Barcelona 23 october 1959

Prof. R. P. Boas
Evanston

Dear Professor Boas:

I received your kind letter of the 11 october together with the manuscript of my paper, Rahman's ~~Txxx~~ correction and Srivastava's paper. Thank you for the confidence in commissioning me to act as referee regarding the publication of the latter.

I quite agree with you respecting the modifications you introduced in my manuscript so I am sending it back to you as indicated. I also return you Rahman's correction the results of which seem to me thoroughly correct.

Regarding the paper by Srivastava I should make the following remarks:

It should be interesting to prove the (1,5) or to give a bibliographic reference. Furthermore, the left hand member of (1,5) should be changed from "λ_n" to "λ".

The first part of the proof of theorem 1, i.e., the proof that f(s) is an integral function, is not correct, however we can prove that f(s) is an integral function as follows: If σ_0, σ' and σ'' are the abcissae of absolute convergence of the series $f(s)$, $f_1(s)$ and $f_2(s)$ respectively, since $\lambda_n \sim \lambda_{1,n} \sim \lambda_{2,n}$ we have

$$\limsup_{n\to\infty} \frac{\log n}{\lambda_n} = \limsup_{n\to\infty} \frac{\log n}{\lambda_{1,n}} = \limsup_{n\to\infty} \frac{\log n}{\lambda_{2,n}} = 0$$

and therefore

$$\sigma_0 = -\limsup_{n\to\infty} \frac{\log |a_n|}{\lambda_n}, \qquad \sigma' = -\limsup_{n\to\infty} \frac{\log |a_{1,n}|}{\lambda_{1,n}}, \qquad \sigma'' = -\limsup \frac{\log |a_{2,n}|}{\lambda_{2,n}}$$

and since $\sigma' = \sigma'' = \infty$, $\lambda_n \sim \lambda_{1,n} \sim \lambda_{2,n}$ and $|a_n| \sim |a_{1,n}|\,|a_{2,n}|$ it follows $\sigma_0 = \infty$. With this correction the proof of theorem 1 is correct.

The proof of theorem 3 is not correct because from

$$\log \frac{\mu(\sigma, f^{(n)})}{\mu(\sigma, f)} = \int_{\sigma_0}^{\sigma} [\lambda_{\nu(t,f^{(n)})} - \lambda_{\nu(t,f)}]\,dt + O(1)$$

it follows

$$\log \frac{\mu(\sigma, f^{(n)})}{\mu(\sigma, f)} = [\lambda_{\nu(\sigma,f^{(n)})} - \lambda_{\nu(\sigma,f)}]\sigma - [\lambda_{\nu(\sigma_0,f^{(n)})} - \lambda_{\nu(\sigma_0,f)}]\sigma_0 - \int_{\sigma_0}^{\sigma} t\,d[\lambda_{\nu(t,f^{(n)})} - \lambda_{\nu(t,f)}] + O(1)$$

where the integral is an Stieltjes's one, and therefore we cannot affirm that necessarily

$$\int_{\sigma_0}^{\sigma} t\, d\left[\lambda_{\nu(t,f^{(n)})} - \lambda_{\nu(t,f)}\right] = O(1)$$

The proof of (iii) of theorem 4 is not correct because it depends of the proof of theorem 3 which is not correct. The proof of (iv) is also not correct, since from (6,3) and

$$e^{-\sigma\delta(\sigma)} \log M(\sigma) = 1$$

it does not necessarily follow

$$\lim_{\sigma\to\infty} e^{-\sigma\rho(\sigma)} \log M(\sigma) = 1.$$

In short, the theorems ~~xxxxx~~ 1 and 2 are interesting, and the theorems 3 and 4 are doubtful because the proof are not correct.

In consequence of a new postal disposition my address has changed, therefore when writing me please put instead of "Barcelona (8)" the indication "Barcelona - 17".

With best wishes

yours sincerely