Kahane – 3 juliol 1961



Monsieur,

J'ai eu l'opportunité de consulter votre Mémoire à la Bibliothèque, mais il me serait très utile pour mon travail, particulièrement pendant les vacances, d'en avoir un exemplaire chez moi.

Je vous serais donc très reconnaissant si vous vouliez bien vouloir m'envoyer un tirés-à-part de la Mémoire cité.

Recevez, Monsieur le Professeur l'expression de mes sentiment très respectueuse of overconvergence is thus related to the general theory of entire functions and a generalization is immediately suggested. Our hypothesis need not demand $c_n = 0$ for all $n$ in $n_k < n \leqslant N_n$ but only for sufficiently many such values of $n$. It is convenient to define « occasional density » for a sequence of integers. In the special case of the regions just described we then have the generalised assertions (i) *If the occasional density of zero coefficients is positive there is overconvergence to* $f(z)$ *at regular points of* $|z| = 1$ (ii) *If* $\lambda$ *is sufficiently large and the occasional density of zero coefficients in th egaps is sufficiently near unity then overconvergence extends to a prescribed point in the region of regularity and* (iii) *If* $N_k|n_k$ *tends to infinity and almost all* $c_n$ *vanish in the gaps then overconvergence extends to the whole interior of* $\mathfrak{D}$. These results extend Theorem E y 4.

The methods used do not conveniently extend to general domains but other special cases can be handled. We can prove for example the following theorem.

If
$$f(z) = \sum_0^\infty c_n z^n$$
is regular on an arc $C$ of length $r$ on its circle of regularity $|z| = 1$ and if the occasional density of zero $c_1$ exceeds $1 - r|2\pi$ then there is overconvergence on $C$. From this theorem follow the gap theorems of Fabry and Polya [5, 626] just as Hadamard's gap theorem is commonly obtained from Oslrowshi's overconvergence theorem [2, 13].

2. In this paragraph $\mathfrak{D}$ is one of a family of domains bounded by spirals and containing the whole of $|z| = 1$ encept for a single point. It is evident that this point can be taken as $z = 1$ without loso of generality. The stalements of 1 are proved by a series of lemmas most of which are well known.

Lemma 1. *If* $f(z)$ *is regular in the whole plane less the segment* $z \geqslant 1$ *of the positive real axis then*
$$f(z) = \sum_0^\infty G(n)\, z^n$$
*where* $G(z)$ *is an entire function of exponential type and is moreover of zero type in the angle* $|\arg z| < \alpha$ *where* $\alpha < \frac{1}{2}\pi$ *may be arbitrarily near* $\frac{1}{2}\pi$. $G(z)$ *will depend on the choice of* $\alpha$.