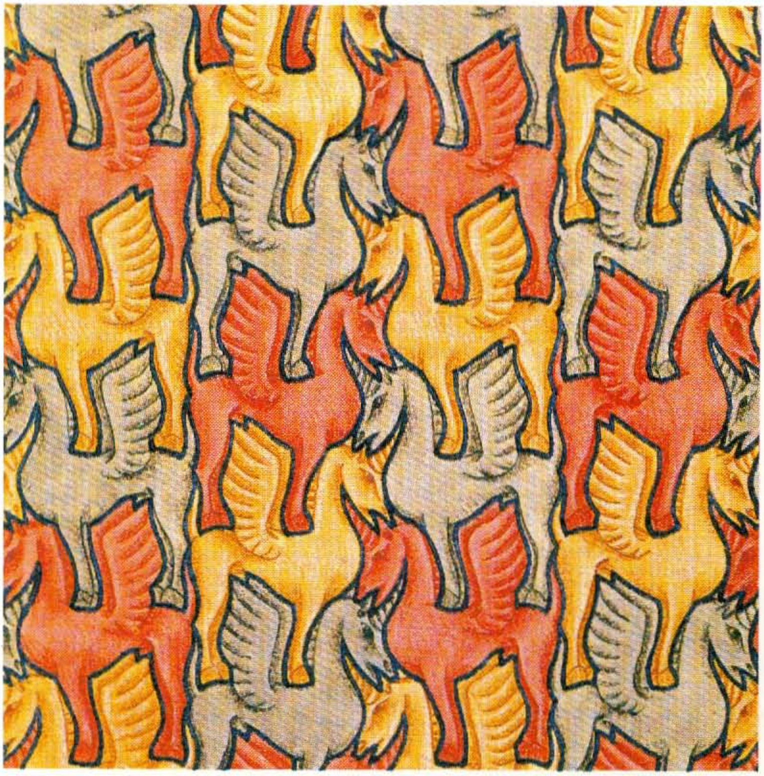
PETER HILTON
DOCTOR HONORIS CAUSA



Universitat Autònoma de Barcelona

UNIVERSITAT AUTÒNOMA DE BARCELONA

DOCTOR
HONORIS CAUSA

PETER HILTON

DISCURS LLEGIT A LA CERIMÒNIA
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PRESENTACIÓ DE PETER HILTON

PER

MANUEL CASTELLET I SOLANAS

Excel.lentíssim i Magnífic Senyor Rector,
Digníssimes Autoritats,
Estimats Col.legues,
Senyores i Senyors:

Quan a l'octubre de 1964 el Dr. Josep Teixidor ens recomanava com a llibre de text per a l'assignatura de Topologia de 5è curs el manual *Homology Theory* de P. Hilton i S. Wylie, poc em podia imaginar aleshores que avui, gairebé vint-i-cinc anys després, em trobaria a la Universitat Autònoma de Barcelona, presentant davant el seu Col.legi de Doctors, precisament Peter Hilton.

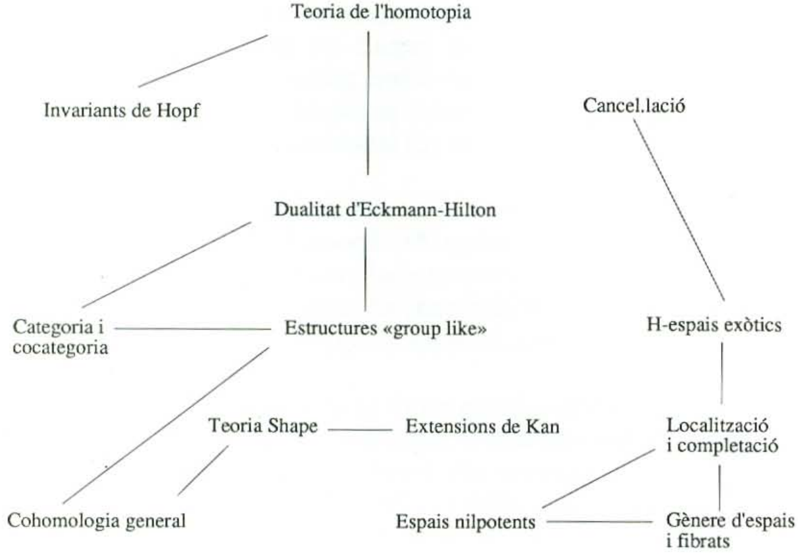
No m'ho podia imaginar per diverses raons, alguna de tan trivial com que l'Autònoma no existia ni en el món de les idees; però fonamentalment perquè en aquells moments jo no sabia què era la topologia algebraica ni sabia qui era Peter Hilton. La meva coneixença amb Hilton ha seguit un procés d'aproximacions succesives, començant per un llibre, passant per articles de recerca i concloent en un coneixement personal i en una amistat que m'honora que ens ha portat la col.laboració científica.

Però si per a mi Hilton era un desconegut l'any 1964, no ho era, evidentment, per als topòlegs de l'època. Des de les darreries de la segona guerra mundial, Hilton treballava en topologia algebraica sota la direcció d'Henry Whitehead; ell mateix confessa que fou una experiència meravellosa, excitant i exigent al mateix temps.

L'any 1964 Hilton havia escrit ja quatre llibres, entre els quals figuraven *An Introduction to Homotopy Theory* -quantes vegades no l'hem arribat a remenar els topòlegs!-, i l'esmentat *Homology Theory*. Entre els anys 1965 i 1970 veuen la llum tres llibres més, el més destacat dels quals és *Homotopy Theory and Duality*, i des de 1971 encara 8 llibres més, si no me n'oblido cap; per cert, un d'ells, *A course on Modern Algebra,* escrit conjuntament amb Y.-G. Wu, acaba d'ésser reeditat per John Wiley & Sons dins la sèrie

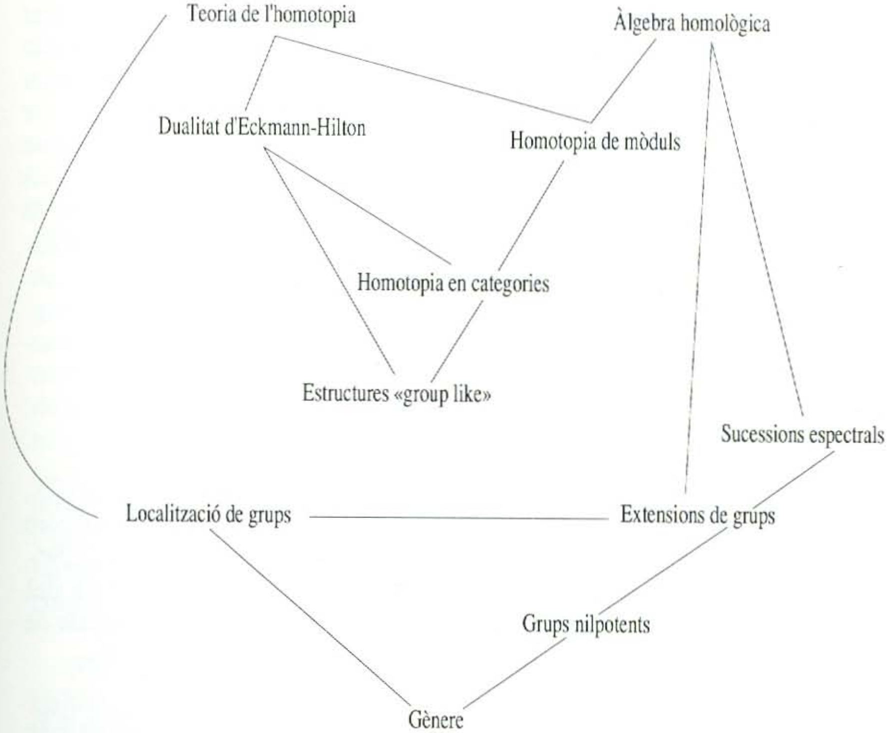
Biblioteca de Clàssics. Per a un topòleg d'aquesta darrera època, però, són eines essencials *General Cohomology Theory and K-Theory*, *A course in Homological Algebra* i *Localization of nilpotent groups and spaces.*

Els llibres publicats per Peter Hilton són fruit de la seva intensa activitat investigadora. Més de tres-cents articles de recerca que arrenquen de la topologia i passen per l'àlgebra i la teoria de categories. Permeteu-me que reprodueixi l'esquema que va exposar Guido Mislin a la Memorial University of Newfoundland l'any 1983, en ocasió del 60è aniversari de Hilton, i que il.lustra les diverses àrees de la topologia en les quals Hilton ha desenvolupat el seu treball.



Esquema 1

Aquest esquema cal complementar-lo amb un altre de dedicat a la seva contribució a l'àlgebra, una àlgebra certament amb motivacions i aplicacions a la topologia, però amb contingut propi.



Esquema 2

En tots els camps, Peter Hilton ha deixat resultats fonamentals, però jo ara, aquí, vull comentar-ne només tres; tres que corresponen a tres èpoques diferents: abans de 1964, al voltant d'aquest any i després de 1970.

Els grups d'homotopia d'una unió d'esferes

L'article «On the homotopy groups of the union of spheres» publicat a les Actes del Congrés Internacional d'Amsterdam de 1954, és l'exemple perfecte de claredat d'exposició i de capacitat d'utilitzar amb eficiència la interrelació de l'àlgebra i la geometria. En aquest treball, escrit sota la influència de Jean Pierre Serre, hom utilitza per primera vegada les àlgebres de Lie a la teoria de l'homotopia. Si X és una unió puntual d'esferes de diverses dimensions, Hilton calcula els grups d'homotopia d'X, π_* (X), i els expressa en termes de grups d'homotopia d'esferes, tot utilitzant la relació entre Ω_* (X) i l'homologia de l'espai de llaços d'X, H_* (ΩX), el producte de Whitehead a π_* (X) i el producte de Pontryagin a H_* (ΩX).

Aquest teorema, que estableix que si

$$X=S^{n_1} \vee \dots \vee S^{n_r},\ \pi_n (X)= \bigoplus_{w(\pi)} \pi_n (S^{n_w}),$$

on w (π) $\in \pi_*$ (X) recorre tots els productes de Whitehead bàsics, fou generalitzat el 1958 per John Milnor, en el que ara es coneix com a Fórmula de Hilton-Milnor: «Si X és la unió puntual dels CW-complexos X_i,... X_k, l'espai de llaços de la suspensió d'X és del mateix tipus d'homotopia que un producte infinit feble dels espais de llaços de la suspensió de certs espais obtinguts a partir dels X_j».

El teorema de Hilton-Milnor ha esdevingut, en paraules d'E. Dror, la pedra de toc de la teoria de l'homotopia moderna. Molts dels avenços més recents l'utilitzen com a punt de partida.

La dualitat d'Eckmann-Hilton

La col.laboració de Beno Eckmann, de l'ETH de Zuric, amb Peter Hilton, col.laboració que des del punt de vista estrictament científic va durar més de 15 anys, va donar lloc no només a una profunda amistat, sinó també a algunes de les pàgines més boniques i interessants de les matemàtiques d'aquesta segona meitat de segle: el que es coneix com a «Dualitat d'Eckmann-Hilton». La interrelació entre topologia i àlgebra, amb el suport de la teoria de categories, els permeté passar de l'homotopia d'espais a l'homotopia de mòduls i, tot dualitzant els resultats algebraics obtinguts, passar altre cop a la topologia.

Eckmann i Hilton van elaborar una dualitat interna a la categoria homotòpica puntejada. Partien del conjunt d'homotopia [X, Y], que pot ésser considerat com un functor en X, o dualment com un functor en Y; introduïen de forma autodual els grups d'homotopia $\pi_n (X; Y) \cong [\Sigma^n X, Y] \cong [X, \Omega^n Y]$, generalitzant simultàniament els grups de cohomologia i els d'homotopia. En relativitzar respecte a una de les variables, obtenien successions exactes llargues duals, i el mot «cofibració» apareixia dual del de «fibració». Davant d'aquesta dualitat, un parell d'espais pot ésser considerat com una aplicació i una tríada com un diagrama $X \rightarrow Y \rightarrow Z$. Aquest punt de vista porta a successions de triples duals a homotopia i cohomologia, que són utilitzades pels autors per definir les descomposicions homotòpiques i homològiques d'una aplicació. En treballs posteriors, Eckmann i Hilton estudiaren la dualitat en categories arbitràries i analitzaren els conceptes de grups i cogrups.

Jo tinc un especial record de la teoria de la dualitat d'Eckmann-Hilton, perquè quan vaig arribar a Zuric el 1971 per treballar amb Eckmann, acabaven d'explotar tots aquests resultats. No vaig conèixer Hilton personalment fins a 1973, però ja m'era familiar sentir parlar del «Pope» i el «Copope» (parodiant grup i cogrup, fibració i cofibració, homotopia i cohomotopia, categoria i cocategoria,...), com els anomenaven els estudiants de doctorat. No he sabut mai, però, qui era l'un i qui l'altre.

Vint anys després de la publicació de la sèrie d'articles i llibres que configuraven la dualitat d'Eckmann-Hilton, el seu contingut ha esdevingut una part usual dels coneixements dels matemàtics. És probablement un dels millors comentaris que es pot fer d'un treball científic.

La localització de grups i espais

L'estudi dels grups topològics des d'un punt de vista homotòpic fou iniciat per Heinz Hopf el 1941. Quan, més de vint-i-cinc anys després, Hilton i Joseph Roitberg troben el primer exemple d'espai finit de Hopf diferent dels clàssics (grups de Lie, esfera de dimensió 7, espai projectiu real de dimensió 7 i llurs productes), l'estudi dels H-espais (espais de Hopf) esdevé un camp extraordinàriament actiu de recerca.

La construcció d'aquest nou H-espai forma part d'una sèrie de treballs que Hilton va publicar amb Roitberg, i més endavant amb Guido Mislin, sobre qüestions relacionades amb el fenomen de la no-cancel.lació en la teoria de l'homotopia. Hi ha dos tipus de cancel.lació: es pot considerar la unió de dos espais amb un punt comú (és com la suma) i preguntar-se sobre la possibilitat de la no-cancel.lació respecte a aquesta suma. Ells van demostrar com construir sistemàticament exemples en els quals es poguessin agafar dos espais diferents, sumar el mateix espai a cadascun i obtenir espais homotòpicament equivalents. Però també van construir exemples reemplaçant la unió pel producte; aquest fou un problema molt més difícil que va tenir fortes implicacions no només en la teoria de H-espais, com ja he esmentat, sinó també en la teoria de la localització i en l'estudi del gènere d'espais i grups.

Les tècniques de localització, que Hilton va contribuir a posar a punt amb una llarga sèrie d'articles, però molt especialment amb el llibre *Localization of nilpotent groups and spaces* (conjuntament amb G. Mislin i J. Roitberg), han estat una eina bàsica de treball per als topòlegs algebraics a partir de 1975. L'estudi dels espais nilpotents, categoria amb la qual la teoria d'homotopia i la localització es comporten especialment bé; dels objectes «hopfians» en la categoria homotòpica, tot generalitzant resultats de la teoria de grups; l'estudi de les pseudo-identitats, en el qual vaig tenir el plaer de col.laborar, amb aplicacions en endomor-fismes d'espais nilpotents i en fibracions homològicament nilpo-tents, etc., han estat algunes de les línies de recerca de Peter Hilton en els darrers anys. La nostra col.laboració, la dels topòlegs de Bar-celona amb Hilton, ha estat fructífera i, sens dubte, molt beneficiosa per a nosaltres. No és cap casualitat que el darrer treball de Peter Hilton, «On nilpotent groups which are finitely generated at every prime», sigui un article conjunt amb el nostre company Carles Casacuberta.

En topologia i àlgebra o en àlgebra i topologia, però fins i tot en algun dels treballs més abstractes, de teoria de categories, hi ha una referència a temes de l'escola primària. I és que és impossible d'apreciar el matemàtic Peter Hilton, com deia Urs Stammbach, sense tenir en compte totes les seves contribucions a l'educació en matemàtiques, sigui a l'escola primària, a la secundària o a la superior.

És força excepcional que un matemàtic de la seva condició i el seu prestigi es dediqui tan intensivament a aquests temes, i, per tant, no és sorprenent que els seus encertats consells siguin sol.licitats per universitats i institucions d'arreu. Hilton ha participat en projectes educatius a Amèrica del Sud, Àfrica, Xina, Singapur, etc., a més dels diversos programes dels Estats Units i el Canadà.

I és que Hilton és capaç de trobar interessants qüestions matemàtiques obertes a tots els nivells de l'ensenyament. Recordo la fruició amb què m'explicava el 1983, en una curta visita a la nostra Universitat, com es pot construir, doblegant un paper, qualsevol polígon regular de forma molt simple. I això era fruit d'una bonica combinació de geometria i de teoria de nombres.

Les seves publicacions en el camp de l'educació, de les quals només vull citar aquí *Fear no more* i *Build your own polyhedra* (amb Jean Pedersen) i les seves conferències posen en evidència la convicció que ensenyar matemàtiques és només una part de fer matemátiques. Aquesta convicció, que el porta a fer accessible allò que realment és difícil, fa exclamar, més d'una vegada, en sentir-lo exposar: «Això ho hauria pogut fer jo!». Però no, el que passa és que Hilton, com Eckmann o com Hopf, tenen la capacitat de fer fàcil allò que és més difícil.

Però Peter Hilton no és només un subtil investigador en matemàtiques; és un gran mestre i un gran mentor i la col.laboració professional ha esdevingut sovint una duradora amistat. Em sento honorat i feliç que més d'una persona del Departament de Matemàtiques d'aquesta Universitat puguem comptar-nos entre aquesta gent afortunada. Gràcies, Peter.

THE JOY OF MATHEMATICS

PER

PETER HILTON

First, let me express my deep appreciation of the honour you have done me in bestowing on me the degree of Doctor Honoris Causa; I will endeavour, during the rest of my academic career, to remain worthy of your trust.

Now let me move to the substance of my address today. I would like to open by recalling an occasion some years ago when I was invited to speak to a group of elementary and secondary school teachers of mathematics in Salt Lake City, Utah. I chose as my theme «Reforming the Curriculum and Pedagogy of Mathematics Education», and, in the course of my remarks on effective pedagogical strategies, I expressed the view that mathematics education could not be successful unless the students were intrigued by the subtleties of the mathematics and regarded doing mathematics as fun. In the course of the discussion which followed my remarks - indeed, as the first contribution to that discussion- a secondary school teacher took issue with me, saying that mathematics was surely a serious subject and should not be regarded by the students, nor presented by the teacher, as fun.

I maintained then -very tactfully, of course,- and I maintain now that that teacher was labouring under the widespread but mistaken belief that there is a fundamental conflict between the delight and the seriousness of mathematics. This is a false dichotomy, comparable in many respects with that other notorious false dichotomy between pure and applied mathematics.

Certainly, mathematics is important, indeed, increasingly important. Certain areas of human enquiry -for example, sociology, psychology, biochemistry- hitherto immune to all but the crudest mathematical influence are now employing sophisticated mathematical techniques and ideas. Certain parts of mathematics, hitherto ineffably pure -for example, classical invariant theory, finite field theory, homology theory- are now being applied to important contemporary real-world problems. The ubiquitous computer is calling into prominence -even sometimes calling into existence- exciting aspects of present-day mathematics. Yes, mathematics is enormously important to modern society; yet I am compelled in all honesty to admit that its importance is not the primary reason why we mathematicians so enthusiastically and so assiduously practise our art. I remember my teacher and friend, Henry Whitehead, the eminent British topologist, once saying in this connection, «Nothing would give me greater pleasure than to wake up one morning to be told that one of my theorems had rendered war utterly obsolete. Yet I would still have to admit that that outcome had nothing to do with my reasons for trying to prove the theorem».

Our reason for doing mathematics is that it fascinates us. It stimulates both our intellectual curiosity and our aesthetic sensibilities. It poses deep, significant questions, whose answers, if we are fortunate enough to obtain any, provide an immediate spiritual reward which quickly gives way to a new wave of curiosity, a new set of questions.

Now there is the possibility of an argument at a deep philosophical level that mathematics fascinates us because of its importance. However, if we are talking of importance, as we have been, in a social, extrinsic sense, then I do not believe this to be true. For the philosophical connection -if it exists, and this is a matter for thoughtful, sustained study, not glib speculation- would surely be between the intellectual appeal of mathematics and its intrinsic importance as a primary area of human endeavour.

Of course I believe profoundly that mathematics is important in this sense. If I may again quote my mentor Henry Whitehead, I recall his trying to persuade a young colleague who was contemplating a career in government service to remain, instead, a university mathematician (he did not succeed in convincing the young man, though his case seemed to me extremely strong!). Whitehead argued that there were, in life, a few things which were worth doing even if one was not absolutely «in the top flight» because of their intrinsic worth, and I recall his citing music, mathematics and the making of good shoes. «Rather», said

Whitehead[1], «be second-rate in a first-rate occupation than first-rate in a second-rate occupation».

However, as I have already indicated, the importance which today's statesmen, parents and educators attach to mathematics is not of this intrinsic kind, but refers to its role in ensuring the prominence and well-being of the society to which they belong. It does therefore relate indirectly to the relevance of mathematics to science and engineering; but this is as near as most people come to understanding the role of mathematics in a civilized society in all its aspects.

The importance of mathematics is, I repeat, generally understood in the social sense of the term; in this sense, it explains why we are paid to do mathematics but not why we do it -and certainly not why we, or, at least, some of us, some of the time, do it well. To understand this we have to penetrate deeply into the nature of mathematics itself.

Mathematics is systematized thought, supported by a beautifully adapted language and notation. It is characterized by the discovery and creation of patterns and by establishing subtle connections between its apparently very dissimilar parts. It is not a set of distinct subdisciplines, but a unity, drawing on a diverse but interrelated repertoire of concepts and techniques. It is not a set of facts; and mathematical understanding is not to be tested by tests of knowledge and memory.

«I attempt to climb Everest», said Mallory, «because it is there». Just so do mathematicians attempt to do mathematics. Mathematics is there because our experience of the outside world leads us to formulate questions which can only be properly asked and answered in the framework of mathematics. «Nature speaks to us in the language of mathematics», said the physicist Richard Feynman. But

[1] One might perhaps add that Whitehead, being first-rate in a first-rate occupation, was faced with no such agonizing dilemma.

mathematics is also there because it is the natural mode of mathematical progress that questions are suggested by recent advances and forms the stimulus to new advances. Thus, as I have often claimed, pure and applied mathematics are alike in their dynamics, as they are alike, too, in their proper practice.

We mathematicians should be envied for the joy which mathematics brings us. This joy comes to us through our awareness of the great power of a fertile mathematical idea, and through our sense that such an idea has been expressed in a form capable of perfection. Often, in reading a passage from a Shakespeare play, we have a sense that the underlying idea could no have been better expressed -«My thoughts fly up, my words remain below. Words without thoughts never to heaven go»- and, less often but just as certainly, we have a sense that a mathematical concept has reached perfection and a mathematical argument has achieved a transcendental beauty and elegance.

Let me attempt to give you 3 examples of this quality of elegance in mathematics. The first is the famous theorem of Euclid that there are infinitely many prime numbers. To prove this, we suppose instead that we can enumerate the prime numbers as $p_1, p_2 ,..., p_n$. Set $N=p_1 p_2 ..., p_n+1$. Then N is bigger than any of $p_1, p_2 ,..., p_n$ and hence not prime. It must therefore be divisible by some prime number. However, it leaves the remainder 1 when divided by any prime p_i , so we have arrived at a contradiction, and the theorem of Euclid is established.

Second, let us consider the assertion that there is no rational number whose square is 2. We again assume the opposite to obtain a contradiction. Suppose that a/b is a rational number whose square is 2. We can assume that a/b is a reduced fraction, that is, that the greatest common divisor of a and b is 1. Now $a^2 /b^2 =2$ or $a^2=2b^2$; thus a^2 is even. Since the square of an odd number is odd, a is even, say a=2c. Then $2b^2 =4c^2$, so $b^2 =2c^2$. As before, we conclude that b is even. But, with a,b both even, a/b cannot be a reduced fraction, and our contradiction is achieved.

For our third example we consider the famous story about the young Gauss. It seems that the teacher had set the class the awesomely dreary and tedious task of adding up all the numbers from 1 to 100. Gauss argued that the sum could be so represented:

$$\begin{array}{r} 1 + 2 + 3 + \ldots + 48 + 49 + 50 \\ + 100 + 99 + 98 \ldots + 53 + 52 + 51 \end{array}$$

Then there are 50 vertical sums and each yields 101. Thus the required sum is 50 x 101 = 5050.

In all these examples the argument reveals an understanding which does much more than compel belief in the conclusion; it shows us why the statement under consideration is true. There is also a beautiful economy about the argument -one cannot imagine it being significantly shortened. It is austerely spare, with no superfluous burden to carry. Certainly the last two examples carry the conviction that much more is true than has been asserted -that the method of argument could readily be adapted to prove other, related assertions. Indeed, Gauss' argument, slightly modified, is now the standard procedure for summing any arithmetic progression.

As we have already suggested, the arguments above all give us insight into the real structure of our mathematical system; and I maintain that our strongest motivation in doing mathematics is our desire to achieve such insights. It is often a strenuous, demanding stuggle to do so; often we seem to have failed and then, in a blinding flash of inspiration following many hours, perhaps many days of apparently fruitless effort, the essence of the problem is laid bare and all becomes wonderfully, magically clear.

Here I wish to make a point about such insights in mathematics which, though purely empirical and experimental in nature, seems to me of fundamental importance, and to constitute a primary reason for choosing to make the study of mathematics a major activity in one's life. I maintain that the thrill one derives from these rare flashes of insight does not depend on one having oneself originated the idea in question -to rediscover is an experience comparable whith the thrill of original discovery. One's experience of discovery in other sciences, admittedly vicarious and second-hand, suggests that mathematicians are very special in this regard. James Watson's candid account, in his book «The Double Helix», of his unravelling, with Frances Crick, of the nature of DNA, suggests that the strongest motivation, among the competing scientists, to solve this riddle was precisely the urge to «get there first»; in fact, Watson tells us how stimulating to Crick and himself was the awareness that Linus Pauling was working hard on the same problem. I also recall a conversation with the eminent biologist and Nobel prize laureate Peter Medawar who said that, in his experience, the thrill of original discovey in science was not communicable and that mere understanding of someone else's work was not to be compared with the excitement of making significant progress in one's own research. It is my experience that, in mathematics, the thrill of understanding, of real understanding, is comparable with the thrill of original discovery; and it is further my experience that mathematicians take genuine pleasure in the triumphs of others and, in a sense, recreate the act of discovery in mastering the intricacies of another's work. Indeed, one may say that real understanding in mathematics consists in making the ideas and techniques one's own -not, of course, in the crude sense of appropiating those ideas, but in the deeper sense of being able to use them to iluminate and advance one's own thinking.

Allow me then to give you an example of a really beautiful mathematical idea, due to the French mathematician Désiré André whose work was published about 100 years ago. There was current at that time the so-called ballot problem, which may be described as follows. We suppose two candidates, X and Y, in an election; and we suppose further that X wins the election, obtaining a votes against the b votes received by Y. We wish to know the probability that, throughout the counting of the votes, X is always ahead of Y. In fact, this problem had already been solved by Bernard when André published his paper -which he entitled «Solution directe d'un problème resolu par M. Bernard»- but what I want to emphasize is the elegance of André's solution and the insight it reveals and conveys.

André translated the problem into one about the integral lattice in the coordinate plane. If P and Q are two points with integral coordinates, then a path from P to Q is a sequence of points P_o, P_1, ...,P_x such that P_o =P_1, P_x =Q and P_{i+1} is obtained from P_i by stepping one unit north or east. The number of paths from (c,d) to (a,b) is easily seen to be the binomial coefficient[2].

$$\binom{a+b-c-d}{a-c}$$

Then the ballot problem may be translated into that of counting the paths form (0,0) to (a,b) which stay below the line y=x except at their initial point. Thus we consider the following more general problem: we assume that (c,d) and (a,b) are below the line y=x, that is, that c>d, a>b, and we wish to count the paths from (c,d) to (a,b) which stay below the line y=x. Let us call such a path «good», and a path from (c,d) to (a,b) which meets the line «bad». Let us call the endpoints P and Q.

We wish, as I say, to count the good paths; André's first nice touch is to count, instead, the bad paths. Now a bad path must meet the line y=x for the first time at some point R. If we reflect the portion PR of our bad path in the line y=x we get a path $\overline{P}R$ where $\overline{P}$ is the point (d,c). Combining $\overline{P}R$ with the portion RQ of our original bad path yields a path $\overline{P}Q$ from (d,c) to (a,b). It is then easy to see that this reflection manoeuvre sets up a one-to-one correspondence between bad paths from (c,d) to (a,b) and paths from (d,c) to (a,b) - it is only necessary to note that any path from (d,c) to (a,b) must cross the line y=x. Thus the number of bad paths from (c,d) to (a,b) is the binomial coefficent

$$\binom{a+b-c-d}{a-d}$$

[2] Assuming, of course, that there are paths from (c,d) to (a,b), that is, that $c \leq a$ and $d \leq b$.

so that the number of good paths from (c,d) to (a,b) is

$$\binom{a+b-c-d}{a-c} - \binom{a+b-c-d}{a-d}$$

Returning to our ballot problem, we now need only count the good paths from (1,0) to (a,b), which, by our formula above, reduces to

$$\frac{(a+b-1)!}{a!\,b!}\,(a-b);$$

so that the required probability is obtained by dividing this by,

$$\binom{a+b}{a},$$

namely, it is

$$\frac{a-b}{a+b}$$

Before we continue, let us draw attention to an important element present in the solution of the ballot problem which contributes greatly to the fun, and the joy, of mathematics, namely, the element of surprise. The formula for the probability, $(a - b)/(a + b)$, is remarkably simple, considering the subtlety of the argument; moreover it shows that the probability that X is always ahead of Y during the counting of the votes depends only on the ratio of the votes they received and not on their actual number. To me, at any rate, this fact is not intuitively obvious. I believe this element of surprise is also present in Gauss' ingenious method for adding the numbers from 1 to 100 -suddenly, a horrendous exercise is reduced to the easy multiplication 50 x 101. It is among the many virtues of the proper teaching of geometry that it should provide this element of sudden revelation, so that one has the sensation of unexpected insight. Unfortunately, geometry today does not usually play its proper role in the curriculum; it is either badly neglected or treated merely as a vehicle for conveying the idea of logical proof- -with the assertion to be proved being either monumentally dull or obvious, or both! But this is another story- albeit a vitally important one- and I must return to my theme.

We said that the reasoning following the reflection manoeuvre was easy - true, but the insight consisted in, first, translating the problem into a combinatorial problem about paths on the integral lattice, and then solving the problem by introducing the reflection method. It is a pity that we are not able to spare the time, in our teaching, to explain first the long, difficult method by which the problem was originally solved, in order the clearer to exhibit the wonderful advantages of the superior method which has replaced it! I am sure that students would then have no difficulty in appreciating the contrast.

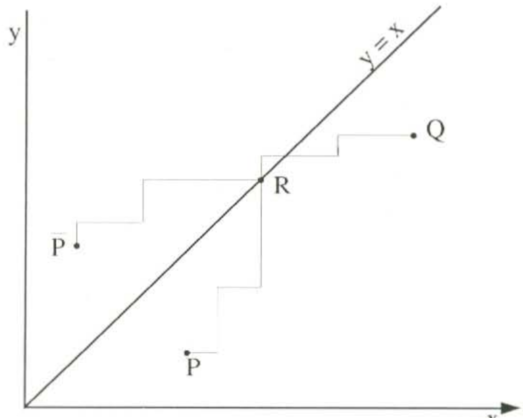


Figure 1. The André Reflection Method[3].

[3] For a fuller account of André's works and further developments, see a forthcoming article by Peter Hilton and Jean Pedersen on Catalan numbers in the Mathematical Intelligencer.

It is my hope that the four examples I have given of superb mathematical reasoning will have given my audience pleasure and, indeed, aesthetic satisfaction; it is my expectation that any bright student, with the necessary mathematical background, will readily appreciate, at least at an intuitive level, their beauty and quality. Indeed, they might well be used to discriminate between those students who would benefit from continuing their mathematical eduation and those who would be better advised to turn to more lucrative but otherwise less rewarding pursuits. The Greeks understood both the importance and the beauty of mathematics; but few of today's administrators, bureaucrats and politicians understand this complementarity in the nature of mathematics. By all means, let us rejoice ifour prominent citizens and leading industrialists encourage us to do our mathematics; but let us accept from them neither the choice of problems to be worked on -for the formulation would be almost certainly imprecise and the problem almost certainly unsolvable- nor the justification for doing mathematics. For, thinking only of materialistic outcomes, they would know nothing of the joy of mathematics.

CURRICULUM VITAE

DE

PETER HILTON

El professor Peter Hilton és actualment «Distinguished Professor» de Matemàtiques a l'State University of New York a Binghamton.

Va obtenir els títols de M.A. i D. Phil. a la Universitat d'Oxford, Anglaterra i el Ph.D. a la Universitat de Cambridge, Anglaterra. De 1952 a 1955 fou «Lecturer» a la Universitat de Cambridge, de 1956 a 1958 «Senior Lecturer» a la Universitat de Manchester, Anglaterra; de 1958 a 1962 «Mason Professor» de Matemàtiques Pures a Birmingham, Anglaterra.

Peter Hilton fou Professor de Matemàtiques a la Cornell University de 1962 a 1971. De 1971 a 1973 va ostentar sengles nomenaments de «Fellow» al Battelle Seattle Research Center i de Professor de Matemàtiques a la Universitat de Washington. L'1 de setembre de 1972 va ocupar la càtedra Louis D. Beaumont al Case Institute of Technology mantenint alhora la seva situació al Battelle Seattle Research Center. Des de 1983 és «Distinguished Professor» de Matemàtiques a la State University of New York a Binghamton.

P. Hilton fou Professor Visitant a l'Eidgenössische Technische Hochschule de Zuric els cursos 1966-67, 1981-82 i 1988-89 i al Courant Institute of Mathematical Sciences, New York University, el 1967-68.

P. Hilton és membre de l'American Mathematical Society; de la Mathematical Association of America, de la qual fou primer vice-president de 1978 a 1980; de la London Mathematical Society; de la Cambridge Philosophical Society; de la Canadian Mathematical Society; de la Royal Statistical Society, i membre honorari de la Societé Mathématique de Belgique i de Phi Beta Kappa.Els principals camps de recerca de P. Hilton són: la topologia algebraica, l'àlgebra homològica, l'àlgebra categòrica i l'ensenyament de les matemàtiques. Ha publicat 16 llibres i més de 350 articles en aquestes àrees, alguns d'ells conjuntament amb col.legues seus.

Peter Hilton és destacat al llibre «Mathematical People», (Birkhäuser, Boston, 1985) com un dels més prominents matemàtics de la segona meitat del segle XX.

Càrrecs actuals

Membre, American Mathematical Society Committee on Human Rights of Mathematics.
Membre, Editorial Board, Expositiones Mathematicae.
Membre, Editorial Board, Handbook of applicable Mathematics.
Membre, Phi Beta Kappa Speakers Panel.
Consultant, John Wiley and Sons, Inc.
Consultant, Open Court Publishing Company.
Consultant, Childern's Television Workshop.
Editor, Publicacions Matemàtiques, UAB.

Càrrecs recents

President, Mathematical Association of America Committee on Award for Distinguished Service.
President, Mathematical Association of America Committee on Award of Chauvenet Prize.
Membre, Mathematical Association of America Panel on Remediation.
Membre, Mathematical Association of America Panel on Public Representation.
Membre, Advisor Committee on Mathematics and Science, Council for Basic Education.
Secretari, International Commission on Mathematical Instruction.
Editor, NICO (Bruxelles).
Consultant, National Institute of Education, Department of Health, Education and Welfare.
President, United States National Research Council Committee on Applied Mathematics Training.
Membre, United States Commission on Mathematical Instruction, National Research Council.
President, Mathematical Association of America Committee on National Awards.
Editor principal, Ergbnisse der Mathematik Series, publicat per Springer Verlag.
Editor, Educational Studies in Mathematics.

President, National Advisory Board, Comprehensive School Mathematics Project.
Membre, Committee on the Undergraduate Program on mathematics, Mathematical Association of America.
President, National Research Council Committee on Graduate and Postdoctoral Training Council.
President, United States Commission on Mathematical Instruction, National Research Council.
Membre, Teacher Training Panel, Committee on the Undergraduate
Program in Mathematics, Mathematical Association of America.
President, Cambridge Conference on School Mathematics.
Membre, National Advisory Committee, Boston University Mathematics Project.
Membre, Committee on Films, Mathematical Association of America.
Membre, Subcommittee on Translations, Mathematical Association of America.
Membre, Committee on Postdoctoral Fellowships, American Mathematical Society.
President, New York State Department of Education Panel on Ph.D. Program in Mathematics (Setembre, 1976).
Editor, Journal of Pure and Applied Algebra.
President, Committee to Select Phi Beta Kappa Book Award in Science.

Distincions honorífiques

Silver Medal, Universitat d'Helsinki, 1975.
Doctor of Humanities (hon. causa), N. Michigan University, 1977.
Membre correspondent, Acadèmia de Ciències del Brasil, 1979.
Doctor of Science (hon. causa), Memorial University of Newfoundland, 1983.
Centenary Medal, John Carroll University, 1986.
Convocation Citation, Ohio State University, 1987.

L'agost de 1983 tingué lloc un congrés internacional de topologia algebraica, sota els auspicis de la Canadian Mathematical Society, per commemorar el 60è aniversari de P. Hilton. El professor Hilton fou obsequiat amb un volum d'articles dedicats a ell (London Mathematical Society Lecture Notes Volume 86, 1983).

L'American Mathematical Society ha publicat les actes del congrés amb el títol «Conference on Algebraic Topology in Honor of Peter Hilton» (Contemporary Mathematics 37, AMS, 1985).

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