Departament d'Economia Aplicada

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DOCUMENT DE TREBALL

04.04

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Unemployment, growth and fiscal policy: new insights on the hysteresis hypothesis[*]

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April 23, 2004

Abstract

We develop a growth model with unemployment due to imperfections in the labor market. In this model, wage inertia and balanced budget rules cause a complementarity between capital and employment capable of explaining the existence of multiple equilibrium paths. Hysteresis is viewed as the result of a selection between these different equilibrium paths. We use this model to argue that, in contrast to the US, those fiscal policies followed by most of the European countries after the shocks of the 1970's may have played a central role in generating hysteresis.

Key Words: unemployment, hysteresis, multiple equilibria, economic growth, fiscal policy.

JEL Classification Numbers: E24, E62, O41.

1. Introduction

This paper takes a new look at the hysteresis hypotheses and provides new insights to evaluate the European unemployment problem. More precisely, the aim of this paper is to provide an explanation of the following two empirical regularities that have not received sufficient attention: i) the close correlation between the unemployment rate trajectory and the growth rate of the capital stock; and ii) the existence of two regimes (this being the central feature of the hysteresis hypotheses) in the unemployment rate and the growth rate of capital stock. In contrast with most of the existing literature, we take a dynamic general equilibrium approach and explain these empirical regularities as the result of equilibria selection in an endogenous growth model with wage inertia, where direct taxes are set by the government to balance its budget constraint.

The fact that European labor markets have never recovered the full employment levels which characterized the 1960s and first 1970s remains as one of the main puzzles in economics. Among the major conceptions of the labor market analysis, the hysteresis hypothesis tackles this puzzle outlining the role played by the extremely persistent effects of the temporary shocks occurred in the 1970s. Within the studies explaining unemployment hysteresis, we should differentiate between those arguing that temporary shocks have persistent effects on unemployment because the speed of convergence is extremely low, and those arguing that temporary shocks have persistent effects because these shocks make agents coordinate to another equilibrium path, where the economy remains when the shock is over. Our paper belongs to the latter line of research, and explains the patterns of unemployment as a result of equilibrium selection.

Blanchard and Summers (1988) argued that it was necessary to go beyond the natural rate hypothesis and concluded that "theories of fragile equilibria [a concept to highlight the sensitive dependence of unemployment on current and past events] are necessary to come to grip with events in Europe". Despite this claim, the work on multiple equilibria has not played a major role in the literature. Two main contributions in this area are Diamond (1982) and Mortensen (1989), but in the context of search and matching models, which fall well apart from the dynamic general equilibrium approach we propose in this paper. From the more traditional perspective of the demand-supply side analysis, Manning (1990 and 1992) argued in favor of models with multiple equilibria to explain the postwar behavior of unemployment. Nonetheless, the mainstream literature on unemployment in the 1990s has kept apart from the multiple equilibria perspective and, following the work by Layard, Nickell and Jackman (1991), has focused mainly on the NRU/NAIRU (i.e., a unique unemployment equilibrium rate), leaving also the hysteresis hypothesis a secondary role. Some work is, of course, being done on the hysteresis hypotheses, but mainly with an empirical concern.[1] Part of this literature is related with the finding of multiple equilibria in unemployment rates, generally by the use of Markow regime switching models. For example, in León-Ledesma-McAdam (2003) the presence of a high and low equilibria in most of the Central and Eastern European countries is observed; Akram (1998 and 1999) applies this analysis to Norway and, finally, Bianchi and Zoega (1998) find that the observed persistence in the

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[1]Some examples are Cross (1988 and 1995), and some papers therein that relate the hysteresis hypothesis with the NRU; Jaeger and Parkinson (1994) and, recently, Piscitelli, Cross, Grinfeld and Lamba (2000), Hughes Hallet and Piscitelli (2002) and León-Ledesma-McAdam (2003), on the empirical testing of the hysteresis hypotheses.

unemployment rate of 15 OECD countries is consistent with multiple equilibria models.

Given the empirical bias of this work, the main sets of candidates for explaining hysteresis are still the ones initially proposed in Blanchard and Summers (1986a and 1986b), a first one pointing to insider-outsider arguments, a second one to capital accumulation (either in the form of physical or human capital) and a third one to fiscal policy. In contrast with the extensive literature on the insider-outsider argument (see among many others Lindbeck and Snower (2001)), the other two explanations have received little relative attention in the theoretical literature. In particular, on the one hand, Coimbra, Lloyd-Braga and Modesto (2000) and Ortigueira (2003) are among the few exceptions arguing that a low accumulation of capital may explain persistent high unemployment rates.[2] On the other hand, Den Haan (2003) and Rocheteau (1999) show, in the framework of a matching model without capital accumulation, that balanced budget rules may yield multiple steady states. Actually, these authors develop the original argument by Blanchard and Summers (1986b) and show that balanced budget rules may turn the effects on unemployment of a temporary shock persistent, and eventually, permanent.

In contrast to the mainstream theoretical literature, generally overlooking the close relation displayed by the data between capital accumulation and unemployment,[3] we outline the central role of fiscal policies in the framework of an endogenous growth model where, because of wage inertia, capital stock growth is found to have permanent effects on employment: labor demand is continuously shifted up by capital accumulation, thereby causing a permanent effect on the employment rate because the wage, due to its inertia, does not fully adjust. In this framework, we show that balanced budget rules may provide an explanation of the hysteresis hypothesis in the patterns of employment and economic growth.

We consider a simple one-sector endogenous growth model with a linear production technology where, for simplicity, we let a union set the wage as a mark-up over a reservation wage, assumed to be a weighted average of past labor income and, thus, taking account of wage inertia. By means of direct taxes, the government finances public spending and subsidies, aiming to maintain a balanced budget rule. To this end, either government spending or direct taxes must be endogenous and adjust to keep the budget constraint balanced. When government spending is treated as endogenous, direct tax rates are constant and, hence, exhibit an acyclical behavior. In contrast, when direct tax rates are considered endogenous, we expect them to be countercyclical; i.e., to be high in bad times and low in good times. This is simply a result from the fact that government expenditures, such as unemployment benefits, rise in bad times and shrink in good times.

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[2] Like us, Coimbra, et. al. (2000) argue in favor of multiple steady states, but with an Overlapping Generations Model with strong increasing returns to scale, which are at odds with the empirical evidence (see Basu and Fernald (1997)). In turn, our approach also differs from Ortigueira (2003), whose analysis is based on a model of labor search with frictional unemployment and human capital accumulation.

[3] Some empirical literature show that there is a close relationship between these two variables. This is outlined by Rowthorn (1999) who suggests "that a major factor behind persistent unemployment may also be inadequate growth in capital stock". Henry, Karanassou and Snower (2000) point to the importance of the role of capital stock in influencing the UK unemployment trajectory, but it is in Karanassou, Sala and Snower (2003) where a reappraisal of the causes of european unemployment is provided, and capital stock is shown to be an important determinant (if not the leading one) of the movements in the European unemployment rate.

When direct tax rates are assumed to be exogenous and constant, the higher is economic growth the higher capital accumulation and the more the labor demand shifts up. Thus, employment is enhanced, provided the rise in labor demand does not fully translate into wage increases, which happens when wage inertia is sufficiently strong. In Section 4, we show that the equilibrium path of this simple model with exogenous taxes is unique and conclude that fails to explain hysteresis.

When direct taxes are assumed to be endogenous, these taxes introduce a complementarity between capital accumulation and employment able to make agents' expectations self-fulfilling and, hence, generate multiple equilibrium paths. To see it, assume that agents coordinate into an expectations of high net interest rate. If agents are willing to substitute consumption intertemporally, the savings rate will be large and so will be the growth rates of capital stock and labor demand. When there is wage inertia, the latter implies high values of the employment rate and, thus, strong economic activity, implying large government revenues and low government expenditures. Obviously, the endogenous direct tax rate will be low and hence the equilibrium interest rate net of taxes will be large, which ensures that agents' expectations hold in equilibrium. This explains the existence of an equilibrium path corresponding to an economic regime of high economic activity and, analogously, it can also explain the existence of another equilibrium path corresponding to a low regime. In Section 5, we show that the assumption of endogenous taxes may cause the existence of two different equilibrium paths converging to different steady states. One of them corresponds to a high regime characterized by high employment, savings and growth rates, and low direct tax rates, whereas the other one is a regime characterized by low employment, growth and savings rates, and high tax rates. Along these two equilibrium paths, government spending as a fraction of income is constant, thus, both paths converge to different steady states that belong to different sides of the same Laffer curve. In this context, we interpret hysteresis as the result of equilibrium selection between these two paths belonging to the same Laffer curve.

The assumptions on the fiscal policy drive the transition. When tax rates are exogenous, employment and the savings rate are negatively related, as a larger employment rate causes a positive wealth effect that reduces the savings rate. In contrast, when tax rates are endogenous, employment and the savings rate display, along the two equilibrium paths, a positive correlation due to a substitution effect. In that case, a larger employment rate implies a lower direct tax rate and, hence, larger net interest and savings rates.

The model allows us to derive a number of necessary conditions to generate hysteresis. These are: i) Strong wage rigidities; ii) Endogenous (countercyclical) tax rates; and iii) Large willingness to substitute consumption intertemporally. These conditions point to the relevance of the link between labor market institutions and fiscal policy. According to our model, hysteresis may only occur when institutions introduce strong wage inertia and direct tax rates follow a countercyclical pattern.

Our model matches remarkably well some observed regularities explained in Section 2. In particular, using Kernel density functions, we show that most of the European economies display high and low regimes in unemployment and the growth rate of capital stock, whereas the US economy displays a unique regime in unemployment. Interestingly, direct taxes seem to have been acyclical in the US economy, in contrast with most of the European ones, where they have tended to be countercyclical. This suggests that

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the experience of the US corresponds to our scenario of exogenous taxes and a unique equilibrium path, whereas the European experience seems to fit with the case where direct tax rates are used to balance the government budget constraint and different equilibria exist. Thus, we are able reinterpret the different consequences of the shocks suffered by these two areas in the 1970s, whose main expression was a temporal downturn in total factor productivity (TFP). In the US, direct tax rates were kept constant and the TFP downturn produced a temporary fall in savings, economic growth and employment, which progressively recovered to reach the original equilibrium. There were no permanent consequences, as the model explains when tax rates are exogenous. In contrast, the European experience seems to correspond to a case where direct tax rates are endogenous and two equilibrium path exist. In that case, the shocks of the 1970's and the resulting temporal TFP downturn may have caused agents to coordinate into a low regime equilibrium, hence keeping permanent the effects of these temporary shocks.

The structure of the paper is the following. Section 2 provides an evaluation of the regime changes in unemployment, which we find closely related with the trajectory of the capital stock growth rate. A countercyclical behavior of the direct tax rates is also identified for most of the European countries. Section 3 describes a simple growth model. The equilibrium is characterized in the following two sections, but in two different cases: when direct tax rates are exogenous (Section 4) and when they are endogenous (Section 5). Section 6 summarizes our findings and concludes.

2. Empirical evidence underlying our theoretical modelling

In this section we provide evidence on the differences between the European economies and the US in the path of unemployment and the capital stock growth rates. As the model highlights the role of fiscal policies to explain hysteresis, we also study the behavior of the direct tax rates.

The analysis we undertake next is inspired in Bianchi and Zoega (1998) and relies on the estimation of Kernel density functions to identify regime changes in the time series of unemployment and the capital stock growth rates. When a time series displays different regimes, the density of the frequency distribution of that series will be multimodal, with the number of modes corresponding to the number of regimes. Our identification criteria is the following. We will consider that a regime exists when the first derivative of the Kernel density function is zero and the second derivative is negative. This point indicates the regime mean value, which can be seen as a local maximum (i.e., a point with the highest density). When two or more regimes exist, a 'valley point' (the first derivative is zero and the second one is positive) divides the data points in the sample. Those observations with values above the 'valley point' will belong to the upper regime, whereas those with values below will belong to the lower regime.

We consider two type of regimes shifts. First, *temporary*, in response to transitory or persistent shocks, which means that a set of data points remain in the same regime at most during four consecutive periods. Second, *permanent*, in response to irreversible shifts or permanent shocks, which are all shocks that lasted at least five years. This allows to disentangle temporary movements from regime shifts.

Our database is the same used in Karanassou, Sala and Snower (2003), containing annual data on unemployment, business capital stock, GDP and direct taxes, all pro-

vided by the OECD, for 11 European countries starting in the 1960s (Austria, Belgium, Denmark, Germany, Finland, France, Italy, Netherlands, Spain, Sweden and the United Kingdom).

Figure 1 pictures the sharp contrast between the unemployment rate trajectory in Europe and the US.

[Insert Figure 1]

In Europe there is a neat regime shift, placed in 1980 by our Kernel density analysis, which shifts the regime mean upwards from 2.5% to 9.7%. In contrast, the US analysis reveals a unique regime, only altered at the beginning of the 1980s by what seems to have been a one-off shock. The country-specific analysis, presented in Figure 2, gives additional evidence on this matter.[4]

[Insert Figure 2]

These plots are obtained from the kernel density analysis depicted in Figure 3.

[Insert Figure 3]

It seems clear, thus, that the European has experienced a permanent change, whereas the US series is characterized by a stationary pattern.

Next, we argue that the European countries experienced a permanent change in capital formation, with a regime mean shift that corresponds to the regime shift in unemployment. Given the existence of several particular cases, we refer first to the country-specific results. In particular, the results of the Kernel density analysis for the individual countries, pictured in Figures 4 and 5 below, are presented in Table 1.

[Insert Table 1]

Note that in Finland, Netherlands and UK only one regime is identified, whereas the rest of countries display two regimes.[5] All the regime changes take place in the mid 1970s, when the unemployment rates in these countries started to rise sharply.

[Insert Figures 4 and 5]

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[4]In Figures 2 and 5 we present only what we consider permanent regime changes (i.e., temporary regime changes are not plotted, as in Figures 1 and 4) using the criteria explained at the beginning of this section.

[5]In Finland and the Netherlands we only observe a regime because of the lack of data in the 1960s (the series start in 1970 and 1969, respectively), which prevents the Kernel density analysis to consider the few data points with high values as a separate regime (see figure 5). In Finland, the unique regime diplays a mean of 2.9%, but from 1970 to 1977 capital stock growth is above 3%. In the Netherlands the regime mean is at 2.3%, but from 1969 to 1979 takes values above 2.5 percentage points in all years except 1976.

With respect to the aggregate capital stock series for the whole European countries, there is no long time-series directly provided by the OECD. Thus, we need to aggregate the series corresponding to the pool of countries under consideration, which involves two important requirements: first, to establish an accurate criterion to assign country weights; second, to avoid any noise derived from exchange rates fluctuations, given that the capital stock series are expressed in national currencies. The connection between capital stock and output point at GDP as the relevant measure to weight the individual capital stock series. Moreover, GDP series are generally available since the 1960s and they allow us to compute a yearly weight. To reach the second criterion, we use a series of real GDP in Purchasing Power Parities. Since we are not interested in the European levels of the capital stock, but on its growth rate, what we finally construct is an aggregate series of the growth rate.[6]

[Insert Figure 6]

With the aggregate European and US series we conduct a Kernel density analysis and obtain the results displayed in Figure 6. In Figure 6c a first regime is identified for Europe, lasting from 1963 to 1974, and having a mean capital stock growth rate of 4.9%. The second one starts in 1975 and lasts up to 1999, with a regime mean of 2.7%. The only exceptional data point in this regime occurs in 1991, when the series comes across the German unification consequences, in the form of a sudden rise in the growth rate of capital stock. The analysis for the US yields a different picture. Despite two regimes are identified (Figure 6d), they differ by just 1.1 percentage points. Following our criterion to qualify the type of regimes, we would identify a high regime mean up to 1985 (with two temporary negative shocks corresponding to the oil price crises), followed by a low regime mean which ends by an upwards shift. We interpret this low regime as a temporal response to a persistent shock that we identify with the anti-inflationist monetary policy of the Volcker era, from 1979 to 1987, which shifted real interest rates upwards.

Beyond this quantitative analysis, the general picture that emerges is the following. In Europe there is a permanent mean shift, which is expressed in an upwards unemployment regime shift of 7.2 percentage points that, perhaps taking too far our analysis, corresponds with a 2.2 percentage points reduction in the mean growth rate of physical capital stock. On the contrary, there is no such a permanent shift in the US unemployment and capital stock. The appropriateness of a multiple equilibria model for Europe, assigning a relevant role to capital formation, seems clear.

Finally, let's turn our attention to the path of the direct tax rates. Figure 7 relates the trajectory of the direct tax rate (as percentage of GDP) to economic growth. As stated before, we interpret that the negative relationship of these two series corresponds to our scenario of endogenous tax rates, which seems to fit the experience of most of the European countries. In particular, the coexistence of what could be taken as a high economic growth regime mean in the 1960s and first 1970s with a low direct tax rate

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[6]For Austria, Belgium, Denmark, Germany and Italy we have data on capital stock since 1960 (on the growth rate since 1961). Nevertheless, the growth rate of the aggregate capital stock series starts in 1963 because since 1962 we also have data for France and the UK, and since 1963 for Spain, all countries with substantial weight in the EU. The rest of the countries are progressively taken into account, the weights being amended correspondingly: data for Sweden start in 1965, for the Netherlands in 1968 and for Finland in 1969.

regime mean (and the opposite in the 1980s and 1990s) is apparent in all the European countries with the sole exception of the UK, where these two series display a very mild negative correlation, just as in the US. Furthermore, in the latter case, there are signs of procyclical tax rates since the second half of the 1980s.

[Insert Figure 7]

In form of scattered diagrams, the plots of Figure 7 would show the negative correlation between the direct tax rate and the growth rate. Table 2 presents the estimates of this correlation, which is significant at the 1% significance level in all countries except Germany (at 8%), Finland (6%) and, of course, the UK and the US, where it is not significant.[7]

[Insert Table 1]

When focussing on the cyclical behavior of the fiscal policy, the literature also reveals differences between the fiscal policies in the US and in most of the European economies. For example, Buti, Franco and Ongena (1997) provide evidence of an initial countercyclical reaction in Europe after the shocks of the 1970s.[8]

It seems, thus, that there is a different fiscal policy pattern in Europe and the US, which leads us to think that the fiscal policy, mainly the pattern of the direct tax rates, may be a relevant factor underlying these two areas' different labor market performance. This is taken into account in the theoretical model presented in Section 3.

3. The Economy

In order to provide an explanation of the empirical regularities just described, in this section we develop a simple one sector endogenous growth model with labor market frictions.

3.1. Labor market

The production function takes the following functional form:

$$Y(t) = AK(t)^{\alpha} L(t)^{1-\alpha} \overline{k}(t)^{1-\alpha}, \ A > 0, \ \alpha \in (0,1),$$

where $Y(t)$ is the gross domestic product (GDP), $K(t)$ is the aggregate stock of capital, $L(t)$ is the number of employed workers, and $\overline{k}(t) = \frac{K(t)}{L(t)}$ is the average stock of capital per employee. The total factor productivity (TFP) is determined by the technological parameter A and the path of the average stock of capital.

Perfect competition and profit maximization imply that the competitive factors payment are

$$r(t) = \alpha AK(t)^{\alpha-1} L(t)^{1-\alpha} \overline{k}(t)^{1-\alpha},$$

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[7]These results consist on a very simple regression of the sort of the ones presented in Fatás and Mihov (2001), which take the following form: $z_t = \alpha + \beta \Delta y_t + \nu_t$, where z_t is the fiscal variable and y_t is GDP. Table 2 presents the estimated β for the European countries and the US.

[8]For further evidence, see also Fatás and Mihov (2001b) and Calmfors et al. (2003).

and

$$w(t) = (1 - \alpha) AK(t)^\alpha L(t)^{-\alpha} \overline{k}(t)^{1-\alpha}.$$

The latter equation implicitly defines the non-equilibrium labor demand

$$L^d \left(w(t), K(t), \overline{k}(t) \right) = \left(\frac{(1 - \alpha) AK(t)^\alpha \overline{k}(t)^{1-\alpha}}{w(t)} \right)^{\frac{1}{\alpha}}.$$

Along a symmetric equilibrium (i.e. when $k(t) = \overline{k}(t)$), the production function per employee is

$$y(t) = Ak(t),$$

where $y(t) = \frac{Y(t)}{N(t)}$, $k(t) = \frac{K(t)}{N(t)}$, and $N(t)$ is the aggregate labor supply. The competitive factor payments along a symmetric equilibrium are

$$r(t) = \alpha A, \tag{3.1}$$

and

$$w(t) = \frac{(1 - \alpha) Ak(t)}{l(t)}, \tag{3.2}$$

where $l(t) = \frac{L(t)}{N(t)}$ is the employment rate.

From (3.2), we obtain the equilibrium labor demand

$$l^d \left(w(t), \widetilde{k}(t) \right) = \frac{(1 - \alpha) Ak(t)}{w(t)}, \tag{3.3}$$

Note that the capital stock growth rises the labor demand, which enhances the employment rate provided there is wage inertia.

Wage inertia arises from the following simple model of firm-level wage setting, where a firm-level union sets the wage in order to maximize:

$$\underset{w(t)}{Max} V = [(1 - \tau(t)) w(t) - w^s(t)]^\gamma L^d \left(w(t), K(t), \overline{k}(t) \right),$$

where $w^s(t)$ is a reference wage, $\tau(t)$ is the direct tax rate and $\gamma > 0$ provides a measure of the weight of the wage gap in the unions' objective function. Since the unions take the labor demand as given, this is a right to manage model. The solution to this maximization problem characterizes the wage equation

$$w(t) = \frac{w(t)^s}{(1 - \tau(t)) \left(1 + \frac{\gamma}{\varepsilon(t)} \right)},$$

where $\varepsilon(t)$ is the inverse of the price elasticity of the labor demand. When the union does not take into account the externality, $\varepsilon(t) = -\frac{1}{\alpha}$ and the wage equation simplifies

to[9]

$$w(t) = \frac{w(t)^s}{(1 - \tau(t))(1 - \alpha\gamma)}. \tag{3.4}$$

The reference wage is a controversial variable in the literature. For example, Blanchard and Wolfers (2000) argue that it depends on the unemployment benefit, past wages and social security benefits, among other variables. For the sake of simplicity, we assume that it depends on the following weighted average of the workers' past labor income:

$$w^s(t) = w^s(0) e^{-\theta t} + \theta \int_0^t e^{-\theta(t-i)} x(i) \, di, \tag{3.5}$$

where $w^s(0)$ is the initial value of the reference wage, $x(t)$ is the workers' average labor income and $\theta > 0$ provides a measure of the rate of wage adjustment. Note that the higher θ, the lower is the weight of the past average labor income in determining the reference wage, that is, the lower is the wage inertia. Actually, if the parameter θ diverges to infinite, then the reference wage coincides with the current average income, thus excluding wage inertia.

It is important to note that there is an initial condition on $w^s(0)$, as this variable is determined by past average labor income. Moreover, $w^s(0)$ determines the initial wage that is set by the unions, $w(0)$. Finally, given the initial wage and the initial stock of capital, the initial employment rate is obtained from the equilibrium labor demand $l(0) = l^d(w(0), k(0))$. Thus, when there is wage inertia, the employment rate is a state variable whose transition is driven by the degree of wage inertia.

Differentiate (3.5) with respect to time to obtain

$$\dot{w}^s(t) = \theta(x(t) - w^s(t)), \tag{3.6}$$

where the average labor income is assumed to be

$$x(t) = (1 - \tau(t)) l(t) w(t) + \lambda (1 - \tau(t))(1 - l(t)) w(t) - jw(t), \tag{3.7}$$

with $\lambda \in (0, 1)$, $j > 0$, $\lambda (1 - \tau(t)) w(t)$ being the unemployment benefits, and $jw(t)$ a tax on the wage.[10] From now on, we assume that $\lambda (1 - \tau(t)) - j > 0$, since otherwise the labor income of the unemployed workers would be negative.

Because the current average labor income is proportional to the wage and, hence, to per capita GDP, the wage set by the unions rises with economic growth. When there is no wage inertia, the rise in the labor demand due to economic growth fully translates into wage increases preventing employment growth. In contrast, when there is wage inertia, labor demand increases do not fully translate into higher wages and hence economic growth causes employment to rise. Since sustained growth implies that the labor demand grows permanently, wage inertia limits the wage adjustment even in the long run, implying a positive relation between economic growth and the employment rate even in the long run. To show this positive relation, next we obtain the path of

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[9]Alternativelly, one could assume a national level union that sets the wage taking into account capital externalities, that is considering the equilibrium labor demand $L^d\left(w(t), \widetilde{k}(t)\right)$. In this case, the wage equation would be $w(t) = \frac{w(t)^s}{(1-\tau)(1-\gamma)}$.

[10]In our model, these taxes amount to any tax allowing the government to finance its expenditures. For simplicity, they are modelled as proportional to the wage.

the employment growth rate. Combine (3.4), (3.6) and (3.7) to get

$$\frac{\dot{w}^s(t)}{w^s(t)} \equiv \xi(l(t), \tau(t)) = \theta \left[\frac{l(t) + (1 - l(t))\lambda}{1 - \alpha\gamma} - \frac{j}{(1 - \tau(t))(1 - \alpha\gamma)} - 1 \right].$$

Log-differentiate (3.4) with respect to time

$$\frac{\dot{w}^s(t)}{w^s(t)} = \frac{\dot{w}(t)}{w(t)} - \frac{\dot{\tau}(t)}{1 - \tau(t)} = \xi(l(t), \tau(t)), \tag{3.8}$$

where $\xi(l(t), \tau(t))$ is the growth rate of the after tax wage. Differentiate (3.3) with respect to time,

$$\frac{\dot{l}(t)}{l(t)} = \frac{\dot{k}(t)}{k(t)} - \frac{\dot{w}(t)}{w(t)},$$

and combine it with (3.8) to obtain the employment growth rate

$$\frac{\dot{l}(t)}{l(t)} = \frac{\dot{k}(t)}{k(t)} - \frac{\dot{\tau}(t)}{1 - \tau(t)} - \xi(l(t), \tau(t)). \tag{3.9}$$

The time-path of the employment rate depends on the difference between two growth rates: the capital stock one and the one of wages before taxes. As (3.3) shows, capital stock growth drives the growth of the labor demand, whereas wage growth provides a measure of the corresponding rise in the labor cost. Thus, (3.9) implies that the employment rate grows when the rise in the labor demand is larger than the rise in the unit cost of labor.

3.2. Consumers

Assume that there is a unique infinitely lived dynasty in the economy. Let $N(t)$ be the number of members of this dynasty that inelastically supply one unit of labor so that the aggregate labor supply is equal to $N(t)$. This dynasty maximizes the discounted sum of the utility of each member

$$\int_0^\infty e^{-(\rho - n)t} \left(\frac{c(t)^{1-\sigma} - 1}{1 - \sigma} \right) dt, \ \rho - n > 0, \ \sigma > 0,$$

subject to the budget constraint[11]

$$c(t) + \dot{k}(t) = ((1 - \tau(t))r(t) - n - \delta)k(t) + x(t),$$

where $\rho > 0$ is the subjective discount rate, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $n > 0$ is the constant population growth rate, and $\delta > 0$ is the constant depreciation rate.

The solution to the dynasty maximization problem is characterized by the growth rate of consumption per capita

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[11]Note that consumers' revenues accrue from capital income and from average labor income. We introduce the average labor income because we assume a large dynasty.

$$\frac{\dot{c}(t)}{c(t)} = \frac{(1 - \tau(t)) r(t) - \delta - \rho}{\sigma}, \tag{3.10}$$

and the transversality condition

$$\lim_{t \to \infty} e^{-(\rho - n)t} k(t) c(t)^{-\sigma} = 0. \tag{3.11}$$

Denote the growth of the per capita consumption by $\mu(t) = \frac{\dot{c}(t)}{c(t)}$. By using (3.1) and (3.10), it follows that, along a symmetric equilibrium,

$$\mu(\tau(t)) = \frac{(1 - \tau(t)) \alpha A - \delta - \rho}{\sigma}. \tag{3.12}$$

Having shown that consumption and employment growth depend on fiscal policy, we conclude the description of the economy by characterizing fiscal policy.

3.3. Government

Assume that the government follows a balanced budget rule, so that its budget constraint is given by the following equation:

$$\tau(t) Y(t) + jN(t) w(t) = G(t) + (N(t) - L(t)) \lambda (1 - \tau(t)) w(t).$$

Government revenues accruing from taxes are used to finance non-productive government spending, $G(t)$, and the unemployment benefit. Denote by $g(t) = \frac{G(t)}{Y(t)}$ the fraction of GDP devoted to government spending and rewrite the government budget constraint as follows

$$\tau(t) - g(t) = (1 - \alpha) \left(\frac{(1 - l(t)) \lambda (1 - \tau(t)) - j}{l(t)} \right).$$

We consider two different fiscal policies. First, assume that $g(t) = g$ is constant and exogenous, and the government sets the value of the direct tax rate to balance its budget constraint in each period. In that case, the path of the direct tax rate is endogenous and it is determined by the government budget constraint as the following function of the employment rate:

$$\tau(l(t)) = \frac{gl(t) + ((1 - l(t)) \lambda - j)(1 - \alpha)}{l(t) + (1 - l(t)) \lambda (1 - \alpha)}. \tag{3.13}$$

Note that

$$\tau'(l(t)) = (1 - \alpha) \left(\frac{(g - 1) \lambda + [1 - \lambda(1 - \alpha)] j}{[l(t) + (1 - l(t)) \lambda (1 - \alpha)]^2} \right) < 0,$$

as $\lambda(1 - \tau) > j$ implies that $(g - 1) \lambda + [1 - \lambda(1 - \alpha)] j < 0$. Note that $\tau'(l(t)) < 0$ implies that the endogenous tax rate follows a countercyclical path.

As a second fiscal policy, assume that $\tau(t) = \tau$ is constant and exogenous. In this case, the government sets the path of government spending to balance its budget constraint. This path is the following function of the employment rate:

$$g\left(l\left(t\right)\right) = \tau - \left(\left(1 - l\left(t\right)\right)\lambda\left(1 - \tau\right) - j\right)\left(\frac{1 - \alpha}{l\left(t\right)}\right). \tag{3.14}$$

Note that

$$g'\left(l\right) = \frac{\left(1 - \alpha\right)\left(\lambda\left(1 - \tau\right) - j\right)}{l^2} > 0,$$

as we assume that $\lambda\left(1 - \tau\right) > j$.

3.4. Employment and savings rate

To characterize the equilibrium path, use the resource constraint to derive the path of savings that summarizes the consumers' behavior. To this end, use the resource constraint.

$$C\left(t\right) + G\left(t\right) + S\left(t\right) = Y\left(t\right),$$

where $S\left(t\right)$ are the savings of the economy that correspond to gross investment. Let $s\left(t\right) = \frac{S(t)}{Y(t)}$ be the savings rate and rewrite the resource constraint as

$$s\left(t\right) = 1 - g\left(t\right) - \frac{C\left(t\right)}{Y\left(t\right)} = 1 - g\left(t\right) - \frac{c\left(t\right)}{Ak\left(t\right)},$$

to obtain

$$\frac{c\left(t\right)}{k\left(t\right)} = \left(1 - s\left(t\right) - g\left(t\right)\right)A. \tag{3.15}$$

Differentiating this equation with respect to time,

$$\dot{s}\left(t\right) = \left(1 - s\left(t\right) - g\left(t\right)\right)\left(\frac{\dot{k}\left(t\right)}{k\left(t\right)} - \mu\left(t\right)\right) - \dot{g}\left(t\right). \tag{3.16}$$

The growth rate of capital is obtained from the resource constraint

$$C\left(t\right) + \dot{K}\left(t\right) + \delta K\left(t\right) = \left(1 - g\left(t\right)\right)Y\left(t\right),$$

which can be rewritten in per capita terms as follows

$$c\left(t\right) + \dot{k}\left(t\right) + \left(n + \delta\right)k\left(t\right) = \left(1 - g\left(t\right)\right)Ak\left(t\right).$$

The growth of the per capita stock of capital is then

$$\frac{\dot{k}\left(t\right)}{k\left(t\right)} = \left(1 - g\left(t\right)\right)A - \frac{c}{k\left(t\right)} - \left(n + \delta\right),$$

which, by using (3.15), becomes

$$\frac{\dot{k}\left(t\right)}{k\left(t\right)} = As\left(t\right) - n - \delta. \tag{3.17}$$

Combine (3.16) with (3.17), to obtain a differential equation that drives the equilibrium path of the savings rate

$$\dot{s}(t) = \widetilde{s}(s(t), \tau(t), g(t), \dot{g}(t)) \tag{3.18}$$
$$= (1 - s(t) - g(t))[As(t) - n - \delta - \mu(\tau(t))] - \dot{g}(t),$$

Finally, use (3.9) and (3.17) to obtain the differential equation that drives the equilibrium path of the employment rate

$$\dot{l}(t) = \widetilde{l}(s(t), l(t), \tau(t), \dot{\tau}(t)) \tag{3.19}$$
$$= l(t)\left(s(t)A - n - \delta - \xi(l(t), \tau(t)) - \frac{\dot{\tau}(t)}{1 - \tau(t)}\right).$$

Note that the equations characterizing the equilibrium depend on the nature of the fiscal policy (i.e., the tax rate being endogenous or exogenous). This distinction is important because we associate economies exhibiting acyclical tax rates (like the US one) with the scenario of exogenous taxes, and economies exhibiting countercyclical taxes (like most of the European ones) with the scenario of endogenous taxes. Next two sections describe the equilibrium path of an economy with exogenous tax rates (Section 4) and with endogenous tax rates (Section 5).

4. The equilibrium when tax rates are exogenous

Assume that the tax rate is constant, so that $\tau(t) = \tau$ and hence $\dot{\tau}(t) = 0$. As a consequence, (3.19) simplifies to

$$\dot{l}(t) = \widetilde{l}(s(t), l(t)) = l(t)(s(t)A - n - \delta - \xi(l(t), \tau)), \tag{4.1}$$

and, by using (3.14), (3.18) can be rewritten as

$$\dot{s}(t) = \widetilde{s}(s(t), l(t)) = (1 - s(t) - g(l(t)))\left[s(t)A - n - \delta - \mu - \frac{g'(l(t))\dot{l}(t)}{1 - s(t) - g(l(t))}\right]. \tag{4.2}$$

Definition 4.1. *Given $\{l_0, k_0; \tau\}$, an equilibrium with exogenous tax rates is defined by $\{l(t), s(t), g(t)\}_{t=0}^{\infty}$ such that solves (3.14), (4.1), and (4.2), satisfies the transversality condition (3.11) and the following constraints: $l(t) \in [0, 1]$, $s(t) \in [0, 1]$ and $g(t) \in [0, 1]$, for all $t \geq 0$.*

To characterize the path of the dynamic equilibrium, first obtain the Balanced Growth Path (BGP), which is defined as a path along which $l(t)$ and $s(t)$ remain constant, and consumption, capital and GDP grow at the constant growth rate μ. By using $\dot{l}(t) = 0$ and $\dot{s}(t) = 0$, it is straightforward to show that the employment rate along a BGP must satisfy the following equation:

$$Q(l) = \xi(l) - \mu = 0.$$

Thus, along a BGP, the long run economic growth rate coincides with the growth rate of wages. In this simple model, this growth rate is equal to the long run growth

rates of capital and, as follows from (3.3), of the labor demand. Thus, the employment rate attains a BGP when the growth rates of the labor demand and of the wage coincide.

It can be shown that $Q(l) = 0$ has a unique root, which is the unique BGP of the economy if it belongs to the close interval $[0, 1]$, and if the corresponding savings rate and fraction of GDP devoted to government spending also belong to this interval. The BGP value of the employment rate is

$$l^* = \left(\frac{1 - \alpha\gamma}{1 - \lambda}\right)\left(\frac{\mu^*}{\theta} + 1\right) - \left(\frac{\lambda(1 - \tau) - j}{(1 - \tau)(1 - \lambda)}\right), \tag{4.3}$$

where the long run growth rate, obtained from (3.12), is

$$\mu^* = \frac{(1 - \tau)\alpha A - \delta - \rho}{\sigma},$$

and the long run savings rate, obtained from $\dot{s}(t) = 0$, is

$$s^* = \frac{\mu^* + n + \delta}{A}.$$

Note that when there is wage inertia (i.e., when θ does not diverge to infinite), economic growth increases the long run employment rate. This relation drives some of the results in the following proposition:

Proposition 4.1. *Assume that $l^* \in [0, 1]$ and $s^* \in [0, 1]$. Then,*

a) $\frac{\partial l^*}{\partial A} > 0$, $\quad \frac{\partial l^*}{\partial \sigma} < 0$, $\quad \frac{\partial l^*}{\partial \theta} < 0$, $\quad \frac{\partial l^*}{\partial \lambda} < 0$, $\quad \frac{\partial l^*}{\partial \gamma} < 0$, $\quad \frac{\partial l^*}{\partial \tau}?.$

b) $\frac{\partial \mu^*}{\partial A} > 0$, $\quad \frac{\partial \mu^*}{\partial \sigma} < 0$, $\quad \frac{\partial \mu^*}{\partial \theta} = 0$, $\quad \frac{\partial \mu^*}{\partial \lambda} = 0$, $\quad \frac{\partial \mu^*}{\partial \gamma} = 0$, $\quad \frac{\partial \mu^*}{\partial \tau} < 0.$

c) $\frac{\partial s^*}{\partial A}?$, $\quad \frac{\partial s^*}{\partial \sigma} < 0$, $\quad \frac{\partial s^*}{\partial \theta} = 0$, $\quad \frac{\partial s^*}{\partial \lambda} = 0$, $\quad \frac{\partial s^*}{\partial \gamma} = 0$, $\quad \frac{\partial s^*}{\partial \tau} < 0.$

P roof. The proof follows from the BGP value of the variables.

As in any AK growth model, the long run growth rate increases with TFP, A, and with the intertemporal elasticity of substitution, $\frac{1}{\sigma}$. When there is wage inertia, the employment rate depends positively on the economic growth rate, which explains the effects of these parameters on the employment rate. Moreover, if there is positive growth, the employment rate increases with the wage rigidity, which is negatively related with the parameter θ. As standard, the employment rate decreases with the replacement ratio, λ, and with the weight assigned by unions to the wage gap, γ. The direct tax rate drives two opposite forces. On the one hand, it increases the wage paid by firms, which causes a negative effect on employment. On the other hand, it decreases the labor income, which reduces wages and enhances employment. The net effect is thus ambiguous. Finally, most of the effects of the parameters on the savings rate follow from the effects on the growth rate. The ambiguous effect of TFP on the savings rate depends on the magnitude of a wealth and a substitution effect.

The acyclical behavior of the direct tax rate in the US can be associated with our scenario of exogenous tax rates. This version of the model and the relevant values (see Table 3) are thus used to characterize the US economy in the long-run, which is taken as the benchmark economy with exogenous taxes. Table 3 quantifies the effect of some parameter increases.

Proposition 4.2. *The BGP is saddle path stable and, hence, the path of the dynamic equilibrium is locally unique.*

P roof. See Appendix.

Figure 8 displays the phase diagram of this economy,[12] which shows that the there is a negative relation between the employment and the savings rate: a larger employment rate increases average labor income, thereby causing a positive wealth effect that deters savings. In the following section, we show that if government spending as a fraction of GDP is constant and direct tax rates are endogenous, then the equilibrium displays a positive correlation between the savings rate and the employment rate. This correlation outlines the complementarity between employment and capital due to the endogenous tax rates.

As there is a unique BGP, the model with exogenous tax rates fits the US experience, but cannot explain the hysteretic behavior of employment in Europe. To see this, consider a reduction in the TFP (a decrease in A in the model). By using the phase lines provided in the appendix, Figure 9 shows the transition induced by this reduction, which initially causes both a substitution and a wealth effect. In the economy characterized in Table 3, the negative wealth effect dominates and hence there is an initial increase in the savings rate. Furthermore, the decrease in TFP deters growth, which causes a decline in the employment rate as wage inertia prevents wage adjustment. This decline causes a further negative wealth effect that explains the rise in the savings rate. Note that the transition in the employment rate is explained by wage inertia. Actually, in the absence of wage inertia, the reduction in economic growth would be fully translated into a reduction in the wage and no effect on the employment rate would occur.

Figure 10 displays the effects on the employment rate, the path of government spending, economic growth and the savings rate of a 5% permanent reduction in A in the benchmark economy displayed in Table 3, assumed to be initially at the BGP. As these effects are transitory when the shock is transitory, we conclude that if tax rates are exogenous, the equilibrium does not exhibit hysteresis.

5. The equilibrium when tax rates are endogenous

Assume now that public spending as a fraction of GDP is constant, i.e. $g(t) = g$, and the government balances its budget constraint by setting the direct taxes endogenously. From (3.13), it follows that $\tau(t) = \tau(l(t))$ and thus

$$\dot{\tau}(t) = \tau'(l(t)) \dot{l}(t),$$

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[12]See the appendix for a discussion on the construction of this phase diagram and the characterization of the policy function.

which can be used to rewrite (3.19) as

$$\dot{l}(t) = \widetilde{l}(s(t), l(t)) = l(t) \left(\frac{s(t) A - n - \delta - \xi(l(t), \tau(l(t)))}{1 + \frac{\tau'(l(t))l(t)}{1 - \tau(l(t))}} \right), \qquad (5.1)$$

and (3.18) as

$$\dot{s}(t) = \widetilde{s}(s(t), l(t)) = (1 - s(t) - g)[s(t) A - n - \delta - \mu(\tau(l(t)))]. \qquad (5.2)$$

Definition 5.1. *Given* $\{l_0, k_0; g\}$, *an equilibrium with endogenous tax rates is characterized by* $\{l(t), s(t), \tau(t)\}_{t=0}^{\infty}$ *such that solves equations (3.13), (5.1), and (5.2), and satisfies (3.11) and the following constraints:* $l(t) \in [0, 1]$, $s(t) \in [0, 1]$, *and* $\tau(t) \in [0, 1]$ *for all* $t \geq 0$.

The BGP of this economy is obtained when $\dot{l}(t) = 0$ and $\dot{s}(t) = 0$, which yield the following equation characterizing the employment rate along a BGP:

$$Q(l) = \xi(l, \tau(l)) - \mu(\tau(l)) = 0,$$

where $\tau(l)$ is obtained from (3.13). Again, along a BGP the growth rates of the labor demand and wages are equal. However, when tax rates are endogenous, $Q(l)$ is a third order polynomial that may have three real roots within the relevant domain, i.e. the close interval $[0, 1]$.[13] These three roots are the three BGPs when the associated savings and tax rate belong to the close interval $[0, 1]$. The existence of these three BGPs is shown by means of numerical examples (see Table 4).

These multiple BGPs arise because the endogenous tax rates generate a complementarity between the employment and the savings decisions, making agents' expectations to be self-fulfilling. To explain this complementarity, assume that agents coordinate into an expectations of high net interest rate. If agents are willing to substitute intertemporally consumption, the savings rate will be large and thus the growth rates of capital stock and labor demand will also be large. When there is wage inertia, the latter implies a high value of the employment rate which, given its impact on economic activity, causes large government revenues and low expenditures. Obviously, the endogenous direct tax rate is low and hence the net of taxes equilibrium interest rate is large in equilibrium. Thus, endogenous tax rates make agents' expectations hold in equilibrium, which explains the existence of an equilibrium path corresponding to a regime of high economic activity. The same argument applies to explain the equilibrium path of a low regime.

Denoting the BGPs by l_1, l_2, and l_3, assume, without loss of generality, that $l_1 < l_2 < l_3$. Along each BGP, the tax rate is obtained from (3.13) as a function of the employment rate, $\tau(l_i)$, for $i = 1, 2, 3$. As the tax rates are countercyclical, they satisfy the following relations: $\tau(l_1) > \tau(l_2) > \tau(l_3)$. From (3.12), it follows that the economic growth rate is

$$\mu(l_i) = \frac{(1 - \tau(l_i))\alpha A - \rho - \delta}{\sigma}, \qquad (5.3)$$

which is negatively related to the tax rate implying that, along the BGP, $\mu(l_1) <$

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[13] The functional form of $Q(l)$ is shown in the proof of Proposition 5.4, in the appendix.

$\mu(l_2) < \mu(l_3)$. Finally, the savings rate is obtained from $\dot{s}(t) = 0$

$$s(l_i) = \frac{\mu(l_i) + n + \delta}{A}.$$

Because the savings rate is positively related with economic growth, the following relations hold: $s(l_1) < s(l_2) < s(l_3)$. Thus, it follows that BGP 1 (i.e. l_1, $\tau(l_1)$, $\mu(l_1)$, $s(l_1)$) corresponds to a regime of low economic activity and high tax rates, whereas BGP 3 (i.e. l_3, $\tau(l_3)$, $\mu(l_3)$, $s(l_3)$) corresponds to a regime of high economic activity and low tax rates.

Analogously to the US case, the countercyclical behavior of the direct tax rate in Spain is associated with our scenario of endogenous tax rates which, together with the relevant values (see Table 4), characterizes the Spanish economy in the long-run. Since BGP 2 is unstable, we identify BGP 1 with the low regime of the Spanish economy and BGP 3 with the high regime. As Table 4 shows, the model is able to replicate fairly well the two regimes.

[Insert Table 4]

In what follows we look at the effects of the parameters along BGPs 1 and 3, and then characterize the stability of each BGP.

Proposition 5.1. *Let $i = 1, 3$ and assume that $l_i \in [0, 1]$, $s(l_i) \in [0, 1]$, and $\tau(l_i) \in [0, 1]$. Then,*

a) $\frac{\partial l_i}{\partial A} > 0$, $\quad \frac{\partial l_i}{\partial \sigma} < 0$, $\quad \frac{\partial l_i}{\partial \theta} < 0$, $\quad \frac{\partial l_i}{\partial \lambda} < 0$, $\quad \frac{\partial l_i}{\partial \gamma} < 0$, $\quad \frac{\partial l_i}{\partial g} < 0$.

b) $\frac{\partial \mu(l_i)}{\partial A} > 0$, $\quad \frac{\partial \mu(l_i)}{\partial \sigma} < 0$, $\quad \frac{\partial \mu(l_i)}{\partial \theta} < 0$, $\quad \frac{\partial \mu(l_i)}{\partial \lambda} < 0$, $\quad \frac{\partial \mu(l_i)}{\partial \gamma} < 0$, $\quad \frac{\partial \mu(l_i)}{\partial g} < 0$.

c) $\frac{\partial \tau(l_i)}{\partial A} < 0$, $\quad \frac{\partial \tau(l_i)}{\partial \sigma} > 0$, $\quad \frac{\partial \tau(l_i)}{\partial \theta} > 0$, $\quad \frac{\partial \tau(l_i)}{\partial \lambda} > 0$, $\quad \frac{\partial \tau(l_i)}{\partial \gamma} > 0$, $\quad \frac{\partial \tau(l_i)}{\partial \tau} > 0$.

P roof. Part a) follows by using the implicit function theorem on $Q(l) = 0$ and by noticing that $Q(0) = -(\theta + \mu(0)) < 0$, which implies that $Q'(l_1) > 0$ and $Q'(l_3) > 0$. Part b) follows from (3.13) and Part c) follows from (5.3).

Note that the effects of the parameters on the BGP are similar to the effects obtained when the tax rates are exogenous. The intuitions behind them are also similar.

Proposition 5.2. *BGPs 1 and 3 exhibit saddle path stability, whereas BGP 2 may be either unstable or locally stable.*

P roof. See Appendix.

Further numerical examples beyond the one in Table 4 show that the instability of BGP 2 is a robust result. Thus, hysteresis, which we identify with the shift between equilibrium paths converging to different BGPs, may only arise when there are three BGPs. In this case, agents can coordinate into an equilibrium path that converges to BGP 1 or into another one that converges to BGP 3. Proposition 5.4 provides sufficient conditions that prevent the existence of three BGPs, which help to understand how savings decisions, the fiscal policy and the labor market institutions interact to explain hysteresis.

Proposition 5.3. *If* $\theta \to \infty$, $\gamma < \overline{\gamma}$, $\frac{1}{\sigma} = 0$ *or* $g \notin \left(\underline{g}, \overline{g} \right)$, *then at most two BGPs exist.*

P roof. See Appendix.

The results in Proposition 5.3 imply that the existence of three BGPs requires labor market rigidities in the form of: i) wage inertia, which ensures the positive effect of economic growth on employment; and ii) a weight of the wage gap in the unions' objective function sufficiently large, so that there is a large markup. Besides, the intertemporal elasticity of substitution must be sufficiently large, since the complementarity requires that the savings rate increases with the interest rate (note, however, that multiple BGP arise under plausible values of the intertemporal elasticity of substitution, as shown in the example of Table 4). Finally, government spending must belong to a given interval, which can be clearly seen through the long run Laffer curve.

To construct the long run Laffer curve, note that along the BGP, both the exogenous and the endogenous tax rate economies are characterized by the same two equations: the government budget constraint and the equality between the growth rates of wage and economic growth. In (4.3), we have shown that this equality happens when $l^* = l^* \left(\tau \right)$ and, by using the government budget constraint (3.14), we obtain

$$g = g \left(l^* \left(\tau \right), \tau \right),$$

which is the long run Laffer curve, relating the tax rate with the fraction of GDP devoted to government spending. Figure 11 displays the Laffer curve corresponding to the example in Table 4. Note that in the exogenous tax rate economy, given a value of the tax rate, we obtain a unique value of government spending and, thus, a unique BGP. In contrast, in the economy with endogenous tax rates, different tax rates may finance a given value of government spending. These different tax rates are the different BGPs, corresponding to a high tax rate and low economic activity regime (the wrong side of the Laffer curve), and to a low tax rate and high economic activity regime (the right side of the Laffer curve). Figure 11 shows that there are three BGPs only when government spending belongs to a given interval. When government spending is too large, it can only be financed by means of a large tax rate; when too low, it can only be financed by means of a low tax rate.

[Insert Figure 11]

The transitional dynamics along these equilibrium paths, that converge to BGP belonging to the same Laffer curve, are driven by agents' expectations on the tax rate. If they expect tax rates to be large (small), they expect the net interest rate to be low (large) and hence the initial savings and growth rate are low (large). This implies that the equilibrium converges to a low (high) economic activity regime, where tax rates are large (low) in equilibrium. In this way, agents' expectations are fulfilled. In contrast, when tax rates are exogenous, the equilibrium is unique because the government selects the equilibrium path by setting the value of the tax rate.

The transitional dynamics with endogenous taxes are displayed in Figure 12, picturing the phase diagram when there are three BGPs. Note that, given an initial value of the employment rate, agents may coordinate, by means of their savings decisions, into an equilibrium path driving towards the high regime (BGP 3) or into an equilibrium path driving towards the low regime (BGP 1).

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[Insert Figure 12]

As shown in the appendix, the policy functions driving towards BGPs 1 and 3 have a positive slope when tax rates are countercyclical. Thus, if the economy converges to one of these BGPs, the equilibrium exhibits a positive relationship between the employment and the savings rate, which is in stark contrast with the negative relationship obtained when tax rates are exogenous. The reason is that an increase in the employment rate implies lower government expenditures and higher revenues. As a consequence, the endogenous tax rate decreases with the employment rate, thereby increasing the net of taxes interest rate, which rises the savings rate.

With three BGPs, a temporary shock, such as a reduction in TFP, may make agents coordinate into another equilibrium path and thus would have permanent effects and generate hysteresis.

[Insert Figure 13]

Figure 13 displays the phase diagram when we assume that the equilibrium is initially in BGP 3 and there is a TFP shock (a reduction in A) that opens two possibilities of coordination giving rise to two different equilibrium paths. In one of them, agents choose an initial small reduction in the savings rate that makes the economy go back to BGP 3. In the other one, agents choose a large initial reduction in the savings rate, which places the economy in the policy function converging towards BGP 1, i.e. the equilibrium converges towards the wrong side of the Laffer curve, where the tax rate is large and the interest rate is low.

[Insert Figure 14]

Using the benchmark economy discussed in Table 4, Figure 14 shows these two equilibrium paths assuming that the economy is initially in the high regime BGP and suffers a 1% permanent reduction in TFP. Observe that the long run effect of this shock depends on the initial jump in the savings rate that, in turn, depends on agents' expectations. Interestingly, when these expectations make agents coordinate into another equilibrium path, temporary shocks have permanent effects and cause hysteresis. To understand why, note that the reduction in TFP has a direct negative effect in the gross interest rate that makes agents be willing to reduce savings. However, when the tax rate is endogenous, the effect on the net interest rate of a reduction in TFP depends on agents expectations. In particular, if agents coordinate into an expectations of low tax rates, they will expect a small reduction of the net interest rate. As a consequence, they will choose a small initial reduction in the savings rate implying a small reduction in economic growth and the employment rate. In this case, the equilibrium converges to the BGP with high activity. Because employment is large, government expenditures as a fraction of GDP are low, thus, the required equilibrium tax rate is small and expectations are fulfilled in equilibrium. However, agents may also coordinate into an expectation of high tax rates, with agents expecting a large reduction in the net interest rate and thereby choosing a large reduction in the savings rate that causes a large decline in economic growth and, hence, in the employment rate. As a consequence, government expenditures as a fraction of GDP would be large implying

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a large equilibrium tax rate. In this equilibrium, expectations are fulfilled again, but now a temporary reduction in TFP has permanent effects.

Finally, according to our model, the different economic performance of the US and the European countries after the temporary shocks of the 1970's can be explained by a different response in terms of fiscal policies. In the US, direct tax rates were kept constant, and employment and growth suffered a temporary decline. In Europe, tax rates exhibited a countercyclical behavior and employment and growth suffered a persistent decline. The model explains this persistent decline as a result of a coordination into another equilibrium path.

6. Concluding remarks

We have used a growth model with a non-competitive labor market to show that endogenous tax rates generate complementarities between employment and capital yielding the possibility of multiple equilibria. With multiple equilibria, the equilibrium path is the result of a coordination among equilibria with high tax rates and low employment and savings rates; and equilibria with low tax rates and high economic activity. These equilibrium paths converge to different long run equilibria that belong to opposite sides of the Laffer curve.

When tax rates are endogenous, agents may coordinate on either side of the Laffer curve. This coordination failure causes economic instability and, furthermore, agents may coordinate into an equilibrium path that converges to the wrong side of the Laffer curve (the low regime). In contrast, when tax rates are exogenous, the government selects the equilibrium path setting the value of the tax rate. In this way, the government prevents economic instability and may place the economy in an equilibrium path that converges to the right side of the Laffer curve. Thus, according to this model, to set the value of the tax rate is a superior fiscal policy.

The model also nests an interpretation of the different performance of the US and the European economies in the aftermath of temporary shocks such as those occurred in the 1970's. In particular, we find a correspondence between the acyclical behavior displayed by the direct tax rates in the US with our scenario of exogenous tax rates, and their countercyclical behavior in most of the European countries with our scenario of endogenous tax rates. In response to a temporary reduction in TFP, in the first case the model implies a temporary decline in the employment, savings and growth rates, which matches well with the US experience. In the second case, the model predicts the possibility of hysteresis, which also fits the extremely persistent reduction in the path of employment, growth and saving rates occurred in Europe. The model explains this permanent reduction as a coordination of agents into an equilibrium with high tax rates and low employment and savings rates.

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Appendix

Proof of Proposition 4.2. The BGP exhibits saddle path stability when the determinant of the Jacobian matrix formed by (4.2) and (4.1)

$$J = \begin{pmatrix} \widetilde{l}_l & \widetilde{l}_s \\ \widetilde{s}_l & \widetilde{s}_s \end{pmatrix} = \begin{pmatrix} -l\xi'(l) & Al \\ g'(l)\,l\xi'(l) & A\,(1 - s - g - g'(l)\,l) \end{pmatrix}.$$

is negative

$$Det\,(J) = -l\xi'(l)\,A\,(1 - s - g) = -l\theta\left(\frac{1-\lambda}{1-\alpha\gamma}\right)A\,(1 - s - g) < 0.$$

∎

Phase diagram of the economy with exogenous tax rates. Denote by $\widetilde{s}_1\,(l)$ the phase line associated to $\dot{s}\,(t) = 0$. The slope around the BGP is given by

$$\frac{\partial \widetilde{s}_1}{\partial l} = \left(\frac{\xi'(l)}{A}\right)\left(\frac{g'(l)\,l}{g'(l)\,l - (1 - s - g)}\right),$$

which can be either positive or negative. If $\dot{l}\,(t) = 0$ then the phase line is

$$\widetilde{s}_2 = \frac{\xi\,(l) + n + \delta}{A},$$

and

$$\frac{\partial \widetilde{s}_2}{\partial l} = \frac{\xi'(l)}{A} = \left(\frac{\theta}{A}\right)\left(\frac{1-\lambda}{1-\alpha\gamma}\right) > 0.$$

Figure 8 displays the phase diagram when $\frac{\partial \widetilde{s}_1}{\partial l} < 0$. Note that the slope of the policy function is negative. When $\frac{\partial \widetilde{s}_1}{\partial l} > 0$, $\frac{\partial \widetilde{s}_1}{\partial l} > \frac{\partial \widetilde{s}_2}{\partial l}$ and the slope of the policy function is also negative. To see this, use the Jacobian Matrix to obtain the equation of the policy function relating $s\,(t)$ with $l\,(t)$

$$s\,(t) = \left(\frac{l\xi'(l) + \lambda_1}{Al}\right)(l\,(t) - l^*) + s^*,$$

where $\lambda_1 < 0$ is the stable eigenvalue. It can be shown that $|J + l\xi'(l)\,I| < 0$, where I is the identity matrix. This inequality implies that $\lambda_1 < -l\xi'(l)$, and hence the policy function has a negative slope. ∎

Proof of Proposition 5.2. To discuss the stability of each BGP, obtain the elements of the Jacobian matrix formed by equations (5.2) and (5.1)

$$J = \begin{pmatrix} \frac{\partial \widetilde{l}}{\partial l} & \frac{\partial \widetilde{l}}{\partial s} \\ \frac{\partial \widetilde{s}}{\partial l} & \frac{\partial \widetilde{s}}{\partial s} \end{pmatrix} = \begin{pmatrix} -\frac{l\xi'(l,\tau(l))}{1 + \frac{\tau'(l(t))l}{1-\tau(l)}} & \frac{lA}{1 + \frac{\tau'(l(t))l}{1-\tau(l)}} \\ \frac{(1-s-g)\alpha A\tau'(l)}{\sigma} & (1 - s - g)\,A \end{pmatrix},$$

and find its determinant

$$Det\left(J\right) = -\left(\frac{l\xi'\left(l,\tau\left(l\right)\right)}{1+\frac{\tau'\left(l\left(t\right)\right)l}{1-\tau\left(l\right)}}\right)\left(1-s-g\right)A - \left(\frac{lA}{1+\frac{\tau'\left(l\left(t\right)\right)l}{1-\tau\left(l\right)}}\right)\left(\frac{\left(1-s-g\right)\alpha A\tau'\left(l\right)}{\sigma}\right) =$$

$$= -\left(\frac{\left(1-s-g\right)lA}{1+\frac{\tau'\left(l\left(t\right)\right)l}{1-\tau\left(l\right)}}\right)\underbrace{\left[\xi'\left(l,\tau\left(l\right)\right)+\frac{\alpha A\tau'\left(l\right)}{\sigma}\right]}_{Q'\left(l\right)}.$$

By using (3.13), obtain

$$1+\frac{l\tau'\left(t\right)}{1-\tau\left(t\right)} = \frac{\left(1-g\right)l\left(1-\left(1-\alpha\right)\lambda\right)l+j\left(1-\alpha\right)\left[2\left(1-\left(1-\alpha\right)\lambda\right)l+\left(1-\alpha\right)\lambda\right]}{\left[\left(1-g\right)l+j\left(1-\alpha\right)\right]\left[\left(1-\left(1-\alpha\right)\lambda\right)l+\left(1-\alpha\right)\lambda\right]} > 0.$$

This inequality implies that the sign of the determinant is the opposite of the sign of the slope of the function $Q\left(l\right)$. As $Q\left(0\right)=-\left(\theta+\mu\left(0\right)\right)<0$, it must be that $Q'\left(l_{1}\right)>0$, $Q'\left(l_{2}\right)<0$ and $Q'\left(l_{3}\right)>0$. It follows that the determinant is negative at the BGPs 1 and 3, and positive at BGP 2. In BGP 2, stability depends on the sign of the trace, which is given by

$$Tr\left(l,s\right) = -\frac{l\xi'\left(l,\tau\left(l\right)\right)}{1+\frac{\tau'\left(l\left(t\right)\right)l}{1-\tau\left(l\right)}} + \left(1-s-g\right)A.$$

BGP 2 is unstable when $Tr\left(l_{2},s_{2}\right)>0$ and it is locally stable when $Tr\left(l_{2},s_{2}\right)<0$.

∎

Proof of Proposition 5.3. The proof follows from $Q\left(l\right)=0$ which, by using (3.13), can be rewritten as

$$Q\left(l\right) = \left(\frac{l\left(1-\lambda\right)+\lambda}{1-\alpha\gamma}-1+\frac{\rho+\delta}{\theta\sigma}\right)\left(\left(1-g\right)l+j\left(1-\alpha\right)\right)\left(l+\left(1-l\right)\lambda\left(1-\alpha\right)\right)$$

$$-\left(\frac{j}{1-\alpha\gamma}\right)\left(l+\left(1-l\right)\lambda\left(1-\alpha\right)\right)^{2}-\left(\frac{\alpha A}{\theta\sigma}\right)\left(\left(1-g\right)l+j\left(1-\alpha\right)\right)^{2}.$$

Note first that if either $\frac{1}{\sigma}\to0$ or $\theta\to\infty$, then a root of $Q\left(l\right)=0$ is

$$l = -\frac{\lambda\left(1-\alpha\right)}{1-\lambda\left(1-\alpha\right)}<0,$$

which implies that there are at most two BGPs. Given that $Q\left(0\right)<0$, it follows that there are at most two BGPs when $Q\left(1\right)<0$. This happens when

$$\gamma = \overline{\gamma} < \frac{\frac{\mu\left(1\right)}{\theta}+\frac{j}{\left(1-\tau\left(1\right)\right)}}{\left(\frac{\mu\left(1\right)}{\theta}+1\right)\alpha}.$$

Finally, note that

$$Q\left(1\right) = -\frac{\alpha A}{\theta\sigma}\left(1-\tau\left(1\right)\right)^{2}+\left(\frac{1}{1-\alpha\gamma}-1+\frac{\rho+\delta}{\theta\sigma}\right)\left(1-\tau\left(1\right)\right)-\left(\frac{j}{1-\alpha\gamma}\right)<0,$$

25

if $\tau\left(1\right)\notin\left(\underline{\tau}\left(1\right),\overline{\tau}\left(1\right)\right)$ where $\underline{\tau}\left(1\right)$ and $\overline{\tau}\left(1\right)$ are the roots of $Q\left(1\right)=0$. By using (3.14), get $Q\left(1\right)<0$ when $g\notin\left(\underline{g},\overline{g}\right)$, where $\underline{g}=\underline{\tau}\left(1\right)+j\left(1-\alpha\right)$ and $\overline{g}=\overline{\tau}\left(1\right)+j\left(1-\alpha\right)$. Thus, when $g\notin\left(\underline{g},\overline{g}\right)$, there are two BGPs at most. $\blacksquare$

Phase diagram of the equilibrium with endogenous tax rates. The phase lines are the following. If $\dot{s}\left(t\right)=0$,

$$\widehat{s}_1=\frac{\mu\left(l\right)+n+\delta}{A}.$$

If $\dot{l}\left(t\right)=0$,

$$\widehat{s}_2=\frac{\xi\left(l\right)+n+\delta}{A}=\frac{\theta}{A}\left[\frac{l\left(1-\lambda\right)+\lambda}{1-\alpha\gamma}-\frac{j}{\left(1-\tau\left(l\right)\right)\left(1-\alpha\gamma\right)}-1\right]+\frac{n+\delta}{A}.$$

Note that

$$\frac{\partial\widehat{s}_1}{\partial l}=-\frac{\alpha\tau'\left(l\right)}{\sigma}>0,$$

and

$$\frac{\partial\widehat{s}_2}{\partial l}=\frac{\xi'\left(l\right)}{A}=\left(\frac{\theta}{A\left(1-\alpha\gamma\right)}\right)\left[1-\lambda-\frac{\tau'\left(l\right)j}{\left(1-\tau\right)^2}\right]>0.$$

Note also that

$$\widehat{s}_1\left(0\right)=\frac{\mu\left(0\right)+n+\delta}{A}>0,$$

and

$$\widehat{s}_2\left(0\right)=\frac{\xi\left(0\right)+n+\delta}{A}.$$

Therefore, $Q\left(0\right)<0$ implies that $\xi\left(0\right)<\mu\left(0\right)$ so that $\widehat{s}_2\left(0\right)<\widehat{s}_1\left(0\right)$. Thus, $s_1\left(l\right)$ and $s_2\left(l\right)$ are increasing and $\widehat{s}_2\left(0\right)<\widehat{s}_1\left(0\right)$. Using these properties of the phase lines, we can construct the phase diagram displayed in Figure 12.

By using the Jacobian matrix, we obtain the equation of the policy function that converges to either BGP 1 or 3

$$s_i\left(t\right)=\left(l_i\left(t\right)-l_i^*\right)\left(\frac{\frac{\left(1-s_i-g\right)\alpha A\tau'\left(l_i\right)}{\sigma}A}{\lambda_{i,1}-\left(1-s_i-g\right)}\right)+s_i^*,\ \ i=1,3,$$

where $\lambda_{i,1}$ is the stable root associated to BGP 1 or 3. Note that the slope of this equation is positive when $\tau'\left(l_i\right)<0$. $\blacksquare$

Tables

Table 1: Regimes in the European capital stock growth rates. 1960-1999.

Means in annual growth rates expressed in percentage points

	Regime 1			Regime 2	
	Means	Years		Means	Years
Austria	7.0	1961-77		4.2	1978-99
Belgium	4.4	1961-78		2.8	1979-99
Denmark	5.2	1961-74		3.2	1975-99
Germany	6.1	1964-71		3.0	1972-99
Finland		2.9	1970-99		
France	4.3	1963-77		2.4	1978-99
Italy	4.8	1961-74		3.0	1975-99
Netherlands		2.3	1969-99		
Spain	11.2	1964-75		4.3	1976-99
Sweden	4.7	1966-76		2.4	1977-99
United Kingdom		1.7	1963-99		

Table 2: Covariation in the direct tax rate and the business cycle.

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Based on the regression: $\tau_t^\delta = \alpha + \beta \Delta y_t$

where τ^d is the ratio $\frac{\text{direct taxes}}{\text{GDP}}$ and Δy is GDP growth

	β	t-stat.	period		β	t-stat.	period
Austria	-0.37	-3.72*	1964-99	Italy	-1.14	-4.84*	1961-99
Belgium	-0.56	-3.15*	1970-99	Netherlands	-0.31	-2.73*	1969-99
Denmark	-1.04	-2.87*	1961-99	Spain	-0.56	-2.73*	1964-99
Germany	-0.10	-1.79**	1961-99	Sweden	-0.44	-2.70*	1961-99
Finland	-0.19	-1.92**	1970-99	UK	-0.02	-0.10***	1963-99
France	-0.45	-3.49*	1964-99	US	-0.01	-0.14***	1961-99

Note: * stands for significant at 1%   *** stands for non-significant

** stands for significant between 5% and 10%

Table 3. The economy with exogenous tax rates[14]				
	Benchmark	$\Delta A = 10\%$	$\Delta \tau = 10\%$	$\Delta \lambda = 10\%$
l	94.4%	98.15%	94.37%	93.77%
μ	3.37%	3.84%	3.22%	3.37%
s	8.26%	7.88%	8.19%	8.26%
g	22.29%	23.03%	23.6%	21.97%
	Saddle Path	*Saddle Path*	*Saddle Path*	*Saddle Path*

Table 4. The economy with endogenous tax rates[15]					
	Spanish Economy			Model	
	Low regime	High regime	BGP1	BGP2	BGP3
l	80.84%	96.39%	80.36%	89.24%	96.26%
μ	2.48%	6.52%	5.80%	6.20%	6.51%
s	12.42%	14.51%	13.75%	14.19%	14.50%
τ	10.12%	3.59%	10.20%	6.40%	3.63%
			Saddle Path	*Unstable*	*Saddle Path*

[14] As standard in the literature, we assume $\rho = 0.045$ and $\delta = 5\%$; $A = 1.1416$ such that $s = 8.26\%$; $\sigma = 7.17$ such that $\mu = 3.37\%$; $j = 15.4\%$ such that $g = 22.29\%$ when $l = 94.4\%$; $\theta = .165$ such that $l = 94.4\%$ when $\gamma = 1$ and $\lambda = 0.5$. We also set $\alpha = 0.34$, $n = 1.06\%$ and $\tau = 13.23\%$. α is the share of capital income on national income obtained from Garofalo and Yamarik (2002). The following variables are averages from 1960 to 1999 taken from the OECD Economic Outlook: g is the ratio of government spending to GDP; τ the ratio of direct taxes to GDP; n population growth; μ GDP growth; and s the average fraction of family income not devoted to consumption.

[15] Again, $\rho = 0.045$ and $\delta = 5\%$; $A = .84149$ and $\sigma = 3.5201$ such that $\mu_3 = 6.5\%$ and $s_3 = 14.5\%$; $j = 25.43\%$ such that $\tau_3 = 10.12\%$; $\lambda = 0.891$, $\gamma = 1.617$ and $\theta = .0607$ such that l_1, l_2, l_3 are within the range of plausible values. We also set $\alpha = 0.4$, $n = 0.69\%$ and $g = 17.47\%$ which, like the rest of the values, are obtained as explained in Table's 3 footnote.
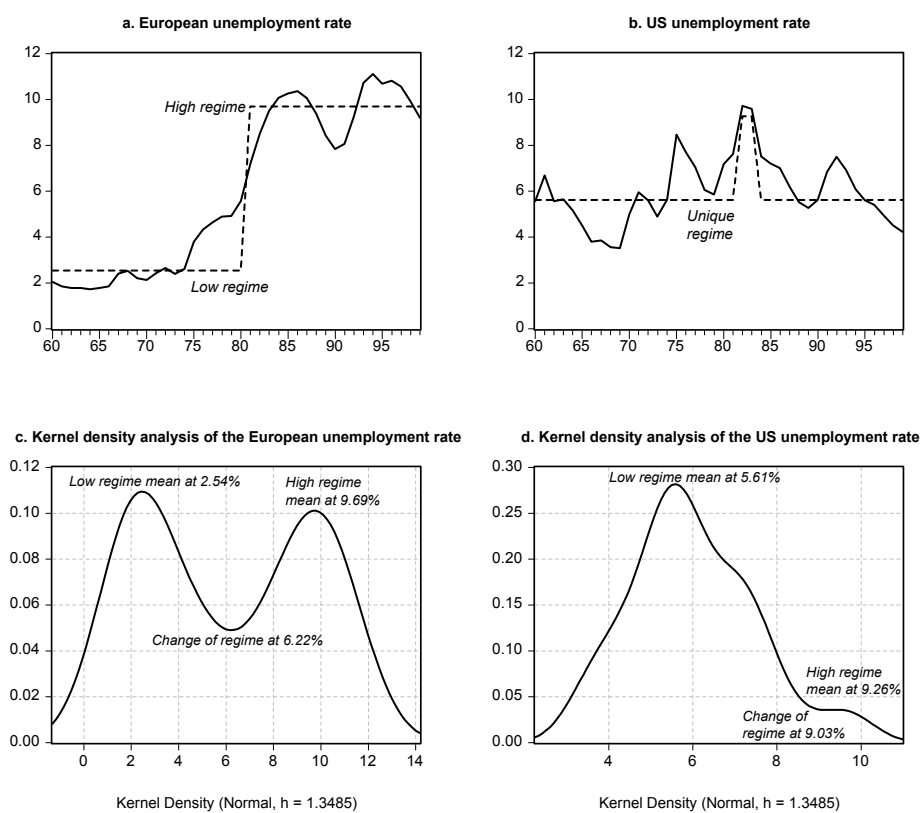
Figures

Figure 1. Unemployment in Europe and the US

a. European unemployment rate

b. US unemployment rate

c. Kernel density analysis of the European unemployment rate

Kernel Density (Normal, h = 1.3485)

d. Kernel density analysis of the US unemployment rate

Kernel Density (Normal, h = 1.3485)

Figure 2. Unemployment rate: actual series and regime means in European countries
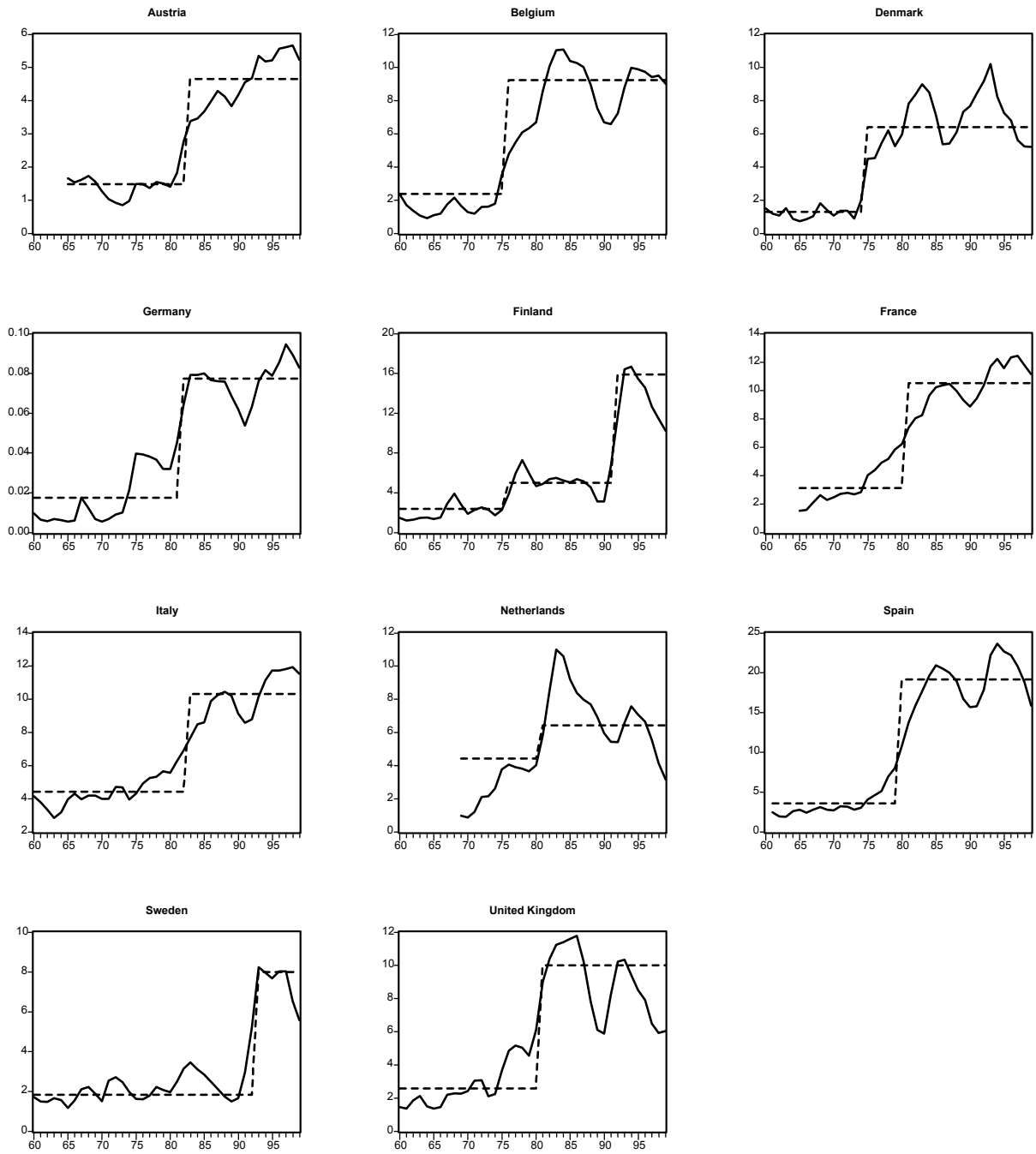
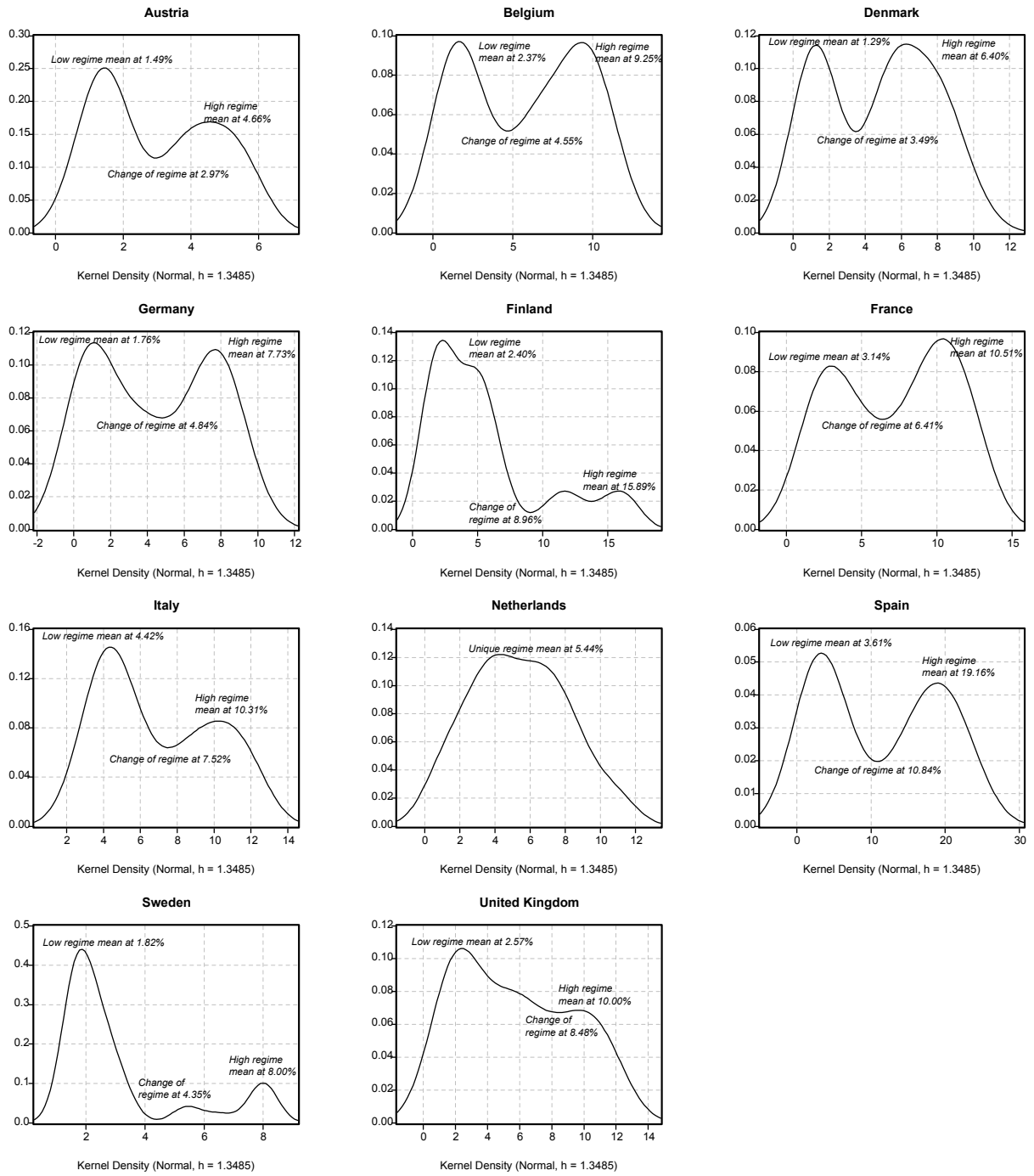**Figure 3. Kernel density analysis of the European countries unemployment rate**

Figure 4. Capital stock growth rate: actual series and regime means in the European countries
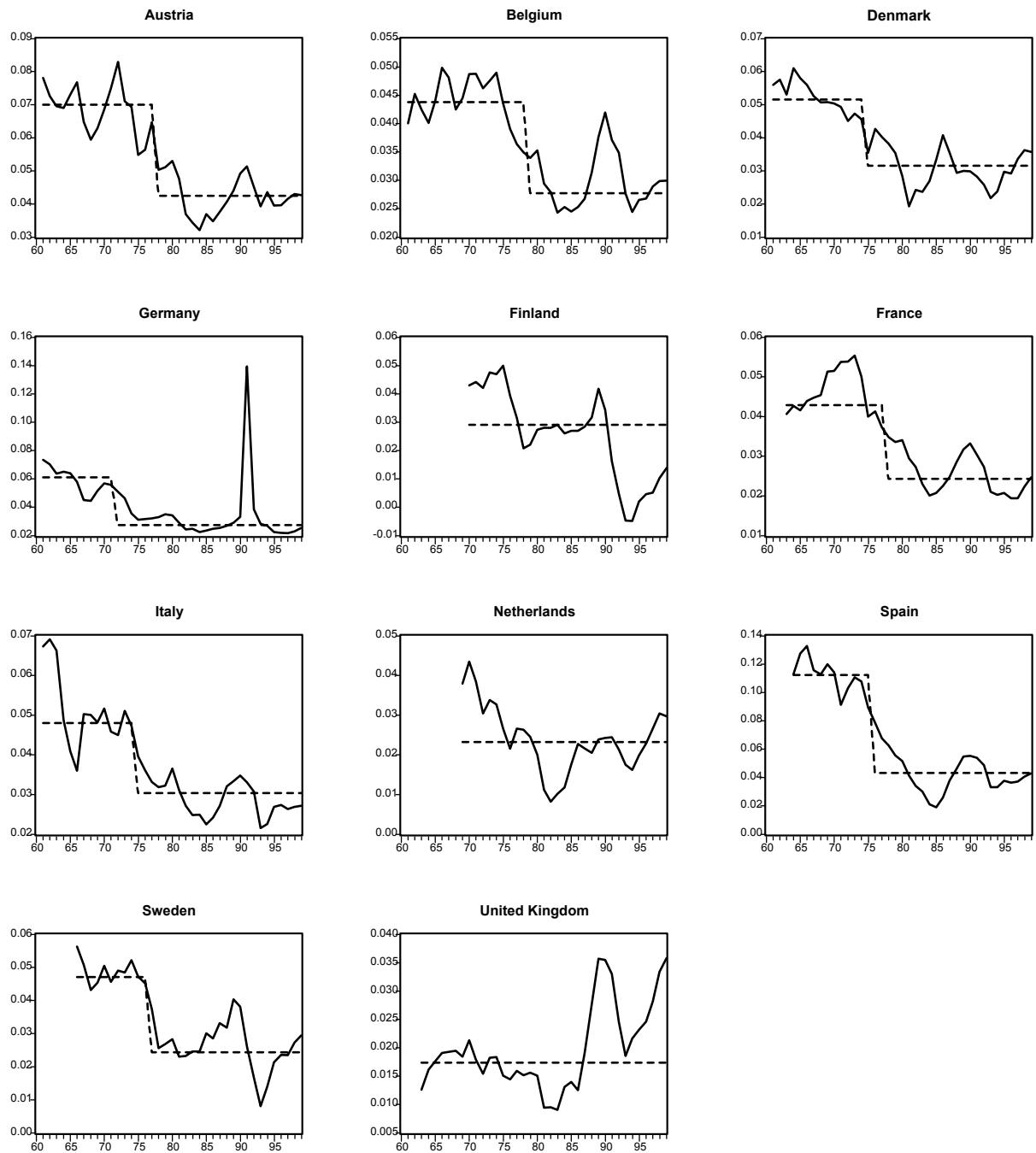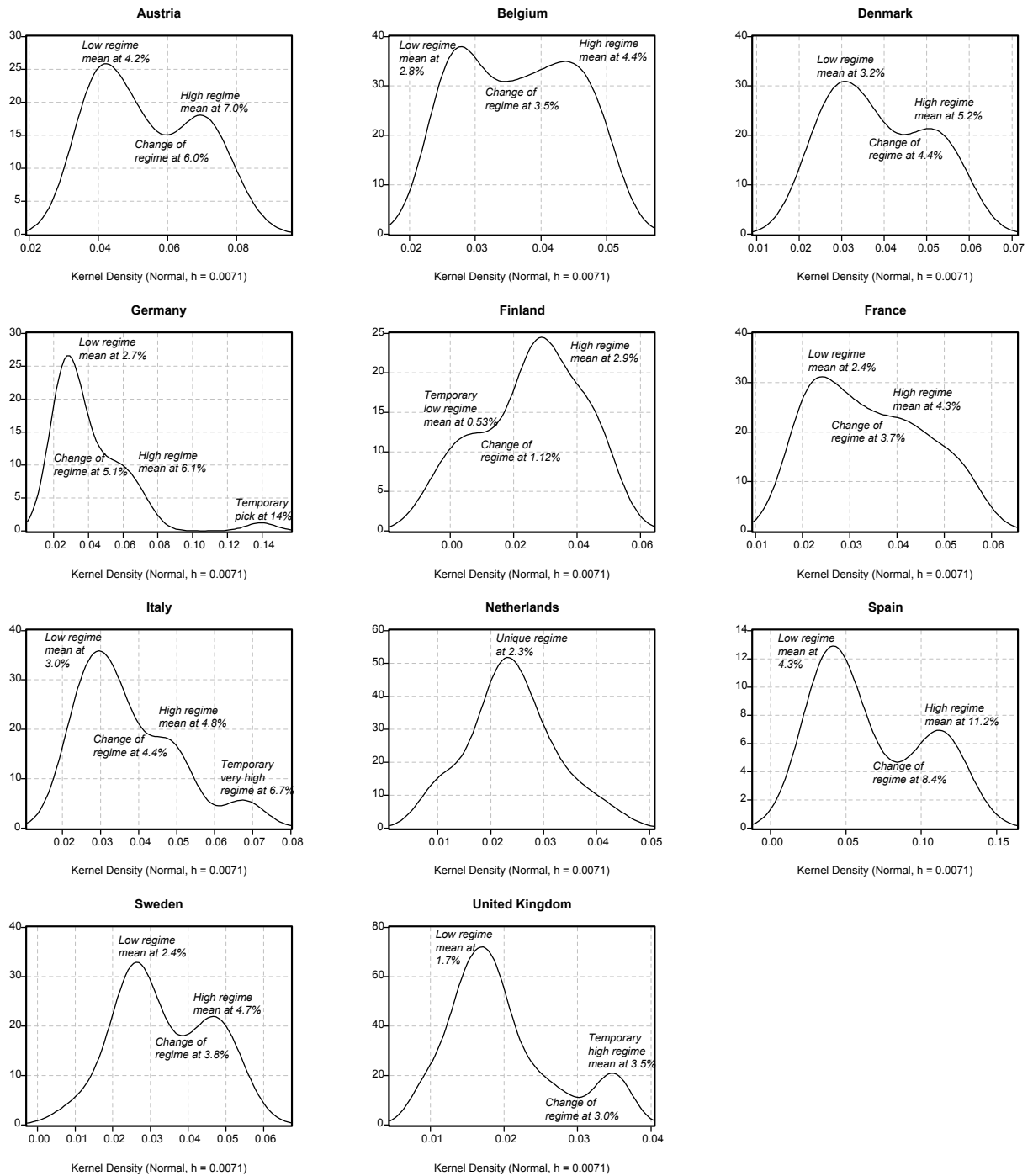
Figure 5. Kernel density analysis of the European countries capital stock growth

Austria — Kernel Density (Normal, h = 0.0071)
Low regime mean at 4.2%; High regime mean at 7.0%; Change of regime at 6.0%

Belgium — Kernel Density (Normal, h = 0.0071)
Low regime mean at 2.8%; High regime mean at 4.4%; Change of regime at 3.5%

Denmark — Kernel Density (Normal, h = 0.0071)
Low regime mean at 3.2%; High regime mean at 5.2%; Change of regime at 4.4%

Germany — Kernel Density (Normal, h = 0.0071)
Low regime mean at 2.7%; Change of regime at 5.1%; High regime mean at 6.1%; Temporary pick at 14%

Finland — Kernel Density (Normal, h = 0.0071)
Temporary low regime mean at 0.53%; Change of regime at 1.12%; High regime mean at 2.9%

France — Kernel Density (Normal, h = 0.0071)
Low regime mean at 2.4%; High regime mean at 4.3%; Change of regime at 3.7%

Italy — Kernel Density (Normal, h = 0.0071)
Low regime mean at 3.0%; Change of regime at 4.4%; High regime mean at 4.8%; Temporary very high regime at 6.7%

Netherlands — Kernel Density (Normal, h = 0.0071)
Unique regime at 2.3%

Spain — Kernel Density (Normal, h = 0.0071)
Low regime mean at 4.3%; High regime mean at 11.2%; Change of regime at 8.4%

Sweden — Kernel Density (Normal, h = 0.0071)
Low regime mean at 2.4%; High regime mean at 4.7%; Change of regime at 3.8%

United Kingdom — Kernel Density (Normal, h = 0.0071)
Low regime mean at 1.7%; Temporary high regime mean at 3.5%; Change of regime at 3.0%

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Figure 6. Capital stock growth in Europe and the US

a. EU capital stock growth

b. US capital stock growth

c. Kernel density analysis of the European capital stock growth

Low regime mean at 2.74%

High regime mean at 4.92%

Change of regime at 4.17%

Kernel Density (Normal, h = 0.0051)

d. Kernel density analysis of the US capital stock growth

Low regime mean at 2.69%

High regime mean at 3.81%

Change of regime at 3.19%

Kernel Density (Normal, h = 0.0051)

34

Figure 7. Direct taxes and business cycles in Europe and the US
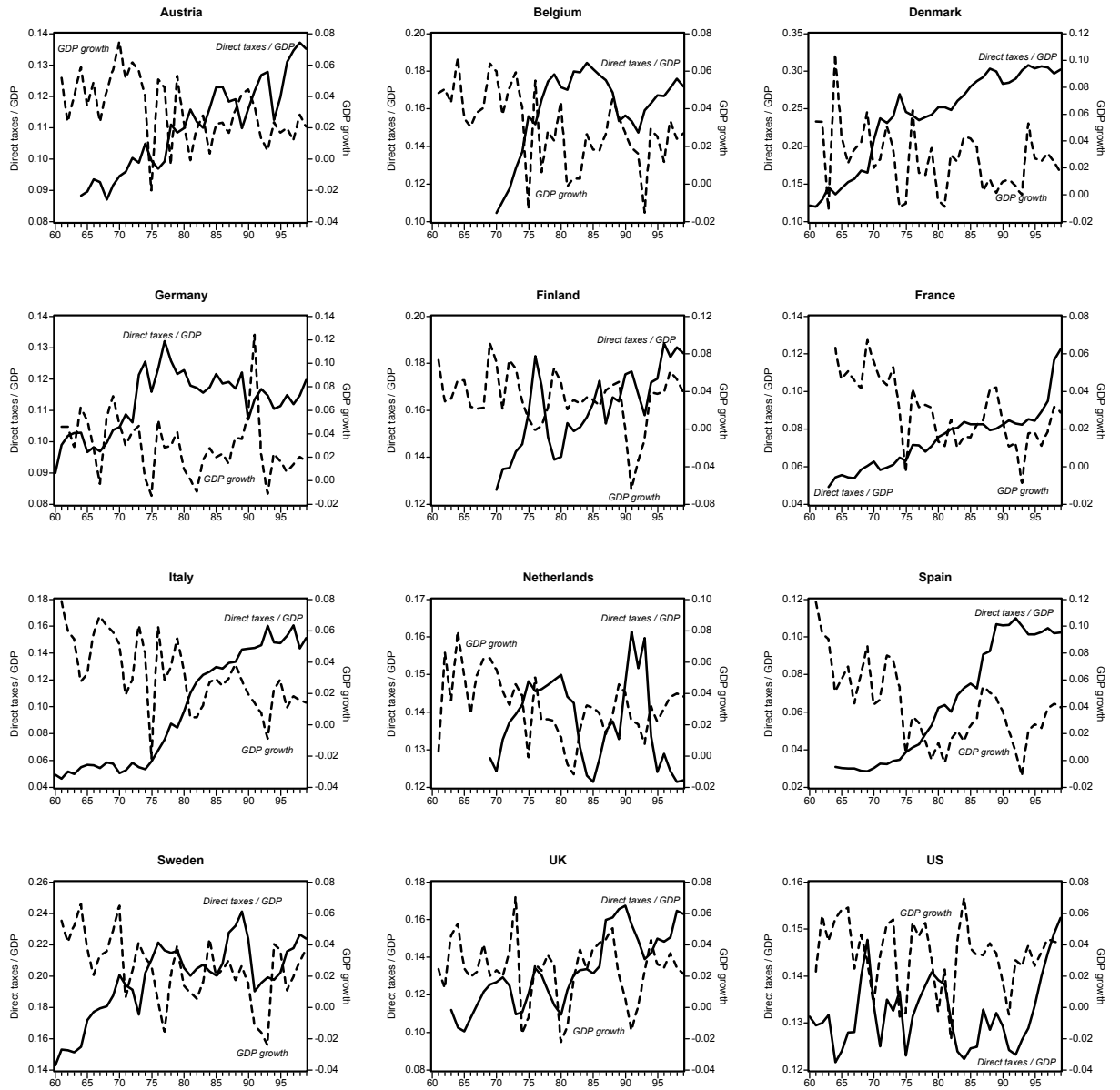
Figure 8. Phase diagram of the economy with exogenous tax rates
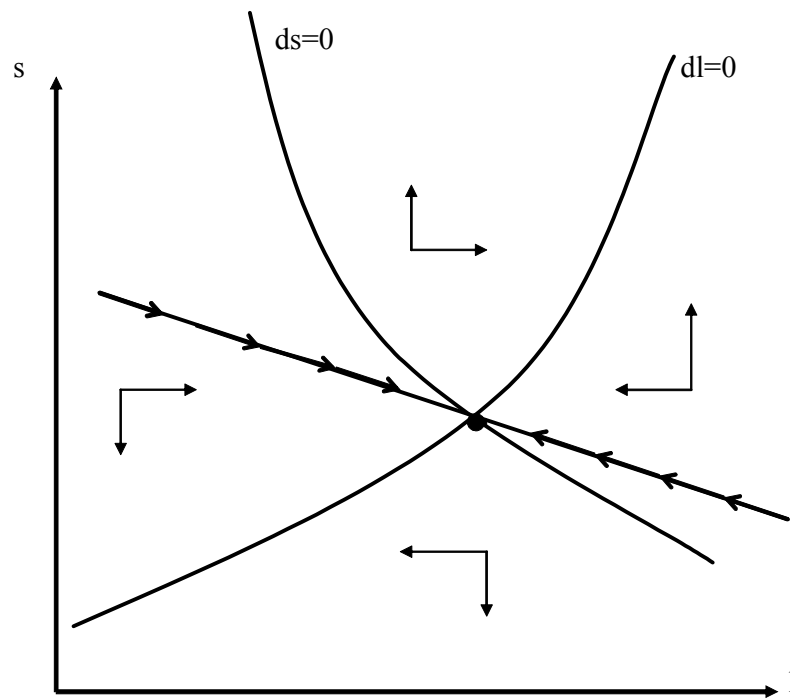
Figure 9. The dynamic effects of a reduction in TFP in an economy with exogenous tax rates
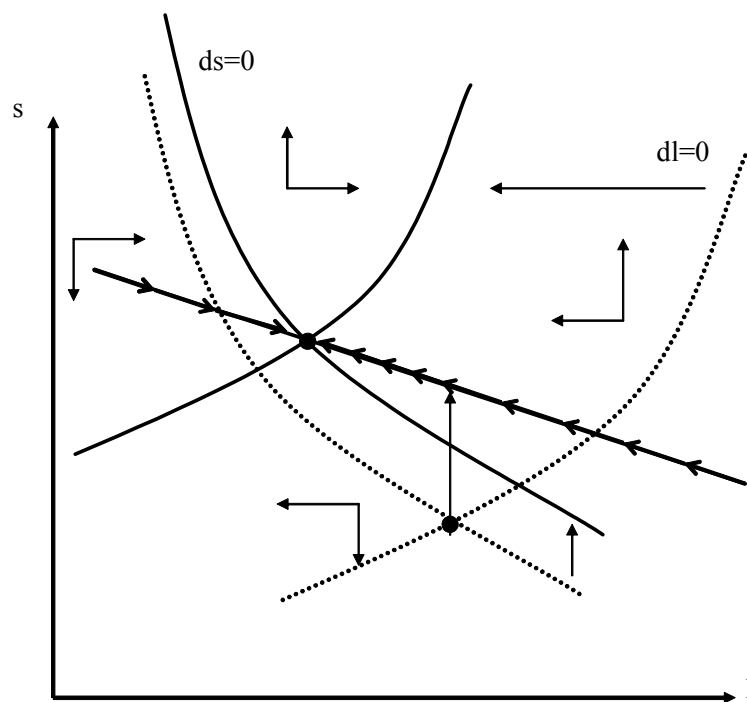
Figure 10. Transition in an economy with exogenous tax rates

The path of the employment rate
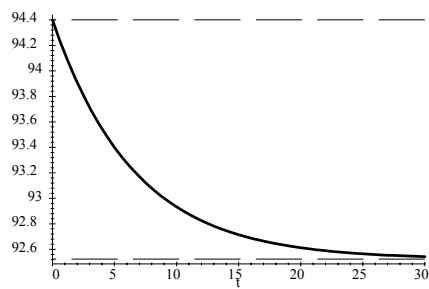
The path of government spending
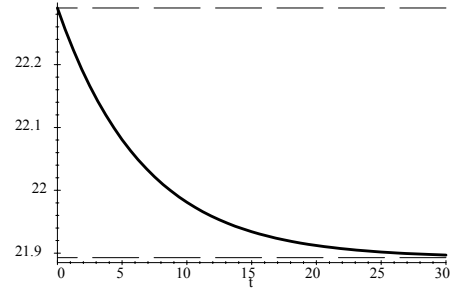
The path of economic growth
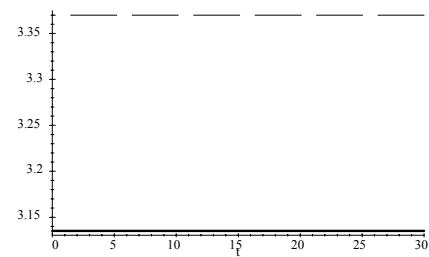
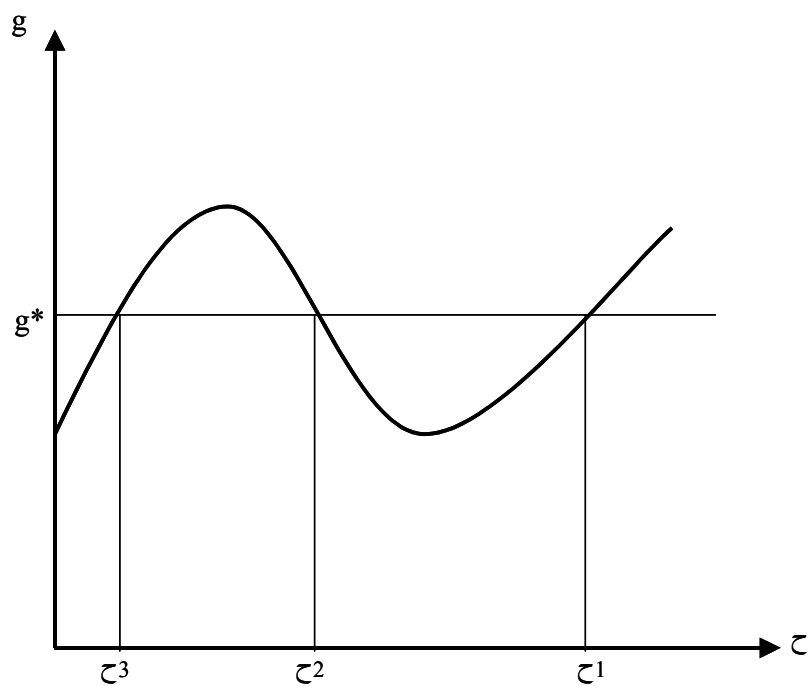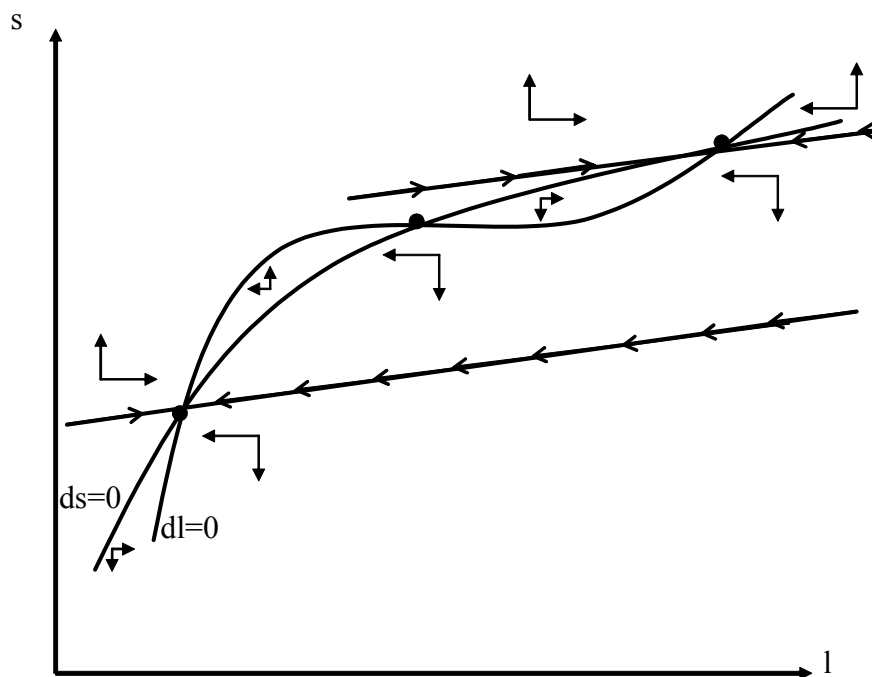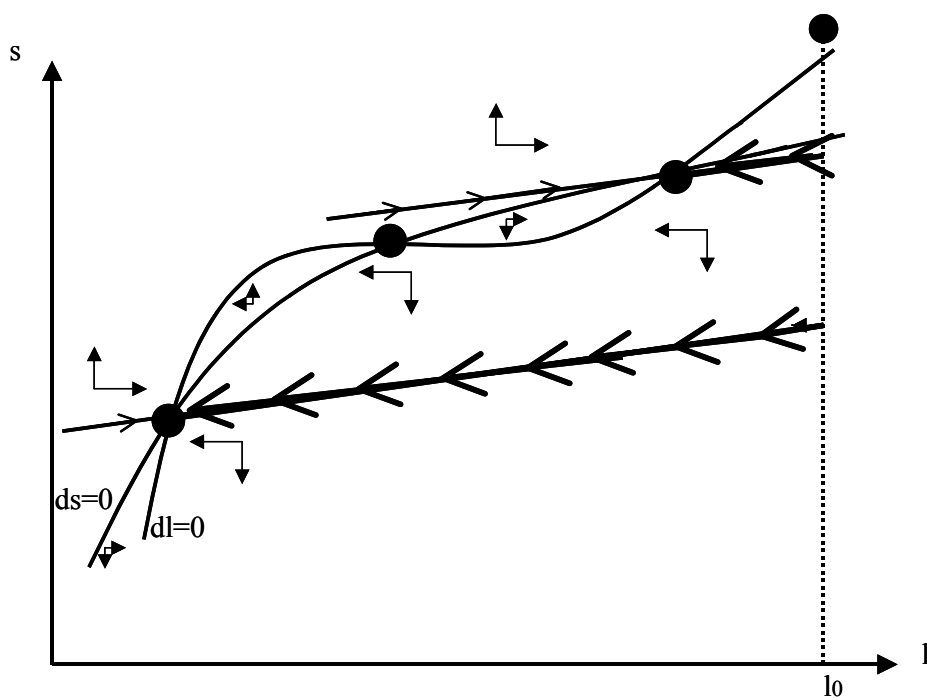The path of the savings rate
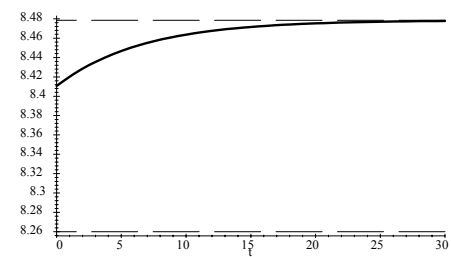
Figure 11. Long run Laffer Curve

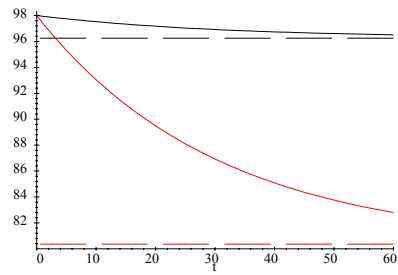**Figure 12. Phase diagram of the economy with endogenous tax rates**

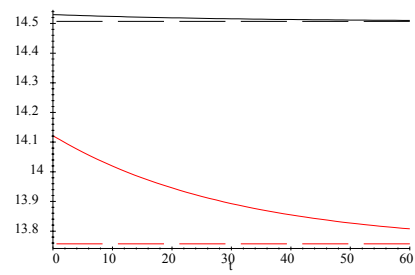**Figure 13. The dynamic effects of a reduction in TFP in an economy with endogenous tax rates**

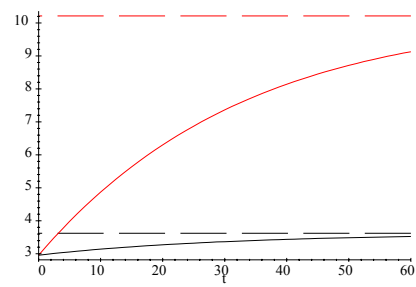## Figure 14. Transition in an economy with endogenous tax rates
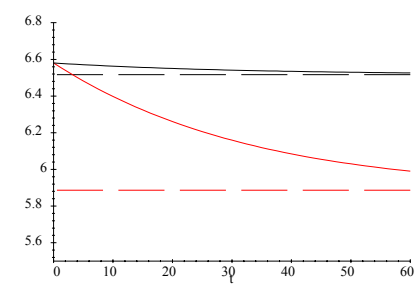
The path of the employment rate

The path of government spending

The path of the tax rate

The path of the growth rate

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