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Dear Professor Kamthan,

As I promised in my letter 26 ult. I have studied the two papers that you last sent me.

Regarding the paper "ON PROXIMATE ORDER (R) OF ENTIRE FUNCTIONS REPRESENTED BY DIRICHLET SERIES (II)" I think it is interesting and it will be published after the paper of the same title that was accepted for publication in "Collectanea Math." However in the manuscript there exists the following small oversights:

On p.2, line 11, instead of

$$\exp\left\{\frac{\rho(\sigma+k)-\rho(\sigma)}{k}\,\sigma\right\}$$

it ought to be written

$$\exp\left\{\frac{\rho(\sigma+k)-\rho(\sigma)}{k}\,k\sigma\right\}$$

On p.3, line 5, instead of

$$E\leq D\left(e^{-\rho\mu}+\rho\mu\right)$$

it ought to be written

$$E\leq C\left(e^{-\rho\mu}+\rho\mu\right)$$

and hence (4.3) will be

(4.3) $\qquad E\leq C$

(from (4.2) and '4.3) as written in the manuscript it would follow that E = F = D)

On p. 3, line 7, instead of

$$\mu = \log(C/D)$$

it ought to be written

$$\mu = \frac{1}{\rho} \log(C/D)$$

On p. 3, line 12, instead of

"......from (6.4) E = 0 ....."

it ought to be written

"......from (6.2) F = 0 ....."

(since $Ce^{-\rho\mu} \to 0$ when $\mu \to \infty$)

Please let me know if you approve these small modifications.

In reference to the paper "THE GENERALIZED LINDELOF'S THEOREMS INVOLVING PROXIMATE ORDER OF ENTIRE FUNCTIONS" I think that in its presend form it cannot be published. In fact owing to the condition (7) of the definition of proximate order the introduction of L(r) is "artificial", since

$$\overline{\lim} \frac{\log M(r)}{r^{\rho(r)} L(r)} = \frac{1}{L}$$

holds alwaya, where $L = \lim_{r \to \infty} L(r)$. And it is sufficient to prove the theorems for L(r) = 1.

On the other hand, if in the definition of proximate order we delete the condition (7) then the most natural thing would be to put $\rho(r) = \rho$, and the theorems will be the Harishanker's theorems. It is a pity since the paper has very elegant proofs. I am returning you here with this paper.

Iam

yours sincerely

$\cancel{1/\rho + \sigma \mp \varphi(\lambda_n/A_2\rho) + \lambda_n}$

$$\frac{1}{\rho} + \sigma \mp \log \varphi(\lambda_n/A_2\rho) \mp \lambda_n \frac{\varphi'(\lambda_n/A_2\rho)}{A_2\rho\,\varphi(\lambda_n/A_2\rho)}$$

$$\rho(\sigma)\sigma - \rho(k\sigma)k\sigma = (\rho(\sigma) - \rho(k\sigma))\sigma + \rho(k\sigma)\sigma(1-k)$$

$$\rho(\sigma)\sigma - \rho(\sigma+k)(\sigma+k) = (\rho(\sigma) - \rho(\sigma+k))\sigma - k\rho(\sigma+k) \to -k\rho$$

$$\lambda(\sigma) \leq O(V(\sigma+k)) = O(V(\sigma))$$

$$|a_n| e^{\lambda_n\sigma} = |a_n| e^{\lambda_n\sigma_1} e^{\lambda_n(\sigma-\sigma_1)} \leq$$

$$\leq |a_{n_1}| e^{\lambda_{n_1}\sigma_1} e^{\lambda_n(\sigma-\sigma_1)} = |a_{n_1}| e^{\lambda_{n_1}\sigma} e^{(\lambda_n-\lambda_{n_1})(\sigma-\sigma_1)} \leq$$

$$\leq \mu(\sigma) e^{-(n-n_1)h(\sigma_1-\sigma)}$$

Finally on the page 3 line 7 ~~10~~ instead of

$$\lim \ldots = (A\rho e)^{1/\rho}$$

to write

$$\lim \ldots = (A_1\rho e)^{1/\rho}$$

On the other hand I ~~tin~~ think that a more precise definition of the function $\varphi(t)$ of the theorem 1 would with:

...... ~~with~~ which is defined as the single solution (when ~~t~~ $t > t_0$) of the equation:

$$\cancel{t = e^{\log\varphi\,\rho(\log\varphi)}}$$

$$t = e^{(\log\varphi)\,\rho(\log\varphi)}$$