing representing or supposing themselves (*supponere pro* — to take the place of), that is, conjunctions, adverbs and prepositions. Firstly, it was considered "syncategoremes" the auxiliary terms in the proposition, that is, those words that only have a signification if in combination with others; but then (towards the end of the thirteenth century), what happened was that some terms came to be considered problematic and to be treated as syncategoremes, and the list increased. *Syncategoremata* are the terms *omnis*, *totus*, *ambo*, *quaslibet*, *et*, *vel*, *an*, *preter*, *neuter*, *nullus*, *non*, *solus*, *tantum*, *si*, *nisi*, *infinitum*, *incipit*, *desinit*, *maximum*, *minimum*, *maius*, *minus*, *tempus*, *instans*, *gradus*, *spacium*, *pars proportionalis*, *divisum*, *velocius*, *motus*, and others, because all of these have the power of affecting the supposition of others in the proposition and, therefore, the meaning of it: either because they are clearly syncategoremes or because they are in *function*. The list of syncategoremes then goes on to include names, adjectives and verbs. The works *Syncategoremata*, the one of Peter of Spain and the one of William of Sherwood, are major milestones in the study of syncategoremes of Parisian and Oxonian tradition of "modern"

6. On the learning *for the domain of Logic* which medieval philosophers extracted *from* Euclidean *geometry*, by the fact of the treatise being an axiomatic system, *vide* Murdoch, 1970.
7. For a concise deepening of the idea of "quantification in medieval physics", *vide* Crombie, 1990: 73-90 (especially 89-90).

logic, respectively. And Oxford authors reject all extra-propositional supposition, they never admitted the existence of the so-called "natural supposition" of Parisian logic. For the Oxonian logic, it only makes sense to speak of the function of signification based on a propositional context.

The theory of *suppositio* is a novelty introduced by scholasticism that would prove fruitful for logical developments, adding, however, more difficulties. The truth or falsity of propositions begins to be analyzed in terms of the supposition of the syncategorematic expressions and, to know it, it is necessary to look at the grammatical position of the parts of the discourse. Thus, it is revealed that some propositions were true although they appeared to be false and others false even though they seemed to be true. There could also be a third alternative: a single proposition to have a proof and a refutation, both plausible. To medieval sophisms (*sophismata*) would be given a fundamental role in this propositional analysis: ambiguous or obscure propositions offered themselves as an illustrative puzzle of problems of supposition, which had to be solved. But "the whole point is not to rule out ambiguities, but to bring them under control" (Novaes, 2005: 36). The critical examination of sophisms requires the leap from natural language to a metalinguistic plan. Throughout this, the logical rules are being applied or discovered, allowing the identification of the causes of the deceit of the sophism or even to show that it does not occur, that is only apparent[8]. Sophisms belonged to the university curriculum in the fourteenth century.

A *sophisma* is not a "sophism" in the usual sense of the term (its identification with the typical reasoning of the sophists), that is, it is not a fallacious proposition through which one seeks to defend something false, trying to confuse and to deceive the opponent. It is rather a kind of "puzzling-sentence" (Libera, 1998: 387), which first requires the elucidation of its difficulties in logical terms so that the false interpretation is rejected. Through sophisms (logical puzzles or paradoxes), one explores problems of the suppositional logic, testing the propositions. Especially in the fourteenth century, sophisms were evaluated on the basis of syntactic and semantic rules. The logical disputes *de sophismatibus* had become a traditional method of work at the Faculties of Arts and had proved to be very useful for clarifying the problems involved by the presence of syncategorematic terms in the propositions, since they may play different roles in them depending on their relative positions. Thus, the distinct kinds of suppositions (*modi supponendi*) were illustrated by the study of sophisms. Fourteenth century Oxonian logicians exalted the idea that the supposition occurred *within* and *by* the propositional context and the sophism becomes a method that extends itself to all areas of knowledge, namely to natural philosophy. The *sophismata* allow to test the validity of logical rules and to make demonstrations. In Oxford, the *Calculatores* discuss the

8. To deepen the medieval notion of "sophism", *vide* Murdoch, 1982c: 85-100; Kretzmann, 1982: 211-240; Yrjönsuuri, 1990: 645-654.

cases, the rules and the arguments from a metalinguistic point of view[9], not allowing thus that philosophical imaginary be limited by impediments of realistic nature. The *sophisma* becomes then a conceptual framework that allows the examination of a question from the point of view of what is logically permissible and not subjugated to what is factually possible.

Speculating about the factors which had led the logic (and the propositional analysis: terms and their context) to assume a dominant position in the framework of knowledge in the late thirteenth — fourteenth century, John Murdoch states that:

> the utilization of sophisms, the application of logic, and metalinguistic analysis constitute something that is generally characteristic of fourteenth-century philosophy […]. These things represent a good part […] of what made the fourteenth century 'analytic'. It is presently difficult to say just why such a way of approaching and resolving problems increased so markedly in this century. Surely, the very central position occupied by logic in medieval university curricula was a contributing factor. Yet if we focus our attention more strictly upon the phenomenon of metalinguistic analysis, then it seems extremely plausible that another cause of the rise of *that* kind of 'being analytic' is to be found in the rapidly growing belief in the absolute contingency of the natural world and in the concomitant concerns with the certitude of knowledge and with the grounding of this certainty, not in contingent *things*, but in *propositions* as the only possible bearers of the requisite universality and necessity characteristic of demonstrative knowledge (Murdoch, 1982b: 197-198)[10].

4. Atomism and infinite divisibility

In a proposition, the term *infinitum* can be used in a categorematic or syncategorematic sense, depending on the position in which it appears in the proposition, differently affecting the name to which it confers a *suppositio determinata* or a *suppositio confusa*. The extension of the categoremes is modified by the presence of the syncategoremes and the exercise of the *sophismata* is, therefore, linked to the resolution of logical-semantic problems. Such syncategorematic expressions have sufficient force to affect the truth or falsity conditions of a proposition. If a syncategoreme appears in Subject position, it is to be taken syncategoremetically; if in Predicate position, categoremetically[11]. The Latin expressions *tantum quod non maius sive tot quod non plura* and *non tantum quin maius sive non tot quin plura* illustrate, respectively, the ideas of categorematic infinity and syncategorematic infinity, relative to continuous or discrete quantities. In fact, they correspond to how scholasticism designated infinity in act and infinity in potency, that Bradwardine expresses in the defi-

9. Absolutely fundamental is the reading of Murdoch, 1981.
10. *Vide* also Murdoch, 1982a: 204-205.
11. The position "subject" or "predicate" is understood to be before or after the copula in the proposition, respectively.

nition of the categorematic infinity (*quantum sine fine*) and the syncategorematic infinity (*quantum et non tantum quin maius*) (Bradwardine, 2013: 76).

The notions of infinity in act and in potency go back to Aristotle's philosophy. The Aristotelian refutation of the actual infinity is presented in the third book of *Physics*, where the philosopher clearly shows that "there is no actually existent infinite body" (Aristotle, *Physics*, III, 5, 206a7). However, the existence of the infinite also can not be denied at all: it occurs naturally[12], in the form of potentialities. The Stagirite distinguishes the (1) infinite by addition of the (2) infinite by division[13]. Both reveal a possible *potential* infinity in both continuous and discrete quantities. Thus, number, time and continuum manifest the infinity by addition, and time and continuum may also reveal the infinity by division (in the latter case, the number no longer appears in the list, since it has no maximum but it has a minimum). The infinite in potency occurs when, for example, a continuum is divided into parts[14] or when this, by composition, can always continue to receive a new part[15]. The "infinite" is therefore an attribute of number and of magnitude, it is predicate of quantity. Now, "the infinite cannot exist actually" (Aristotle, *Metaphysics*, XI, 10, 1066b11), "it has no independent existence as the finite has" (*Idem*, *Physics*, III, 6, 206b14-15)[16]. The potential infinite is dynamism, in contrast to an actual infinity that would be completeness, that is, implying the presence of a limit[17]. Hence the categorematic infinite is the infinite in absolute (*simpliciter*), a maximum, while the syncategorematic infinite is a relative infinite (*secundum quid*), that is, it points out to the possibility of being always conceived a greater quantity than the whole given quantity (however great it may be), that is, surpassing it. Thus, the *infinitum secundum quid* admits more and less, which does not happen with the *infinitum simpliciter*. In line with the Aristotelian tradition, "being" has then more than one meaning[18], and a group of these meanings derives from the fact that something can exist in act or in potency. The *infinitum in actu* is "that which there is nothing beyond" and the *infinitum in potentia* "that of which some part is always beyond" (Aristotle, *Physics*, III, 6, 206b34-35)[19].

12. Cf. Aristotle, *Physics*, III, 7, 207b1-3.
13. In Aristotle, *Physics*, III, 6, 206b3-26.
14. One can assure the existence of the potential infinite for the fact that "division never ceases to be possible" (Aristotle, *Metaphysics*, IX, 6, 1048b15-16).
15. It should be noted, however, that Aristotle does not allow the existence of a physical magnitude which, by an infinite additive, surpasses the greatest of the existent things (*vide* Aristotle, *Physics*, III, 6, 206b20-27). And one does not arrive at infinite in act (an infinite magnitude or multitude) by the successive addition of finite parts.
16. Aristotle, *Metaphysics*, IX, 6, 1048b14-15: "But the infinite does not exist potentially in the sense that it will ever actually have separate existence; its separateness is only in knowledge".
17. Cf. Aristotle, *Physics*, III, 6, 207a13-14.
18. Cf. Aristotle, *Physics*, III, 6, 206a19.
19. The actual infinite or in categorematic sense is a determinate infinite, whereas the potential infinite or in syncategorematic sense is an infinite in process, unfinished, always susceptible of receiving something extra. With respect to a given quantity, we can divide it or add new

As aforesaid, because *infinitum* can be either categorematic or syncategorematic, it is possible to form sophisms whose solutions turn on the distinction between the categorematic and the syncategorematic *use* of the term in a given proposition. Let us consider these two propositions as examples:

1) "Continuum divisibile esse in infinitum" (Bradwardine, 2013: 80).
2) "In infinitum continuum est divisibile".

In the first proposition, *infinitum* comes after the copula and so it is to be taken as a categoreme, which means that a continuum can be completely divided into an infinity of parts *in actu*, and so the proposition is false. On the other hand, in the second proposition the term *infinitum* comes in the beginning of it, having there the meaning of to be *in potentia*, expressing potential infinite divisibility; and so the proposition is true, considering the possible number of cuts. In sum, an inverse order of the term *infinitum* within a proposition and the truth value is otherwise. To clarify "exactly why one or the other position of a term like *infinitum* had one result rather than another was, more often than not, a matter of convention" (Murdoch, 1982a: 195). Taking in consideration the theory of supposition within philosophical discussions — using logic as an argumentative and demonstrative method —, we can observe the chains of rigidity and technicity[20] arising, since it demands logical sophistication. Supposition theory is the basis for a theory of thuth conditions.

5. The relation between Mathematics and Physics in scholastic philosophy

It is important to note that, in Bradwardine's *Tractatus de continuo*, by means of a single demonstration — conducted according to the axiomatic method, respecting the rules of logic and the due order of the deductive steps — valid inferences are made in three domains: (1) what is true of a particular case; (2) what is true of any other case of the same class, generalizing the results obtained by means of x methodical procedure (ie: it shall be understood, for instance, "Be a straight line AB…" as the expression of the following idea "Consider the straight line just like any straight line")[21]; and (3) what is true in the realm of physics. In relation to this last level of the generalizing tendency of the demonstration, we shall note that after the statement of Conclusion 57 of the *De Continuo* one reads that: "duplex est scientia speculativa, scilicet realis et sermocinalis. Realis est triplex, ut patet 6° Methaphysice, scilicet mathematica, naturalis et divina" (Bradwardine, 2013: 158).

parts to it successively; beginning to enumerate the parts, as such, they exist and are finite, and those which remain to be enumerated, infinite.

20. For more technical details about the medieval theory of supposition, *vide* Parsons, 2014: 184-226.
21. The demonstrations that Bradwardine presents are about particular cases of continua and the guarantee that such proofs have a generalizable validity to all cases of the same kind is related to *Suppositio* 3 of the treatise.

It should be noted that, according to Aristotle, mathematics was included in a group designated as "real sciences", where physics and metaphysics also stood. Scholastic authors, relying on Aristotelian philosophy, believed that what is said about the mathematical continuum is equally true for the physical continuum (which is also explicable by the understanding of mathematics as *scientia realis*). For Aristotle, mathematical beings are "structural modes of being of sensible things" (Reale, 1999: 185). As George Molland states, "Bradwardine's *realist* view of geometry also led him to take the ontological consequences seriously" (1978: 135 [*our italics*]). Taking the Aristotelian way, as the objects of geometry have a potential existence in nature, although the demonstration of Bradwardine is made within the framework of geometric continua, the results obtained still apply to the physical continua[22]. To know, "the only reason that allowed Bradwardine to make this erroneous inference lay in a fundamental confusion between the formal, mathematical sciences and those of the physical world" (Murdoch, 1957: 316). This treatise mirrors well the philosophy of science in the fourteenth century.

Let's not conclude, however, that the aim which the medieval atomists had in view, with their theories, was to establish physical laws for natural phenomena. There is nothing in their writings which could indicate that intention[23]. What happens is that these philosophers actually engage in the discussion of questions such as qualitative change, the propagation of light, etc., but they do so because they need to find the variables for the mathematical approach of a more general problem. Using these resources, they present new conceptions about the continuum, in an inevitable offensive, thus, against the anti-indivisibilist doctrine of Aristotle. And for such they use mathematics because the atom of which they speak about has no extension, being idealized as a mathematical point. And "this form of atomism seemed natural to medieval thinkers. For this atomism resulted not from a physical exploration of nature, but from a purely intellectual reaction to the abstract analysis that Aristotle had made of the continuity of magnitudes" (Murdoch, 1974: 27).

Bibliographical references

Aristotle, Physics: *Aristotle's Physics*. A Revised Text with Introduction and Commentary by W. D. Ross. Oxford: Clarendon Press, 1979.

— *Metaphysics*, transl. by W. D. Ross. In: Barnes, Jonathan (ed.). *The Complete Works of Aristotle. The Revised Oxford Translation*, vol. II. Princeton: Princeton University Press, 1985, 1552-1728.

22. Murdoch, 1957: "Bradwardine was firmly convinced that whatever he had to say concerning the mathematical — for him strictly geometrical — continuum applied also to the physical continuum" (p. 316); "he undoubtedly intends to infer that whatever be the structure of the mathematical continuum, so must be the structure of the physical continuum" (p. 324).
23. "The Oxford indivisibilists do not seem to have been anxious to promote physical atomism" (North, 1992b: 149).

Courtenay, William (1987). *Schools & Scholars in Fourteenth-Century England*. New Jersey: Princeton University Press.
Crombie, Alistair (1990). *Science, Optics, and Music in Medieval and Early Modern Thought*. London—Ronceverte: The Hambledon Press.
Grant, Edward (1996). *The Foundations of Modern Science in the Middle Ages: their Religious, Institutional, and Intellectual Contexts*. Cambridge: Cambridge University Press.
<https://doi.org/10.1017/CBO9780511817908>
Grellard, Christophe (2011). "Thought experiments in late medieval debates on atomism". In: Ierodiakonou, Katerina & Roux, Sophie (eds.). *Thought Experiments in Methodological and Historical Contexts*. Leiden-Boston: Brill, 65-79.
King, Peter (1991). "Mediaeval Thought-experiments. The Metamethodology of Mediaeval Science". In: Horowitz, Tamara & Massey, Gerald (eds.). *Thought Experiments in Science and Philosophy*. Lanham: Rowman & Littlefield, 43-64.
Kretzmann, Norman (1982). "Syncategoremata, Exponibilia, Sophismata". In: Kretzmann, Norman *et al.* (eds.). *The Cambridge History of Later Medieval Philosophy. From the Rediscovery of Aristotle to the Disintegration of Scholasticism 1100-1600*. Cambridge: Cambridge University Press, 211-245.
Libera, Alain de (1998). *La Philosophie médiévale*. Paris: Presses Universitaires de France.
Molland, George (1978). "An Examination of Bradwardine's Geometry". *Archive for History of Exact Sciences*, 19, 113-175.
<https://doi.org/10.1007/BF00328611>
Murdoch, John (1957). *Geometry and the Continuum in the Fourteenth Century: A Philosophical Analysis of Thomas Bradwardine's Tractatus de Continuo*, Ph.D. dissertation, University of Wisconsin.
— (1970). "The Medieval Euclid: Salient Aspects of the Translations of the Elements by Adelard of Bath and Campanus of Novara". In: *Colloques. Textes des rapports*, vol. 1. Paris: Albert Blanchard, 67-94.
— (1974). "Naissance et développement de l'atomisme au Bas Moyen Âge Latin". In: Allard, G.-H. & Ménard, J. (eds.). *Cahiers d'études médiévales II. La science de la nature: théories et pratiques*, 11-32.
— (1975a). "From Social into Intellectual Factors. An Aspect of the Unitary Character of Late Medieval Learning". In: Murdoch, John & Sylla, Edith (eds.). *The Cultural Context of Medieval Learning. Proceedings of the First International Colloquium on Philosophy, Science and Theology in the Middle Ages*. Dordrecht—Boston: D. Reidel Publishing Company, 271-348.
<https://doi.org/10.1007/978-94-010-1781-7_9>
— (1975b). "A Central Method of Analysis in Fourteenth-Century Science". In: *Proceedings of the* xivth *International Congress of the History of Science, vol. 2. Tokyo:* Science Council of Japan, 68-71.
— (1978). "The Development of a Critical Temper: New Approaches and Modes of Analysis in Fourteenth-Century Philosophy, Science and The-

ology". In: Wenzel, Siegfried (ed.). *Medieval and Renaissance Studies. Proceedings of the Southeastern Institute of Medieval and Renaissance Studies*. Chapel Hill: The University of North Carolina Press.

— (1981). "'*Scientia mediantibus vocibus*': Metalinguistic Analysis in Late Medieval Natural Philosophy". In: Beckmann, Jan *et al.* (eds.), *Sprache und Erkenntnis im Mittelalter*. Acts of the Sixth International Congress of Medieval Philosophy, (*Miscellanea Mediaevalia*, vol. 13/1). Berlin: Walter de Gruyter, 73-106.

— (1982a). "William of Ockham and the Logic of Infinity and Continuity". In: Kretzmann, Norman (ed.). *Infinity and Continuity in Ancient and Medieval Thought*. Ithaca—London: Cornell University Press, 165-206.

— (1982b). "The Analytic Character of Late Medieval Learning: Natural Philosophy without Nature". In: Roberts, Lawrence (ed.). *Approaches to Nature in the Middle Ages*. Binghamton—New York: Center for Medieval & Early Renaissance Studies, 171-213.

— (1982c). "Mathematics and Sophisms in Late Medieval Natural Philosophy". In: *Les genres littéraires dans les sources théologiques et philosophiques médiévales. Actes du Colloque International de Louvain-la-Neuve*. Louvain-la-Neuve: Publications de l'Institut d'Études Médiévales de l'Université Catholique de Louvain, 85-100.

— (1989). "The Involvement of Logic in Late Medieval Natural Philosophy". In: Caroti, Stefano & Murdoch, John (eds.). *Studies in Medieval Natural Philosophy*. Firenze: Leo S. Olschki Editore, 3-28.

North, John (1992a). "Natural Philosophy in Late Medieval Oxford". In: Catto, J.I. & Evans, Ralph (eds.). *The History of the University of Oxford*, vol. II. Oxford: Clarendon Press, 65-102.

— (1992b). "Astronomy and Mathematics". In: Catto, J.I. & Evans, R. (eds.). *The History of the University of Oxford*, vol. II. Oxford: Clarendon Press, 103-174.

Novaes, Catarina (2005). *Formalizations «après la lettre». Studies in Medieval Logic and Semantics*. Alblasserdam: Haveka BV.

Parsons, Terence (2014). *Articulating Medieval Logic*. Oxford: Oxford University Press.
<https://doi.org/10.1093/acprof:oso/9780199688845.001.0001>

Reale, Giovanni (1999). *Guía de lectura de la «Metafísica» de Aristóteles*. Barcelona: Herder.

Sylla, Edith (1997). "Transmission of the New Physics of the Fourteenth Century from England to the Continent". In: Caroti, Stefano & Souffrin, Pierre (eds.). *La nouvelle Physique du xiv*e *siècle*. Firenze: Leo S. Olschki, 65-110.

Tomás Bradwardine (2013). *De continuo.* Introdução, tradução e notas por Lídia Queiroz, texto latino por J. Murdoch (†), prefácio J. Meirinhos. Porto: Afrontamento.

Weisheipl, James (1984). "Ockham and the Mertonians". In: Catto, J.I. (ed.). *The History of the University of Oxford*, vol. I. Oxford: Clarendon Press, 163-213.

Yrjönsuuri, Mikko (1990). "Obligationes, Sophismata and Oxford Calculators". In: Knuuttila, Simo *et al.* (eds.). *Knowledge and the Sciences in Medieval Philosophy. Proceedings of the Eighth International Congress of Medieval Philosophy*, vol. II. Helsinki: Publications of Luther-Agricola Society Series B 19, 645-654.

Lídia Queiroz holds a PhD in Philosophy from the Faculty of Arts of University of Porto. She completed a postgraduate degree in medieval philosophy at that Faculty, obtained the European Diploma in Mediaeval Studies of FIDEM (in Rome) and studied paleography at the Center for the History of Philosophy and Science (in Nijmegen, Holland). She is a postdoctoral researcher at the Institute of Philosophy of the University of Porto and at the Center for Philosophy of Sciences of the University of Lisbon.

Lídia Queiroz es doctora en Filosofía por la Faculdade de Letras de la Universidade do Porto. Concluyó, en la misma Facultad, un posgrado en filosofía medieval. Obtuvo el Diplôme Européen d'Études Médiévales de la FIDEM (en Roma) y estudió paleografía en el Center for the History of Philosophy and Science (en Nijmegen, Holanda). Es becaria de postdoctorado en el Instituto de Filosofia de la Universidade do Porto y en el Centro de Filosofia de las Ciências de la Universidade de Lisboa.