Barcelona 21 April 1967

Prof. P.K. Kamthan
Department of Mathematics
Hamilton College
McMaster University
Hamilton, Ontario
Canada

Dear Professor Kamthan,

Thank you for your letter of the 15 March enclosing the manuscript of your paper "FK-space of entire Diriclet functions". I regret the delay in replying but I have been away for the last three weeks.

With regard to your manuscript I think that in the actual form it cannot be published because there are some oversights.

In the first place, if you can complete the proof I think that theorem 1 ought to be stated in the following form:

Theorem 1: With the usual addition and mutiplication we can define a topology such that $\mathcal{X}$ is a complex FK-space.

Because it is known that if to a linear space we assign two different topologies it can be that the corresponding topological linear space have different properties.

In the second place I think that it would be convenient to suppose that

$$(1) \qquad \lim \frac{\log n}{\lambda_n} = 0,$$

since without some hypothesis on the $\{\lambda_n\}$, the series

$$\sum a_n e^{\lambda_n s}$$

can represent an entire function and simultaneously it can be that it does not converge absolutely at any point of the complex plane. Therefore it is possible that

$$\|f;\ x_i\| = \infty \quad \text{for} \quad i = 1,2,\ldots.$$

On the other hand in the proof of (1.3) you suppose that from

$$d(f_m,\ f_n) < \varepsilon$$

it follows

$$\|f_m - f_n;\ x_i\| < \varepsilon$$

and this is not true, it only follows

$$\|f_m - f_n;\ x_i\|(1 - 2^i\varepsilon) \leq 2^i\varepsilon$$

But the proof can be completed as follows: if $2^i\varepsilon < \theta < 1$ your method gives

$$|a_j^{(m)} - a_j| < \frac{\theta}{1-\theta}\, e^{-x_i\lambda_j}$$

and therefore

$$|a_j| < |a_j^{(m)}| + \frac{\theta}{1-\theta}\, e^{-x_i\lambda_j}$$

and since

$$\lim \frac{\log|a_j^{(m)}|}{\lambda_j} = -\infty$$

it follows

$$\limsup \frac{\log|a_j|}{\lambda_j} \leq -x_i$$

but if $\varepsilon \to 0$ in $2^i\varepsilon < \theta < 1$ we can suppose that $i \to \infty$ and finally it will result

$$\lim \frac{\log|a_j|}{\lambda_j} = -\infty$$

Then from (1) it follows

$$\lim_{m\to\infty} ||f_m - f;\ x_i|| = 0 \qquad \text{and} \qquad \lim_{m\to\infty} d(f_m,\ f) = 0$$

Moreover in a step of the proof of (1.4) you implicitly supposa that

$$\alpha_n = e^{\lambda_n s}.$$

I think that the proof of the theorem in No.3 is not complete because I have not been able to understand how Q_0 does not change when m and n change.

There are some other minor oversights without importance.

With kindest regards

I am,

Yours sincerely

F. Sunyer Balaguer
Angel Guimera 66 pral.2º
Barcelona - 17, Spain