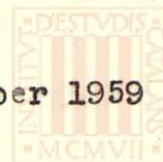


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Prof. R. P. Boas  
Evanston

Dear Professor Boas:

I received your kind letter of the 11 october together with the manuscript of my paper, Rahman's ~~than~~ correction and Srivastava's paper. Thank you for the confidence in commissioning me to act as referee regarding the publication of the latter.

I quite agree with you respecting the modifications you introduced in my manuscript so I am sending it back to you as indicated. I also return you Rahman's correction the results of which seem to me thoroughly correct.

Regarding the paper by Srivastava I should make the following remarks:

It should be interesting to prove the (1,5) or to give a bibliographic reference. Furthermore, the left hand member of (1,5) should be changed from " $\lambda_n$ " to " $\lambda$ ".

The first part of the proof of theorem 1, i.e., the proof that  $f(s)$  is an integral function, is not correct, however we can prove that  $f(s)$  is an integral function as follows: If  $\sigma_0$ ,  $\sigma'$  and  $\sigma''$  are the abscissae of absolute convergence of the series  $f(s)$ ,  $f_1(s)$  and  $f_2(s)$  respectively, since  $\lambda_n \sim \lambda_{1,n} \sim \lambda_{2,n}$  we have

$$\limsup_{n \rightarrow \infty} \frac{\log n}{\lambda_n} = \limsup_{n \rightarrow \infty} \frac{\log n}{\lambda_{1,n}} = \limsup_{n \rightarrow \infty} \frac{\log n}{\lambda_{2,n}} = 0$$

and therefore

$$\sigma_0 = -\limsup_{n \rightarrow \infty} \frac{\log |a_n|}{\lambda_n}, \quad \sigma' = -\limsup_{n \rightarrow \infty} \frac{\log |a_{1,n}|}{\lambda_{1,n}}, \quad \sigma'' = -\limsup_{n \rightarrow \infty} \frac{\log |a_{2,n}|}{\lambda_{2,n}}$$

and since  $\sigma' = \sigma'' = \infty$ ,  $\lambda_n \sim \lambda_{1,n} \sim \lambda_{2,n}$  and  $|a_n| \sim |a_{1,n}| \sim |a_{2,n}|$  it follows  $\sigma_0 = \infty$ . With this correction the proof of theorem 1 is correct.

The proof of theorem 3 is not correct because from

$$\log \frac{\mu(\sigma, f^{(n)})}{\mu(\sigma, f)} = \int_{\sigma_0}^{\sigma} [\lambda_{\nu(\kappa, f^{(n)})} - \lambda_{\nu(\kappa, f)}] d\kappa + o(1)$$

it follows

$$\log \frac{\mu(\sigma, f^{(n)})}{\mu(\sigma, f)} = [\lambda_{\nu(\sigma, f^{(n)})} - \lambda_{\nu(\sigma, f)}] \sigma - [\lambda_{\nu(\sigma_0, f^{(n)})} - \lambda_{\nu(\sigma_0, f)}] \sigma_0 - \int_{\sigma_0}^{\sigma} \kappa d[\lambda_{\nu(\kappa, f^{(n)})} - \lambda_{\nu(\kappa, f)}] + o(1)$$



where the integral is an Stieltjes's one, and therefore we cannot affirm that necessarily

$$\int_{\sigma_0}^{\sigma} K d[\lambda_{\nu(\sigma, f^{(n)})} - \lambda_{\nu(\sigma, f)}] = o(1)$$

The proof of (iii) of theorem 4 is not correct because it depends of the proof of theorem 3 which is not correct. The proof of (iv) is also not correct, since from (6,3) and

$$e^{-\sigma \delta(\sigma)} \log M(\sigma) = 1$$

it does not necessarily follow

$$\lim_{\sigma \rightarrow \infty} e^{-\sigma \delta(\sigma)} \log M(\sigma) = 1.$$

In short, the theorems ~~xxxxx~~ 1 and 2 are interesting, and the theorems 3 and 4 are doubtful because the proof are not correct.

In consequence of a new postal disposition my address has changed, therefore when writing me please put instead of "Barcelona (8)" the indication "Barcelona - 17".

With best wishes

yours sincerely