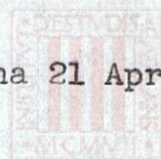


Barcelona 21 April 1967



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Fundació FERRAN SUNYER I BALAGUER

Prof. P.K. Kamthan
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Dear Professor Kamthan,

Thank you for your letter of the 15 March enclosing the manuscript of your paper "FK-space of entire Diriclet functions". I regret the delay in replying but I have been away for the last three weeks.

With regard to your manuscript I think that in the actual form it cannot be published because there are some oversights.

In the first place, if you can complete the proof I think that theorem 1 ought to be stated in the following form:

Theorem 1: With the usual addition and multiplication we can define a topology such that χ is a complex FK-space.

Because it is known that if to a linear space we assign two different topologies it can be that the corresponding topological linear space have different properties.

In the second place I think that it would be convenient to suppose that

$$(1) \quad \lim_{\lambda_n} \frac{\log n}{\lambda_n} = 0,$$

since without some hypothesis on the $\{\lambda_n\}$, the series

$$\sum a_n e^{\lambda_n s}$$

can represent an entire function and simultaneously it can be that it does not converge absolutely at any point of the complex plane. Therefore it is possible that

$$\|f; x_i\| = \infty \quad \text{for} \quad i = 1, 2, \dots$$

On the other hand in the proof of (1.3) you suppose that from

$$d(f_m, f_n) < \varepsilon$$

it follows

$$\|f_m - f_n; x_i\| < \varepsilon$$

and this is not true, it only follows

$$\|f_m - f_n; x_i\| (1 - 2^i \varepsilon) \leq 2^i \varepsilon$$

But the proof can be completed as follows: If $2^i \varepsilon < \theta < 1$ your method gives

$$|a_j^{(m)} - a_j| < \frac{\theta}{1 - \theta} e^{-x_i \lambda_j}$$

and therefore

$$|a_j| < |a_j^{(m)}| + \frac{\theta}{1 - \theta} e^{-x_i \lambda_j}$$

and since

$$\lim \frac{\log |a_j^{(m)}|}{\lambda_j} = -\infty$$

it follows

$$\limsup \frac{\log |a_j|}{\lambda_j} \leq -x_i$$

but if $\varepsilon \rightarrow 0$ in $2^i \varepsilon < \theta < 1$ we can suppose that $i \rightarrow \infty$ and finally it will result

$$\lim \frac{\log |a_j|}{\lambda_j} = -\infty$$



Then from (1) it follows

$$\lim_{m \rightarrow \infty} \|f_m - f; x_i\| = 0 \quad \text{and} \quad \lim_{m \rightarrow \infty} d(f_m, f) = 0$$

Moreover in a step of the proof of (1.4) you implicitly suppose that

$$\alpha_n = e^{\lambda n^S}.$$

I think that the proof of the theorem in No.3 is not complete because I have not been able to understand how α_0 does not change when ~~both~~ m and n change.

There are some other minor oversights without importance.

With kindest regards

I am,

Yours sincerely

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