27.Nov.1951.

Dear Professor Sunyer!

Many thanks for your kind letter of 15.Nov. and for your very interesting reprints I received today. Unfortunately I don't speak your fine and musical language; nevertheless the lecture of your papers caused me not the slightest difficulty. The result I hinted in my letter to dr. Augé is contained in your paper "Propriedades de las funciones enteras representadas por series de Taylor lagunares" as Lema 1.3. As to this I showed in my forthcoming book entitled "Über eine neue Methode der Analysis mit Anwendungen" the following. Let

$$f(z) = \sum_{k=1}^{\infty} a_k z^{\lambda_k}$$

the numbers λ_k being integers with the Fabry-condition

$$\lim_{k\to\infty} \frac{1}{k}\lambda_k = +\infty .$$

We denote as usual

$$\max_{|z|=r} |f(z)| = M(r), \qquad \max_{\substack{|z|=r \\ \alpha \leq \operatorname{arc} z \leq \beta}} |f(z)| = M(r,\alpha,\beta).$$

Then for arbitrary $0<\varepsilon<\frac{1}{2}$ for $r>r_0(f,\varepsilon,\alpha,\beta)$ the inequality

$$M(r)^{1+\frac{\varepsilon}{2}} \leq \frac{48}{\beta-\alpha} M(2r)^{\varepsilon} M(r,\alpha,\beta)$$

holds. In the case of an integral function of finite order this gives not only that $f(z)$ grows "equally fast" in every angle, but that $f(z)$ has also the same order and type with respect to any domain of the form

$$|\vartheta-\vartheta_0| \leq \frac{1}{M(r)^{\varepsilon}}$$

where $z=re^{i\vartheta}$ and ϑ_0 denotes an arbitrary but fixed real number/than with respect to the whole plane. An inequality near akin to yours is proved in Laurent Schwarz's paper " Approximation d'une fonction quelconque... " in "Ann. de la Fac. des Sc. de l'Univ. Toulouse". 1943. p.111-174.

Within short time I shall gladly send to you some of my reprints.

With heartiest greetings to you and also
to Dr. Augé

yours very sincerely

Paul Turán

Us saluda afectuosament

J. Nuyot