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XARXA DE REFERÈNCIA EN ECONOMIA APLICADA

Discrete time Non-homogeneous Semi-Markov Processes applied to Models for Disability Insurance

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Abstract: In this paper, we present a stochastic model for disability insurance contracts. The model is based on a discrete time non-homogeneous semi-Markov process (DTNHSMP) to which the backward recurrence time process is introduced. This permits a more exhaustive study of disability evolution and a more efficient approach to the duration problem. The use of semi-Markov reward processes facilitates the possibility of deriving equations of the prospective and retrospective mathematical reserves. The model is applied to a sample of contracts drawn at random from a mutual insurance company.

1 Introduction

Non-homogeneous semi-Markov processes were defined independently by Hoem (1972) and Iosifescu-Manu (1972). The approach proposed by Iosifescu-Manu was further generalized by Janssen and De Dominicis (1984). Additional results were obtained in discrete time by Vassiliou and Papadopoulou (1992) and by Papadopoulou and Vassiliou (1994).

The theory of semi-Markov processes quickly found applications in finance and insurance problems. The reader can find examples in Janssen (1966), Hoem (1972), CMIR (1991), Carravetta et al. (1981), Balcer and Sahin (1979, 1986) Janssen and Manca (1997) and, more recently, in the book by Janssen and Manca (2007).

A generalization of the transition probabilities of the discrete time non-homogeneous semi-Markov process (DTNHSMP) can be obtained by introducing the initial and final backward times (see D'Amico et al., 2009; D'Amico et al., 2010). The backward process facilitates the possibility of considering the dependence of the transition probabilities on the time of entrance into a given state.

A detailed description of continuous time homogeneous semi-Markov processes with backward time is reported in Limnios and Oprişan (2001) and in Janssen and Manca (2006). The discrete time non-homogeneous semi-Markov reward process with initial backward recurrence time is studied in Howard (1971) using recursive equations and more recently new results have been presented by Stenberg et al. (2007).

In this paper we generalize the results obtained in D'Amico et al. (2009) by introducing the reward structure. The reward structure allows us to determine equations for the prospective and retrospective mathematical reserves and a discrete version of the Thiele differential equation in a semi-Markov environment.

To the best of our knowledge, this is the first time that the general formulae of a DTNHSMP with rewards and initial and final backward times have been presented together with their corresponding mathematical reserves.

The paper is organized as follows. The next section presents a short introduction to DTNHSMP considering initial and final backward times. Section 3 analyzes semi-Markov reward processes with initial and final backward times. Successively, prospective and retrospective reserves are determined. Section 4 describes the disability data from a mutual insurance company from Catalunya and gives the results obtained by the model with these data.

2 Discrete time Non-homogeneous Semi-Markov Processes

We follow the notation given in Janssen and Manca (2006). In a semi-Markov process environment, two random variables run together. $J_n, n \in \mathbb{N}$, with state space I={1, 2, …, m}, represents the state at the n-th transition. $T_n, n \in \mathbb{N}$, with state space equal to $\mathbb{N}$, represents the time of the n-th transition,

$$J_n: \Omega \to I, \ \ T_n: \Omega \to \mathbb{N}.$$

We suppose that the process (J_n, T_n) is a non-homogeneous Markov renewal process and by $X_n = T_{n+1} - T_n$ we denote the sojourn time in state J_n before the *(n+1)*th jump. The kernel $\mathbf{Q} = [Q_{ij}(s,t)]$ associated to the Markov renewal process is defined in the following way:

$$Q_{ij}(s,t) = \mathrm{P}[\, J_{n+1} = j,\ T_{n+1} \le t \mid J_n = i,\ T_n = s],$$

and so:

$$p_{ij}(s) = \mathrm{P}[\, J_{n+1} = j, \mid J_n = i,\ T_n = s] = \lim_{t\to\infty} Q_{ij}(s,t);\ \ i, j \in I,\ \ s, t \in \mathbb{N},\ \ s \le t\ .$$

$\mathbf{P}(s) = [p_{ij}(s)]$is the transition matrix of the embedded non-homogeneous Markov chain.

Furthermore, the probability that the process will leave state i from time s within time t has to be introduced:

$$H_i(s,t) = \mathrm{P}[\, T_{n+1} \le t \mid J_n = i,\ T_n = s].$$

Obviously, it follows that $H_i(s,t) = \sum_{j=1}^{m} Q_{ij}(s,t)$.

Now the distribution function (d.f.) of the waiting time in each state i can be defined, given that the state successively occupied is known:

$$F_{ij}(s,t) = \mathrm{P}[\, T_{n+1} \le t \mid J_n = i,\ J_{n+1} = j,\ T_n = s].$$

The related probabilities can be obtained by means of the following formula:

$$F_{ij}(s,t) \ = \ \begin{cases} Q_{ij}(s,t)\,/\,p_{ij}(s) & \text{if } p_{ij}(s) \ne 0 \\ \quad 1 \quad \text{if } p_{ij}(s) = 0 \end{cases},$$

In a Markov environment, the d.f. $F_{ij}(s,t)$ have to be geometrically distributed. By contrast, in the semi-Markov case the d.f. $F_{ij}(s,t)$ may be of any type.

By means of the $F_{ij}(s,t)$ we can take into account the problem given by the duration inside the states. In the disability context, we know that the transition probabilities depend on the time an individual has remained at a certain state level.

Now, let $N(t) = sup\{n \in \mathbb{N}: T_n \le t\}$ be the number of transitions up to time t, then the DTNHSMP $Z(t)$ can be defined as $Z(t) = J_{N(t)}$ denoting the state occupied by the process at each time.

The transition probabilities are defined in the following way:

$$\phi_{ij}(s,t) = \mathrm{P}\left[Z(t) = j \mid Z(s) = i, T_{N(s)} = s\right].$$

They are obtained by solving the following evolution equations:

$$\phi_{ij}(s,t) = d_{ij}(s,t) + \sum_{\beta=1}^{m} \sum_{\vartheta=s+1}^{t} b_{i\beta}(s,\vartheta)\phi_{\beta j}(\vartheta,t)\,, \qquad (2.1)$$

where

$$b_{ij}(s,t) = P\left[J_{n+1} = j, T_{n+1} = t \mid J_n = i, T_n = s\right] = \begin{cases} Q_{ij}(s,t) - Q_{ij}(s,t-1) & \text{if } t > s \\ \quad 0 \quad \text{if } t = s \end{cases},$$

and

$$d_{ij}(s,t) = \begin{cases} 1 - H_i(s,t) & \text{if } \ i = j \\ \quad 0 & \text{if } \ i \ne j. \end{cases}$$

The first part of formula (2.1) provides the probability that the system does not have transitions up to the time t given that it entered in state i at time s. $d_{ij}(s,t)$, in a disability insurance model represents the probability that the policyholder does not have any new evaluation from time s up to time t. This makes sense if and only if $i=j$.

In the second part of (2.1), $b_{i\beta}(s,\vartheta)$ represents the probability that the system enters state β just at time ϑ given that it entered in state i at time s. After the transition, the system will go to state j following one of the possible trajectories that go from state β at time ϑ, bringing the system into state j at time t.

Well-known algorithms are available for the numerical solution of equation (2.1), see for example Janssen and Manca (2007).

Definition 1: Let $B(t)=t-T_{N(t)}$ be the backward recurrence time process (see Limnios and Oprişan, 2001; Janssen and Manca, 2006).

The backward recurrence time process denotes the time since the occurrence of the last transition.
In D'Amico et al. (2009) the following probabilities were defined:

$$\begin{aligned} {}^{b}\phi_{ij}(l,s;t) &= P[Z(t)=j|Z(s)=i,B(s)=s-l] \\ &= P\big[J_{N(t)}=j\big|J_{N(s)}=i,T_{N(s)}=s,T_{N(s)+1}>s\big], \end{aligned} \tag{2.2}$$

$$\begin{aligned} \phi_{ij}^{b}(s;l',t) &= P[Z(t)=j,B(t)=t-l'|Z(s)=i,B(s)=0] \\ &= P\big[J_{N(t)}=j,T_{N(t)}=l',T_{N(t)+1}>t\big|J_{N(s)}=i,T_{N(s)}=s\big]. \end{aligned} \tag{2.3}$$

Formulae (2.2) and (2.3) represent the semi-Markov transition probabilities with initial and final backward times respectively.
In (2.2) we know that at time *s* the system is in state *i*. We also know that it entered in this state at time *l* and *s-l* represents the initial backward time. Then we are looking for the probability of being in state *j* at time *t*.
In (2.3) we know that the system entered state *i* at time *s*. In this case we are interested in the probability of being in state *j* at time *t* with the entrance in this state at time *l'*. The final backward time is *t-l'*.
Combining these two cases, we obtain the transition probabilities with initial and final backward times (see D'Amico et al., 2009):

$$\begin{aligned} {}^{b}\phi_{ij}^{b}(l,s;l',t) &= \mathrm{P}\big[Z(t)=j,B(t)=t-l'\,|\,Z(s)=i,B(s)=s-l\big] \\ &= \mathrm{P}\big[J_{N(t)}=j,T_{N(t)}=l',T_{N(t)+1}>t\,|\,J_{N(s)}=i,T_{N(s)}=l,T_{N(s)+1}>s\big]. \end{aligned} \tag{2.4}$$

In Figure 1 a trajectory of a DTNHSMP with initial and final backward times is reported. In this figure we have that $N(s)=n$, $N(t)=h-1$, the initial backward time $B(s)=s-T_n=s-l$ and the final backward time $B(t)=t-T_{h-1}=t-l'$.

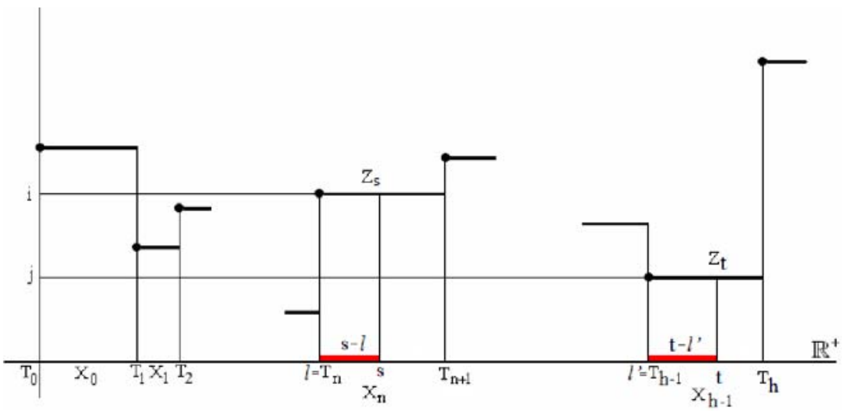

Figure 1: Initial and final backward values

To present the evolution equations of probabilities (2.2), (2.3) and (2.4) we introduce the following notation:

$$d_{ij}(l,s;t)=\begin{cases} \dfrac{1-H_i(l,t)}{1-H_i(l,s)} & \text{if} \quad i=j \\ 0 & \text{if} \quad i\neq j \end{cases},$$

which represents the probability of having no transition from state *i* between times *l* and *t* given that no transition occurred from state *i* between times *l* and *s*.
Moreover by

$$b_{ij}(l,s;t)=\frac{b_{ij}(l,t)}{1-H_i(l,s)},$$

we denote the probability of making the next transition from state i to state j from time l to time t given that the system does not make transitions from state i between times l and s.
The relations (2.5), (2.6) and (2.7) represent the evolution equations of (2.2), (2.3) and (2.4) respectively:

$$ {}^{b}\phi_{ij}(l,s;t) = d_{ij}(l,s;t) + \sum_{\beta=1}^{m}\sum_{\vartheta=s+1}^{t} b_{i\beta}(l,s;\vartheta)\phi_{\beta j}(\vartheta,t), \tag{2.5} $$

$$ \phi_{ij}^{b}(s;l',t) = d_{ij}(s,t)\mathbf{1}_{\{l'=s\}} + \sum_{\beta=1}^{m}\sum_{\vartheta=s+1}^{l'} b_{i\beta}(s,\vartheta)\phi_{\beta j}^{b}(\vartheta;l',t), \tag{2.6} $$

where $\mathbf{1}_{\{l'=s\}} = 1$ if and only if $l' = s$.

$$ {}^{b}\phi_{ij}^{b}(l,s;l',t) = d_{ij}(l,s;t)\mathbf{1}_{\{l'=l\}} + \sum_{\beta=1}^{m}\sum_{\vartheta=s+1}^{l'} b_{i\beta}(l,s;\vartheta)\phi_{\beta j}^{b}(\vartheta;l',t). \tag{2.7} $$

Expression (2.5) provides the probability that the system is in state j at time t given that it was in state i at time s and entered in this state at time l. If in (2.5) $l = s$ then we recover the equation (2.1).
Expression (2.6) gives the probability that the system will enter state j just at time l' and will remain in this state, without any other transition, up to time t given that it entered at time s in state i. The part $d_{ij}(s;t)\mathbf{1}_{\{l'=s\}}$ of (2.6) represents the probability of not having a transition from time s to time t. Consequently, the final backward time $t-l'$ must be exactly equal to $t-s$ and it makes sense only if $i = j$. The second part of (2.6) means that the system does not move from time s to time ϑ and that, just at this time, it jumps to state β . Afterwards, following one of the possible trajectories, the system arrives in state j just at time l' and does not move from this state at least up to time t.

Remark 1. It should be noted that considering all the possible backward values in the final state, we recover the transition probabilities (2.1) that is:

$$ \phi_{ij}(s,t) = \sum_{l'=s}^{t} \phi_{ij}^{b}(s;l',t) \ . $$

Expression (2.7) gives the probability that the system entered in state j at time l' and remained inside this state without any other transition up to time t given that it entered state i at time l and it did not move up to s. The term $d_{ij}(l,s;t)\mathbf{1}_{\{l'=l\}}$ gives the probability of not having transitions from l to t outside state i given that no transition occurred from l to s. This probability contributes only if $i = j$ and $l' = l$. The second part of (2.7) represents the probability of making the next transition from i at time l to whatever state β at whatever time ϑ and then of moving, following whatever trajectory which makes provision for the entrance in j at time l' with no transition up to time t. This probability is conditional on the permanence of the system in i from time l up to time s.
The algorithm used to obtain the numerical solutions for the given equations (2.5), (2.6) and (2.7) is given in D'Amico et al. (2009).

Remark 2. Relation (2.7) is a combination of (2.5) and (2.6). This last evolution equation is the one used to construct the model for the disability insurance. This type of model was suggested by Haberman and Pitacco (1999), but no formulae were included with the problem, or at least they were not presented.

3 Semi-Markov reward processes with backward times

Now we introduce the reward structure. A permanence reward $\psi_i(s,t)$ is paid when the process visits state i at time t for a contract starting at time s. An impulse reward $\gamma_{ij}(s,t)$ is paid due to the transition from state i to state j at time t for a contract starting at time s. We assume that permanence and impulse rewards are amounts of money. They have to be discounted using a discrete time non-homogeneous discount factor $v(s,t)$. To define $v(s,t)$ we introduce the non-homogeneous interest rates $r(s,s+1), r(s,s+2),\ldots, r(s,s+t),\ldots$ and then we can define

$$ v(s,t) = \begin{cases} 1 & \text{if} \quad t = s \\ \prod_{h=s+1}^{t}(1+r(s,h))^{-1} & \text{if} \quad t > s \end{cases} . $$

With the aim of defining the discounted accumulated semi-Markov reward process with initial and final backward times we introduce the random variable

$$ 1_{\{T_{N(s)+1}>t | J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}}, $$

with Bernoulli distribution of parameter

$$p=\Pr\left(T_{N(s)+1}>t \middle| J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\right).$$

This random variable assumes a value of one if the time of the next transition is greater than t given that the system entered state i with last transition at time l and it did not move up to time s. The random variable is also introduced

$$1_{\{J_{N(s)+1}=k, T_{N(s)+1}=\theta | J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}},$$

with Bernoulli distribution of parameter

$$\tilde{p}=\Pr\left(J_{N(s)+1}=k, T_{N(s)+1}=\theta \middle| J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\right).$$

This random variable assumes a value of one if the time of the next transition is equal to θ and the state entered is k given that the system entered state i with last transition at time l and it did not move up to time s.

In line with Stenberg et al. (2007) we define the accumulated reward process with initial and final backward times by means of the following relation:

Definition 2. Let $\xi_{ij}(l,s;l',t)$ be the discounted accumulated semi-Markov reward process with initial and final backward times, defined by

$$\begin{aligned}
\xi_{ij}(l,s;l',t) &\overset{d}{=} \\
&1_{\{T_{N(s)+1}>t | J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}} 1_{\{i=j\}} 1_{\{l'=l\}} \left[\sum_{\tau=s+1}^{t} \psi_i(s,\tau) v(s,\tau)\right] \\
&+\sum_{k\in I} \sum_{\theta=s+1}^{l'} 1_{\{J_{N(s)+1}=k, T_{N(s)+1}=\theta | J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}} \\
&\cdot \left[\sum_{\tau=s+1}^{T_{N(s)+1}} \psi_i(s,\tau) v(s,\tau) + \right. \\
&\left. v(s,T_{N(s)+1}) \left(\gamma_{i,J_{N(s)+1}}(s,T_{N(s)+1}) + \xi_{J_{N(s)+1}j}(T_{N(s)+1},T_{N(s)+1};l',t)\right)\right]
\end{aligned} \tag{3.1}$$

The symbol $\overset{d}{=}$ means that the random variables on the left and on the right have the same distribution. The process $\xi_{ij}(l,s;l',t)$ describes the discounted total amount of money accumulated from time s up to time t considering that the DTNHSMP will be in state j at time t with entrance in this state at time l' (final backward time equal to t-l') given that at time s it was in state i with entrance in this state at time l (initial backward time equal to s-l).

Let us denote by ${}^bV_{ij}^b(l,s;l',t)=E\left[\xi_{ij}(l,s;l',t)\right]$. To compute the expectation of (3.1) we have to consider that:

i)
$$E\left[1_{\{T_{N(s)+1}>t | J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}}\right]$$
$$=\Pr\left(T_{N(s)+1}>t \middle| J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\right)=d_{ij}(l,s,t);$$

ii) $1_{\{J_{N(s)+1}=k, T_{N(s)+1}=\theta | J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}}$ and $\xi_{k,j}(\theta,\theta;l',t)$ are independent random variables because the accumulation process $\xi_{k,j}(\theta,\theta;l',t)$ has the Markov property at transition times. Indeed, it depends only on the future evolution of the DTNHSMP starting from state k at time θ and ending in state j at time t with a final backward time equal to l';

iii)
$$E\left[1_{\{J_{N(s)+1}=k, T_{N(s)+1}=\theta | J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}}\right]$$
$$=\Pr\left(J_{N(s)+1}=k, T_{N(s)+1}=\theta \middle| J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\right)$$
$$=\frac{b_{ik}(l,\theta)}{1-H_i(l,s)}=b_{ik}(l,s;\theta);$$

By taking the expectation in (3.1) we obtain the following equation:

$$
\begin{aligned}
&{}^{b}V_{ij}^{b}\left(l,s;l',t\right)=\chi\left(l'=l\right)d_{ij}\left(l,s;t\right)\left[\sum_{\tau=s+1}^{t}\psi_i(s,\tau)v(s,\tau)\right]\\
&+\sum_{k\in I}\sum_{\theta=s+1}^{l'}b_{ik}\left(l,s;\theta\right)\left[\sum_{\tau=s+1}^{\theta}\psi_i(s,\tau)v(s,\tau)\right. \qquad (3.2)\\
&\left.+\gamma_{i,k}\left(s,\theta\right)v(s,\theta)+{}^{b}V_{kj}^{b}\left(\theta,\theta;l',t\right)v(s,\theta)\right].
\end{aligned}
$$

Equation (3.2) is the evolution equation representing the actuarial value of the total rewards accumulated in the interval [s, t] by imposing constraints on the arrival times in the states occupied at times s and t. Following Stenberg et al. (2007) recursive equations can be derived for the higher order moments of the reward process $\xi_{ij}\left(l,s;l',t\right)$.

It should be noted that equation (3.2) makes provision in a complete way for the duration dependence by using the backward process at initial and final times simultaneously. Moreover, the process also considers the final state j and this is a crucial point for defining the retrospective reserve (see subsection below).

There are some interesting particular cases of equation (3.2). First of all we can ignore the duration effects on the starting state by not considering the initial backward time. In this case, if $B(s)=0\ \Rightarrow l'=s$ and we obtain the following definition:

Definition 3. Let $\xi_{i,j}(s;l',t)\ \coloneqq\ \xi_{i,j}(s,s;l',t)$ be the discounted accumulated semi-Markov reward process with final backward time, defined by

$$
\xi_{ij}(s;l',t)\overset{d}{=}
$$

$$
1_{\{T_{N(s)+1}>t|J_{N(s)}=i,T_{N(s)}=s,T_{N(s)+1}>s\}}1_{\{i=j\}}1_{\{l'=s\}}\left[\sum\nolimits_{\tau=s+1}^{t}\psi_i(s,\tau)v(s,\tau)\right]
$$

$$
\sum_{k\in I}\sum_{\theta=s+1}^{l'}1_{\{J_{N(s)+1}=k,T_{N(s)+1}=\theta|J_{N(s)}=i,T_{N(s)}=s,T_{N(s)+1}>s\}}
$$

$$
\left[\sum\nolimits_{\tau=s+1}^{T_{N(s)+1}}\psi_i(s,\tau)v(s,\tau)+v(s,T_{N(s)+1})\left(\gamma_{i,J_{N(s)+1}}(s,T_{N(s)+1})+\xi_{J_{N(s)+1}j}(T_{N(s)+1};l',t)\right)\right]
$$
(3.3)

Denote by $V_{ij}^{b}\left(s;l',t\right)=E\left[\xi_{ij}\left(s;l',t\right)\right]$, by taking the expectation of (3.3) we have:

$$
\begin{aligned}
V_{ij}^{b}(s;l',t)&=1_{\{l'=s\}}1_{\{i=j\}}\left(1-H_i(s;t)\right)\left[\sum\nolimits_{\tau=s+1}^{t}\psi_i(s,\tau)v(s,\tau)\right]\\
&++\sum\nolimits_{k\epsilon I}\sum\nolimits_{\theta=s+1}^{l'}b_{ik}(s;\theta)(\sum\nolimits_{\tau=s+1}^{\theta}\psi_i(s,\tau)v(s,\tau)\\
&+\gamma_{i,k}(s,\theta)v(s,\theta)+V_{kj}^{b}(\theta;l',t)v(s,\theta)). \qquad (3.4)
\end{aligned}
$$

Formula (3.4) is obtained by substituting for l the value s in expression (3.2), by using the notation $V_{ij}^{b}(s;l',t)\coloneqq\ {}^{b}V_{ij}^{b}(s,s;l',t)$ and by observing that:

$$
d_{ij}(s,s;t)=\frac{1-H_i(s,t)}{1-H_i(s,s)}=1-H_i(s,t),
$$

$$
b_{ij}(s,s;t)=\frac{b_{ij}(s;t)}{1-H_i(s;s)}=b_{ij}(s;t).
$$

If we ignore the duration effects on the arriving state by not considering the final backward value and we similarly ignore the arriving state j we have the process $\xi_i\left(l,s;t\right)$. This represents the accumulated discounted semi-Markov reward process with initial backward time. This process has been defined and analyzed by Stenberg et al. (2007).

3.1 The algorithm

The relations of a discrete time initial and final backward semi-Markov reward process are fully described above. We present this program using a pseudo-language so that the reader can follow the algorithm used to obtain the numerical solution for the given process.

The computer program used in the application was written in Mathematica code.

Inputs:
T: time horizon considered,
m: number of states,
P(t): matrix of the non-homogeneous embedded Markov chain,
F(s,t): matrix of the waiting time d.f.,
Γ(s,t): matrix of the impulse rewards,
Ψ(s,t): matrix of the permanence rewards,
R(s): matrix of the non-homogeneous interest rates,

```
(* discount factors construction*)
FOR s = 0, s < T, s++,
     FOR t = s + 1, t ≤ T, t++,
          v(s,t)= v(s,t-1) /(1+r(s,t));
    END FOR;
END FOR;

(* kernel construction*)
FOR s = 0, s ≤ T, s++,
     FOR t = s + 1, t ≤ T, t++,
          Q(s,t)= P(s) *F(s,t)
    END FOR;
END FOR;

(* probability of exiting from state i *)
FOR s = 0, s ≤ T, s++,
     FOR t = s + 1, t ≤ T, t++,
          FOR i= 1, i ≤ m, i++,
               H(s,t) = Q(s,t) · 1 ;
          END FOR;
    END FOR;
END FOR;

(* probability of exiting from state i with initial backward time s-u *)
FOR u = 0, u ≤ T, u++,
     FOR s = u, s ≤ T, s++,
          FOR t = s + 1, t ≤ T, t++,
               FOR i= 1, i ≤ m, i++,
                    d_ii(u,s;t)=(1-H_i(u,t))/ (1-H_i(u,s)) ;
               END FOR;
          END FOR;
    END FOR;
END FOR;

(* probability of going from state i to state j just at time t with an initial backward time s-u *)
FOR u = 0, u ≤ T, u++,
     FOR s = u, s ≤ T, s++,
          FOR t = s + 1, t ≤ T, t++,
               FOR i= 1, i ≤ m, i++,
                    FOR j= 1, j ≤ m, j++,
                         b_ij(u,s;t)=(Q_ij(u,t)- Q_ij(u,t-1))/ (1-H_i(u,s)) ;
                    END FOR;
               END FOR;
          END FOR;
    END FOR;
END FOR;

(* solution of DTNHSMRP evolution equation with initial and final backward times *)
```

(* construction of the accumulated discounted permanence reward matrix $\widetilde{\mathbf{\Psi}}(s,t)$ and of the discounted transition reward matrix **a**(s,t)*)

```
FOR  s = 0, s < T, s++,
     FOR  t = s + 1, t ≤ T, t++,
          ψ̃(s,t) = ψ̃(s,t − 1) + ψ(s,t) * v(s,t);
          a(s,t) = γ(s,t) * v(s,t);
     END FOR;
END FOR;
```

(* construction of the matrix of the kernel multiplied by the rewards *)

```
FOR  u = 0, u ≤ T, u++,
     FOR  s = u, s ≤ T, s++,
          FOR t = s + 1, t ≤ T, t++,
               Br(u,s;t) = b(u,s;t) * [ ψ̃(s,t) + a(s,t)];
          END FOR;
     END FOR;
END FOR;
```

(* step 1 – computation of expected reward with initial and final backward times equal to zero *)

```
FOR h= T + 1, h > 0, h−−,
     ᵇVᵇ(h,h,h,h) = ψ̃(h,h) * d(h,h,h);
     FOR k= T + 1, k > h, k−−,
          ᵇVᵇ(h,h,k,k) = ψ̃(h,k) * d(h,h,k);
          FOR z= k, z > h, z−−,
               ᵇVᵇ(h,h,k,k) += b(h,h,z) · ᵇVᵇ(z,z,k,k) * v(h,z);
               A += Br(h,h,z);
          END FOR;
          ᵇVᵇ(h,h,k,k) += A · 1;
     END FOR;
END FOR;
```

(* step 2 – computation of expected reward with only initial backward times *)

```
FOR u= T, u > 0, u−−,
     FOR h= T + 1, h > u, h−−,
          ᵇVᵇ(u,h,h,h) = ψ̃(h,h) * d(u,h,h);
          FOR k= T + 1, k > h, k−−,
               ᵇVᵇ(u,h,k,k) = ψ̃(h,k) * d(u,h,k);
               FOR z= k, z > h, z−−,
                    ᵇVᵇ(u,h,k,k) += b(u,h,z) · ᵇVᵇ(z,z,k,k) * v(h,z);
                    C += Br(u,h,z);
               END FOR;
               ᵇVᵇ(u,h,k,k) += C · 1;
          END FOR;
     END FOR;
END FOR;
```

(* step 3 – computation of expected reward with only final backward times *)

```
FOR k= T + 1, k > 1, k−−,
     FOR lpr= k − 1, lpr > 0, lpr−−,
          ᵇVᵇ(lpr,lpr,lpr,k) = ψ̃(lpr,k) * d(lpr,lpr,k);
          FOR h= lpr − 1, h > 0, h−−,
               ᵇVᵇ(h,h,lpr,k) = b(h,h,lpr) · (ψ̃(lpr,k) * d(lpr,lpr,k));
               FOR z= lpr, z ≥ h, z−−,
                    ᵇVᵇ(h,h,lpr,k) += b(h,h,z) · ᵇVᵇ(z,z,lpr,k) * v(h,z);
                    E += Br(h,h,z);
               END FOR;
               ᵇVᵇ(h,h,lpr,k) += E · 1;
          END FOR;
```

END FOR;
END FOR;

(* step 4 – computation of expected reward with initial and final backward times *)
FOR k= $T+1, k>1, k--$,
FOR lpr= $k-1, \text{lpr}>1, \text{lpr}--$,
FOR l= $\text{lpr}-1, l>1, l--$,
${}^{b}\mathbf{V}^{b}(l,\text{lpr},\text{lpr},k)$+= $\widetilde{\boldsymbol{\psi}}(\text{lpr},k) * \mathbf{d}(l,\text{lpr},k)$;
END FOR;
END FOR;
FOR lpr= $k-1, \text{lpr}>1, \text{lpr}--$,
FOR h= $\text{lpr}-1, h>1, h--$,
FOR l= $h-1, l>0, l--$,
${}^{b}\mathbf{V}^{b}(l,h,\text{lpr},k)$+= $\widetilde{\boldsymbol{\psi}}(\text{lpr},k) * \mathbf{d}(l,\text{lpr},k))$;
FOR z= $\text{lpr}, z>h, z--$,
${}^{b}\mathbf{V}^{b}(l,h,\text{lpr},k) \mathrel{+}= \mathbf{b}(l,h,z)\cdot {}^{b}\mathbf{V}^{b}(z,z,\text{lpr},k) * v(h,z)$;
$\mathbf{G} \mathrel{+}= \mathbf{Br}(l,h,z)$;
END FOR;
${}^{b}\mathbf{V}^{b}(l,h,\text{lpr},k) \mathrel{+}= \mathbf{G}\cdot\mathbf{1}$;
END FOR;
END FOR;
END FOR;
END FOR;

The * means the element-by-element or Hadamard matrix product. The $\cdot$ is the usual row column matrix product. The **1** is a vector of ones of appropriate size. The variable names written in italics are real numbers; those written in boldface are matrices or vectors.
The algorithm shows how to solve the (3.2) and (3.4) evolution equations and the evolution equation in the case of only initial backward time. After reading the inputs, the structure of the discount factor is constructed by using the non- homogeneous interest rates and then the kernel of the process is constructed multiplying the matrices **P** and **F**. Step-4 is the resolution of equation (3.2) which is executed by five down to loops. In Step-1, Step-2 and Step-3 we solve different special cases of equation (3.2).

3.2 Prospective reserves

Let us assume that the policy is issued at time s in state $Z(s)=i$ of the semi-Markov chain with backward time $B(s)=s-l$. Premiums and benefits for the policy are paid by the insured party and by the insurer depending on the state of the degree of disability.
The permanence reward $\psi_i(s,t)$ considers the payment of a premium or a benefit due to the occupancy of state i at time t for a contract starting at time s.
The impulse reward $\gamma_{ij}(s,t)$ considers an insurance benefit or lump sum.
In general, the prospective premium reserve is defined as the expected value of the loss function (see Wolthuis, 2003). In our case the random process $\xi_{ij}(l,s;l',t)$ represents the accumulated discounted reward process with initial and final backward times and expresses the difference between future benefits and premium payments with constraints on the duration in the starting and arriving states. Consequently, $\xi_{ij}(l,s;l',t)$ is a constrained loss function and its expectation represents the prospective reserve with full backward information. A particular case of ${}^{b}V_{ij}^{b}(l,s;l',t)$ is ${}^{b}V_{ij}(l,s;t)$ which is the prospective reserve with initial backward time.
Generally, in life insurance, the policy terminates with the death of the policyholder. Since death occurs at a random time, the prospective reserve is considered for $t\to\infty$ (see, for example, Wolthuis, 2003).
Let us denote by $\xi_i(l,s)=\xi_i(l,s;\infty)$. the accumulated discounted reward process in the interval [s,∞) given that the DTNHSMP entered state i at time l and did not move up to s.
If we consider the random variables $1_{\{T_{N(s)+1}>s+1|J_{N(s)}=i,T_{N(s)}=l,T_{N(s)+1}>s\}}$, and $1_{\{J_{N(s)+1}=k,T_{N(s)+1}=s+1|J_{N(s)}=i,T_{N(s)}=l,T_{N(s)+1}>s\}}$ with Bernoulli distribution function of parameter $\Pr\left(T_{N(s)+1}>s+1|J_{N(s)}=i,T_{N(s)}=l,T_{N(s)+1}>s\right)$ and

$\Pr\left(J_{N(s)+1}=k, T_{N(s)+1}=s+1 \middle| J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\right)$ respectively, we can give the following recursive representation:

$$\xi_i(l,s) =$$
$$1_{\{T_{N(s)+1}>s+1|J_{N(s)}=i,T_{N(s)}=l,T_{N(s)+1}>s\}}[\psi_i(s,s+1)v(s,s+1)+\xi_i(l,s+1)]$$
$$\sum_{k\in I} 1_{\{J_{N(s)+1}=k\ T_{N(s)+1}=s+1|J_{N(s)}=i,T_{N(s)}=l,T_{N(s)+1}>s\}}\, v(s,T_{N(s)+1})$$
$$\cdot\left[\psi_{J_{N(s)+1}}(s,T_{N(s)+1})+\gamma_{i,J_{N(s)+1}}(s,T_{N(s)+1})+\xi_{J_{N(s)+1}}(T_{N(s)+1},T_{N(s)+1})\right] \tag{3.5}$$

Formula (3.5) states that, given the information set $\{J_{N(s)}=i, T_{N(s)}=l, T_{N(s)+1}>s\}$, if no transition occurs up to time $s+1$, the accumulated discounted reward for the interval $[s,\infty)$ can be obtained as the sum of the discounted permanence reward due to the occupancy of state i at time $s+1$, i.e. $\psi_i(s,s+1)v(s,s+1)$ and the accumulated discounted reward for the interval $[s+1,\infty)$ given that the DTNHSMP is in state i where it entered with last transition at time l.
By contrast, if the next transition occurs at time s+1 in state k then the accumulated discounted reward for the interval $[s,\infty)$ is obtained as the sum of two addends:
The first term is the discounted permanence reward due to the occupancy of the state visited at time s+1, the second is the discounted impulse reward due to the transition executed at time s+1, and the third is the remaining accumulated discounted reward for the interval $[s+1,\infty)$ given that the DTNHSMP is in state k where it entered at time $s+1$.
If we denote by $W_i\left(l,s\right)=E\left[\xi_i\left(l,s\right)\right]$, by taking the expectation of (3.5), by applying similar arguments to i), ii) and iii) we obtain

$$W_i(l,s)=\frac{1-H_i(l;s+1)}{1-H_i(l;s)}[\psi_i(s,s+1)+W_i(l,s+1)]v(s,s+1)$$
$$\sum_{k\in I} b_{ik}(l,s;s+1)[\psi_i(s,s+1)+\gamma_{ik}(s,s+1)+W_i(l,s+1)]\,v(s,s+1) \tag{3.6}$$

Equation (3.6) expresses the change of the prospective reserve for state i at time s with duration $s-l$ from time s to time $s+1$. Therefore, it can be seen as a generalization of the Thiele differential equation for a disability insurance contract described by a non-homogeneous semi-Markov chain.
This equation explains that the expected accumulated reward during the whole life of a contract can be computed.

3.3 Retrospective reserves

In general retrospective reserves are defined as the expected discounted value of past premiums minus past benefits.
Different definitions of retrospective reserves have been proposed. Here we consider Norberg's (1990) definition and we adapt it to our general framework. For a general notation on retrospective reserve see Janssen et al. (2009).
Let us denote the conditional retrospective premium reserve relative to the period $\left[s,t\right]$ with initial and final backward times by ${}^bM_{ij}^b\left(l,s;l',t\right)$.
The reserve ${}^bM_{ij}^b\left(l,s;l',t\right)$ is defined over the time interval $\left[s,t\right]$ valued at time and conditioned on $\left\{Z(s)=i, B(s)=s-l\right\}$ and $\left\{Z(t)=i, B(t)=t-l'\right\}$ for $0\le s\le t$ by

$$v(s,t)\,{}^b\phi_{ij}^b\left(l,s;l',t\right)\,{}^bM_{ij}^b\left(l,s;l',t\right)=-{}^bV_{ij}^b(l,s;l',t), \tag{3.7}$$

from which we obtain

$${}^bM_{ij}^b\left(l,s;l',t\right)=-{}^bV_{ij}^b(l,s;l',t)\frac{1}{v(s,t)\,{}^b\phi_{ij}^b\left(l,s;l',t\right)}, \tag{3.8}$$

If ${}^b\phi_{ij}^b\left(l,s;l',t\right)=0$ we set ${}^bM_{ij}^b\left(l,s;l',t\right)=0$.
The definition of the retrospective reserve in a semi-Markov environment demonstrates the need to introduce the reward process $\xi_{ij}(l,s;l',t)$ by considering an initial state and backward time, as well as a final state and backward time.
Notice that it is possible to derive recursive equations for the retrospective reserves by using relation (3.2) and (3.6) for the prospective reserves.

4 Real data numerical example

In this section we apply our model to a sample of real contracts. For the sake of completeness, we first report results for the transition probabilities, as obtained in D'Amico et al. (2009), and, second, we extend the analysis by introducing a reward structure.
The model has the following four states:
W – active; 2) P – pensioner; 3) Di – disabled; 4) De – dead
interrelated as indicated in Figure 2.

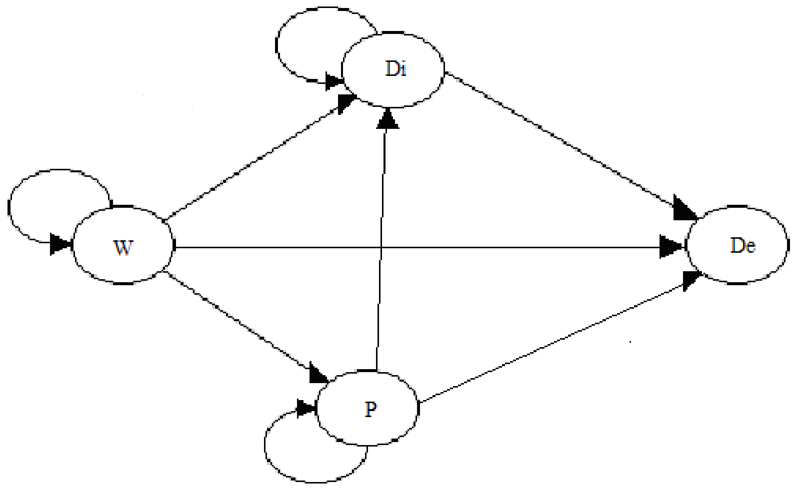

Figure 2: The disability model

It is well known that the transition probabilities from the disabled state are a function of the duration in the current state (see Haberman and Pitacco, 1999). In the SMP environment this aspect is considered, but solution (2.1) is not sensitive to duration. The introduction of the backward times, as in (2.7), allows us to manage transition probabilities that depend on the length of permanence inside the initial and final states.
The data analyzed are taken from a sample of contracts drawn at random from a mutual insurance company in Catalunya. A total of 150,000 insurance contracts are analysed and 2,800 LTC spells are observed for a period extending from 1975 to 2005.
In order to simplify the model, we chose to work with a five-year interval.
Owing to lack of space, we do not show the kernel estimates and other results, but these are available upon request. Note that the transition probability values vary in function of both the initial and final backward times, so the model is sensitive to both backward times (see Figure 3).
In all the histograms W is the starting state. The blue bars report the results in the absence of initial backward time (IBk=0); the red bars report the case with one year of initial backward time (IBk=1). The first observation is that the probability distribution is spread among the final backward times (for example in the south- west histogram FBk=0, 1, 2 and 3) and the arriving states. Indeed, in the north-west there are eight possible events with an arriving time equal to starting time plus one (AT=ST+1), four in the case of a final backward time equal to 1 and four with a final backward time equal to 0. In the north-east, with an arriving time equal to starting time plus two (AT=ST+2), there are twelve possible cases, four for each final backward time and so on. The first blue and red bars in each histogram represent the probability of staying in the starting state; this decreases in function of the arriving time. It is also interesting to observe that the shape of the histograms changes in function of both the initial and final backward times, so the model is sensitive to both backward times.
To facilitate the reading of Figure 3, the first two bars in the first histogram represent respectively the probabilities ${}^{b}\phi_{11}^{b}(0,0;0,1)$, ${}^{b}\phi_{11}^{b}(0,1;1,2)$

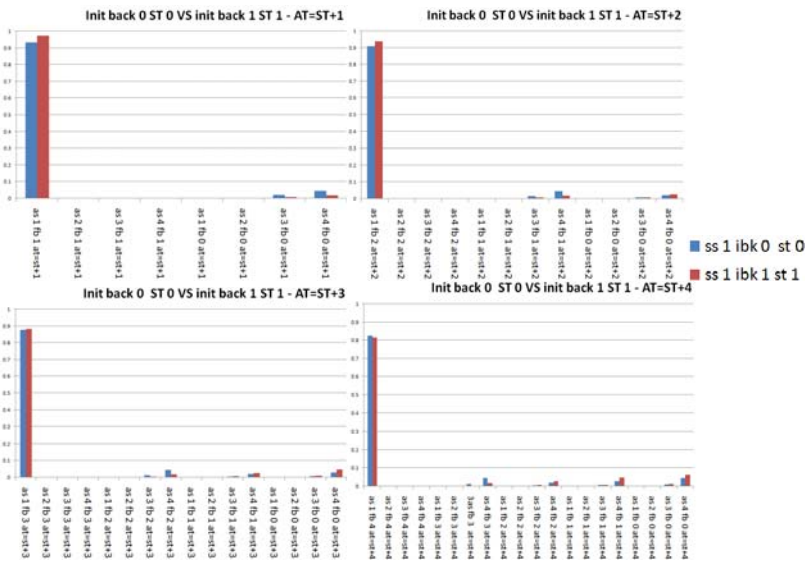

Figure 3: Comparison of initial and final backward times

The same behaviour is translated to the accumulated reward process. Due to missing data on rewards (permanence and impulse), we assumed the following reward structure:

$$\psi_W(s,t) = -5000, \ \psi_P(s,t) = 15000,$$

$$\psi_{Di}(s,t) = 30000, \ \gamma_{W,Di}(s,t) = \gamma_{P,Di}(s,t) = 2000,$$

and we consider a constant interest rate structure *r(s,s+h)=2%* for all *h>0*.
Table 1 shows the dependence of the accumulated reward process in function of the initial and final backward times. In fact, by comparing two columns we see different expected reward values due to different initial backward times. By comparing two rows, we see the effects resulting from different final backward times.

Final backward	$V_1(0,3;l',10)$	$V_1(1,3;l',10)$	$V_1(2,3;l',10)$	$V_1(3,3;l',10)$
l'=3	-4622.85	-8392.82	-11732.10	-13395.30
l'=4	-420821.00	-424019.00	-426845.00	211.25
l'=5	-415040.00	-417212.00	-419330.00	-1537.27
l'=6	-401852.00	-402520.00	-403586.00	-8536.55
l'=7	-371546.00	-370686.00	-369788.00	-20629.00
l'=8	-291318.00	-285725.00	-279689.00	-23346.60
l'=9	-131829.00	-125292.00	-115690.00	-20180.90
l'=10	-3209.33	-11431.30	-15806.30	-17031.90

Table 1: Expected accumulated reward values

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