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Edited By:
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U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Marine Fisheries Service
Southwest Fisheries Science Center

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Edited by Aaron Mamula[1] and John Walden[2]

[1] NOAA National Marine Fisheries Service
SWFSC Fisheries Ecology Division
110 Shaffer Road
Santa Cruz, CA 95060

[2] NOAA National Marine Fisheries Service
Northeast Fisheries Science Center
Woods Hole, MA

NOAA-TM-NMFS-SWFSC-503

U.S. DEPARTMENT OF COMMERCE
Rebecca M. Blank, Acting Secretary
National Oceanic and Atmospheric Administration
Jane Lubchenco, Undersecretary for Oceans and Atmosphere
National Marine Fisheries Service
Samuel D. Rauch, Acting Assistant Administrator for Fisheries

Productivity, Price Recovery, Capacity Constraints and their Financial Consequences*

E. Grifell-Tatjé
Universitat Autònoma de Barcelona

C. A. K. Lovell
University of Queensland

Abstract

Mining and fishing are both extractive industries, although one resource is renewable and the other is not. Miners and fishers pursue financial objectives, although their objectives may differ. In both industries financial performance is influenced by productivity and price recovery. Finally, in both industries capacity constraints influence financial performance, perhaps but not necessarily through their impact on productivity, and both industries encounter external as well as internal capacity constraints.

The objective of this study is to develop an analytical framework that links all four phenomena. We use return on assets to measure financial performance, and the basic analytical framework is the duPont triangle. We measure productivity in two ways, with a theoretical technology-based index and with an empirical price-based index. We measure price change with an empirical quantity-based index. We measure internal capacity utilisation by relating a pair of output quantity vectors, actual output and full capacity output, and we develop physical and economic measures of internal capacity utilisation. External capacity constraints restrict the ability to reach full capacity output. The analytical framework has productivity change, price change and change in capacity utilisation influencing change in return on assets, the latter in two ways, directly and indirectly through its impact on productivity change.

JEL classification: D24

Keywords: duPont triangle, capacity utilization, productivity, price recovery

Productivity, Price Recovery, Capacity Constraints and their Financial Consequences

1. Introduction

Mining and fishing are both extractive industries, although one resource is renewable and the other is not. Miners and fishers pursue financial objectives, although their objectives may differ. In both industries productivity and prices influence financial performance. In both industries capacity constraints influence financial performance, perhaps but not necessarily through their impact on productivity, and both industries encounter external as well as internal capacity constraints.

We offer two relevant illustrations. First, global mining giant Rio Tinto has generated impressive, and volatile, financial results throughout the recent mining boom. Figure 1 shows five-year moving averages of return on assets and its two components, profit margin and asset turnover, from 2007 through 2011.[1] One would like to learn something about the sources of the observed volatility in return on assets that digs deeper than just variation in the profit margin and asset turnover. Variation in productivity, prices and capacity constraints are likely sources.

Second, ABARES (2012) publishes a fisheries surveys report. The report provides detailed boat-level financial information, averaged over boats, within each of two fisheries, and similarly detailed economic information for the fisheries themselves. The boat-level financial information includes alternative measures of profit and return on assets. The fishery economic information includes profit and net economic returns, which adjusts profit in several ways, including the incorporation of the costs of managing the fishery. One would like to know something about the sources of variation in profit and return on assets across boats within a fishery, and the sources of variation in net economic returns through time and across fisheries. Again, variation in productivity, prices and capacity constraints are likely sources.

The objective of this study is to develop an analytical framework that links all four phenomena, financial performance, productivity, prices and capacity constraints. We use return on assets ROA to measure financial performance, and the basic analytical framework is the duPont triangle depicted in Figure 1. We measure productivity change Y/X in two ways, with a theoretical technology-based index and with an empirical price-based index. We measure price recovery change P/W with an empirical quantity-based index. We measure capacity utilization CU by relating a pair of output quantity vectors, actual output and full capacity output, and we develop physical and economic measures of CU. The analytical framework has Y/X, P/W and CU influencing ROA, the latter in two ways, directly and indirectly through its impact on productivity.

The study unfolds as follows. In Section 2 we introduce the duPont triangle as a framework for financial performance evaluation. In Section 3 we attempt to incorporate Y/X and CU into the duPont triangle. We succeed with CU and fail with Y/X because, while CU is an absolute indicator, Y/X is a relative indicator that compares one situation with another. Accordingly, in Section 4 we compare ROA in two time periods by converting the analytical framework to an inter-temporal one, and we seek to exploit the duPont triangle format to attribute ROA change from one period to the next to CU change and productivity change. Once again we succeed with CU change and fail with productivity change. In Sections 2-4 we ignore price change as an ROA driver; we would have failed, for the same reason we fail with productivity change. In Section 5 we develop a pair of analytical frameworks within which CU change, productivity change and price recovery change drive ROA change. In Sections 2-5 CU is an *internal* measure associated with short run fixity of some inputs used by the firm. In Section 6 we introduce *external* capacity constraints resulting from regulation and other sources outside the firm, and we show how these external capacity constraints can render some or all internal capacity constraints redundant. Section 7 concludes.

2. The duPont Triangle

ROA is a widely used measure of financial performance. Bliss (1923), in discussing ROA, claims that "[f]rom the operating point of view as distinguished from the stockholders' point of view, the real measure of the financial return earned by a business is the percentage of operating profits earned on the total capital used in the conduct of such operations…regardless from what sources such capital may have been secured." Two duPont executives, Kline & Hessler (1952), concur, writing that "It is our considered opinion, which has been critically re-examined many times over three decades, that a manufacturing enterprise with large capital committed to the manufacture and sale of goods can best measure and judge the effectiveness of effort in terms of 'return on investment'." Amey (1969) calls ROA "the key index of business 'success'," even though he acknowledges that maximizing ROA and maximizing profit in absolute terms do not generally coincide. Amey continues, "…maximization of profits in absolute terms will be taken as the firm's objective; this can then be *expressed* as a rate of return." (italics in the original) Thus ROA is an observable consequence of the pursuit of a different (indeed, almost any) objective.

ROA sits atop the duPont triangle, a management accounting system developed at duPont and General Motors (GM) early in the 20th century. Even then both duPont and GM were diversified corporations, producing a variety of products in several locations, and management had to decide how to allocate capital investment, as well as other resources and managerial compensation, across product lines and among plants. The allocation criterion duPont and GM used was the return on those investments, ROA. The developers also devised a product pricing formula designed to set product prices that would yield a desired ROA when production was at standard volume, defined at GM to be two shifts per day.

To assist in the resource allocation and product pricing strategies, ROA = π/A was decomposed into a pair of financial ratios that drive π/A. This in turn enabled management to develop strategies intended to enhance either ratio, and hence π/A. The decomposition states that π/A is the product of the profit margin π/R, and asset turnover R/A. π/R indicates how much of sales revenue a firm retains as profit rather than absorbs as expense. An increase in π/R is consistent with an improvement in cost efficiency, the adoption of cost-saving technology, a reduction in input prices or an increase in output prices. R/A indicates the revenue productivity of a firm's assets. An increase in R/A suggests that capital is being allocated to higher-valued uses, or output prices are increasing.[2]

For our purposes it is important to note that the duPont triangle does not contain measures of CU, Y/X or P/W, any one of which is a potential driver of π/R and/or R/A. We incorporate CU in Section 3, we incorporate CU and Y/X in Section 4, and we incorporate CU, Y/X and P/W in Section 5.

3. Capacity Utilization

Incorporating CU into a duPont triangle requires a definition of capacity, and there are several to choose from. A generic approach is to write the triangle as

$$\pi/A = \pi/R \times R/A$$

$$= \pi/R \times (p^Ty/p^Ty^c) \times (p^Ty^c/A), \qquad (1)$$

with output price vector $p \in R^M_{++}$, output quantity vector $y \in R^M_{+}$ and capacity output quantity vector $y^c \in R^M_{+}$. Weighting y and y^c by p maintains the financial structure of the triangle and, more significantly for our purposes, allows M>1. Expression (1) decomposes ROA into the product of three drivers: the profit margin, the rate of capacity utilization, and potential asset turnover, the turnover that would occur at full capacity output. We now consider how to define y^c.

Figure 2 supports three definitions of capacity and its rate of utilization. We observe output vector y and input vector x, with $y \in P(x)$ and feasible set P(x) bounded above by its frontier $P^F(x)$. All $y \in P^F(x)$ are maximum output vectors that can be produced with x and given technology. The technically efficient output vector associated with y is $y^a = y/D_o(x,y)$, with $D_o(x,y)$ an output distance function defined as $D_o(x,y) = \min\{\lambda: y/\lambda \in P(x)$, and the technical efficiency of y is $y/y^a = D_o(x,y) \leq 1$. We next partition x into fixed and variable sub-vectors, so that $x = (x_f, x_v)$, and by fixity of x_f we mean $x_f \leq \bar{x}_f$. Following Gold (1955) and Johansen (1968), we define $P(\bar{x}_f)$ as the set of feasible output vectors obtainable from $x_f \leq \bar{x}_f$ when no constraint is imposed on the availability and use of x_v. $P(\bar{x}_f)$ is bounded above by its frontier $P^F(\bar{x}_f)$, and all $y \in P^F(\bar{x}_f)$ are full capacity output vectors, given $x_f \leq \bar{x}_f$ and technology.[3]

Our first definition of capacity and its rate of utilization follows Gold and Johansen, and solves an output maximization problem. It is independent of prices, and defines capacity output as the largest feasible radial expansion of y. In Figure 2 $y^{GJ} = y/D_o(\bar{x}_f,y) \in P^F(\bar{x}_f)$ is the full capacity output vector associated with actual output vector y, with $D_o(\bar{x}_f,y) = \min\{\lambda: y/\lambda \in P(\bar{x}_f)\}$, and so the rate of capacity utilization is $CU^{GJ} = D_o(\bar{x}_f,y) \leq 1$. The superscript "^{GJ}" honors the two pioneers, Gold and Johansen. CU^{GJ} is measured holding the output mix constant, and so is useful without output price information even when M > 1. CU^{GJ} is a gross measure that can be decomposed into the product of an output-oriented technical efficiency term $[D_o(x,y) \leq 1]$ and a net capacity utilization term $[D_o(\bar{x}_f,y)/D_o(x,y) \leq 1]$. We refer to the two components of CU^{GJ} as *wasted capacity* and *excess capacity*, respectively.[4]

Our second definition follows Segerson & Squires (1995) and Lindebo *et al.* (2007), and solves a revenue maximization problem.[5] It is dependent on the output price vector p, and defines capacity output as the vector $y^r \in P^F(\bar{x}_f)$ that solves the revenue maximization problem $\max_y\{p^Ty: x_f \leq \bar{x}_f\}$, and so the rate of capacity utilization is $CU^r = p^Ty/p^Ty^r \leq 1$. In Figure 2 the vectors $y^a = y/D_o(x,y) \in P^F(x)$ and $y^{GJ} = y/D_o(\bar{x}_f,y) \in P^F(\bar{x}_f)$ divide revenue-based capacity utilization into three components, an output-oriented technical efficiency term $p^Ty/p^Ty^a = D_o(x,y) \leq 1$ and a pair of capacity utilization components, a radial capacity utilization term $p^Ty^a/p^Ty^{GJ} = D_o(\bar{x}_f,y)/D_o(x,y) \leq 1$ and an output mix term p^Ty^{GJ}/p^Ty^r. We refer to the three components as wasted capacity, excess capacity, and *misallocated capacity*, respectively. Wasted capacity and excess capacity have the same interpretations and magnitudes as in the output maximization problem, and misallocated capacity is new. It captures the economic value of an optimizing movement along $P^F(\bar{x}_f)$ to adapt the output mix to prevailing output prices.

Our third definition follows Coelli *et al.* (2002), and solves a variable profit maximization problem, with variable profit $\pi_v = p^Ty - w_v{}^Tx_v$, w_v being the variable input price vector and $w_v{}^Tx_v$ being variable cost. This definition is dependent on two price vectors, p and w_v. It defines capacity output as the output vector $y^{v\pi} \in P^F(\bar{x}_f,x_v^{v\pi})$ that, together with $x_v^{v\pi}$, solves the variable profit maximization problem $\max_{y,x_v}\{p^Ty - w_v{}^Tx_v: x_f \leq \bar{x}_f\}$, so that maximum $\pi_v^{v\pi} = p^Ty^{v\pi} - w_v{}^Tx_v^{v\pi}$. The rate of capacity utilization is $CU^{v\pi} = p^Ty/p^Ty^{v\pi}$. The vectors $y^a = y/D_o(x,y) \in P^F(x)$ and $y^b = y/D_o(\bar{x}_f, x_v^{v\pi},y) \in P^F(\bar{x}_f,x_v^{v\pi})$ divide $CU^{v\pi}$ into an output-oriented technical efficiency term $p^Ty/p^Ty^a = D_o(x,y) \leq 1$ and a pair of capacity utilization components, a radial capacity utilization term $p^Ty^a/p^Ty^b = D_o(\bar{x}_f,x_v^{v\pi},y)/D_o(x,y) \leq 1$, and an output mix term $p^Ty^b/p^Ty^{v\pi}$. As in the revenue maximization problem we refer to the three components as wasted capacity, excess capacity, and misallocated capacity, although excess capacity and misallocated capacity have different magnitudes in the two problems.[6]

We are now prepared to introduce capacity utilization into a duPont triangle. For the output maximization problem we have

$$\pi/A = \pi/R \times R/A$$

$$= \pi/R \times p^T y/[p^T y/D_o(\bar{x}_f,y)] \times [p^T y/D_o(\bar{x}_f,y)]/A, \qquad (2)$$

in which CU^{GJ} is $p^T y/[p^T y/D_o(\bar{x}_f,y)] = p^T y/p^T y^{GJ} = R/R^{GJ} = D_o(\bar{x}_f,y)$, and asset turnover is converted to potential asset turnover, defined as $[p^T y/D_o(\bar{x}_f,y)]/A = p^T y^{GJ}/A = R^{GJ}/A$. Although CU^{GJ} appears to be price-dependent, prices appear in CU^{GJ} to implement the division operator, and to maintain a revenue-based numerator in the potential asset turnover term. As above, CU^{GJ} decomposes into wasted capacity and excess capacity components, and so expression (2) contains four drivers of ROA.

For the revenue maximization problem we have

$$\pi/A = \pi/R \times R/A$$

$$= \pi/R \times p^T y/p^T y^r \times p^T y^r/A, \qquad (3)$$

in which CU^r is $p^T y/p^T y^r = R/R^r$ and potential asset turnover is $p^T y^r/A = R^r/A$. In this case CU^r is price-dependent, and decomposes into wasted capacity, excess capacity and misallocated capacity. Consequently expression (3) contains five drivers of ROA.

For the variable profit maximization problem we have

$$\pi_v/A = \pi_v/R \times R/A$$

$$= \pi_v^{v\pi}/p^T y^{v\pi} \times \pi_v/\pi_v^{v\pi} \times p^T y^{v\pi}/A, \qquad (4)$$

in which the profit margin is converted to a potential profit margin $\pi_v^{v\pi}/p^T y^{v\pi} = \pi_v^{v\pi}/R^{v\pi}$, $CU^{v\pi}$ is $\pi_v/\pi_v^{v\pi}$, and potential asset turnover is $p^T y^{v\pi}/A = R^{v\pi}/A$. $CU^{v\pi}$ remains price-dependent, and decomposes into wasted capacity, excess capacity and misallocated capacity. Expression (4) also contains five drivers of ROA.

The three CU measures are derived from an analytical framework in which $x_f \le \bar{x}_f$, and therefore $C_f = w_f^T x_f \le \bar{C}_f = w_f^T \bar{x}_f$. However it is possible to impose $C_f \le \bar{C}_f$ without imposing $x_f \le \bar{x}_f$, thereby allowing substitution among fixed inputs along a fixed input budget constraint $C_f \le \bar{C}_f$. This formulation is particularly appropriate if information on w_f is unavailable. However if this information is available, then the constraints $x_f \le \bar{x}_f$ collapse to a single constraint $w_f^T x_f \le \bar{C}_f \Leftrightarrow (w_f/\bar{C}_f)^T x_f \le 1$. This strategy allows the construction of three "fixed cost indirect" CU measures corresponding to the three direct measures in expressions (2) – (4). In this case $P(\bar{x}_f)$ is replaced by $P(w_f/\bar{C}_f) \supseteq P(\bar{x}_f)$, and so each indirect CU measure is smaller than its corresponding direct CU measure. Referring to Figure 8.2, $P^F(x)$ remains unchanged, $P^F(\bar{x}_f, x_v^{v\pi})$ expands to $P^F(w_f/\bar{C}_f, x_v^{v\pi})$, and $P^F(\bar{x}_f)$ expands to $P^F(w_f/\bar{C}_f)$. The full capacity output quantity vectors increase accordingly.[7]

The output maximization problem becomes $\max_y\{y: (w_f/\bar{C}_f)^T x_f \le 1\}$, and the associated duPont triangle is

$$\pi/A = \pi/R \times R/A$$

$$= \pi/R \times p^Ty/[p^Ty/D_o(w_f/\bar{C}_f,y)] \times [p^Ty/D_o(w_f/\bar{C}_f,y)]/A, \quad (5)$$

in which the fixed cost indirect CU^{GJ} simplifies to $D_o(w_f/\bar{C}_f,y)$.

The revenue maximization problem becomes $\max_y\{p^Ty: (w_f/\bar{C}_f) \le 1\}$, and the associated duPont triangle is unchanged from that in expression (3), with the proviso that $y^r \in P^F(w_f/\bar{C}_f)$. The variable profit maximization problem becomes $\max_{y,x_v}\{p^Ty - w_v{}^T x_v: (w_f/\bar{C}_f) \le 1\}$, and the associated duPont triangle is unchanged from that in expression (4), with the proviso that $y^{v\pi} \in P^F(w_f/\bar{C}_f, x_v^{v\pi})$.

The direct and fixed cost indirect analyses are structurally similar; the only difference is the expansion of the direct output sets $P^F(x_f, x_v{}^{v\pi})$ and $P^F(x_f)$ to the fixed cost indirect output sets $P^F(w_f/\bar{C}_f, x_v^{v\pi})$ and $P^F(w_f/\bar{C}_f)$, and the corresponding reductions in capacity utilization. The virtues of the fixed cost indirect approach are (i) at the producer level it offers flexibility in the allocation of fixed cost budgets, (ii) at the industry level it offers managers and/or regulators an alternative way of restricting capacity, by assigning quotas to a single variable $C_f \le \bar{C}_f$ rather than several $x_f \le \bar{x}_f$, and (iii) at the analyst level it shrinks the number of direct constraints in an optimization problem.

We have introduced direct and fixed cost indirect measures of capacity utilization into a duPont triangle. We now attempt to introduce productivity into a duPont triangle by extending expression (1) to

$$\pi/A = \pi/R \times p^Ty/p^Ty^c \times p^Ty^c/A$$

$$= [1 - (C/R)] \times p^Ty/p^Ty^c \times p^Ty^c/A, \quad (6)$$

in which C/R is the ratio of cost to revenue, also known as the operating ratio or the expense ratio. Gold argued, convincingly, that productivity was negatively related to C and positively related to R, both of which are positively related to π/R.

Gold's argument is persuasive, but analytically deficient. Y/X does not appear explicitly in expression (6) as a driver of π/R. Its role is played out behind the scenes. There is a reason for its absence. The components of the duPont triangle are absolute variables describing levels. But Y/X is a relative variable describing change from one situation to another. Any attempt to incorporate a relative variable into a relationship among absolute variables is destined to fail. Incorporating productivity into a duPont triangle requires construction of a pair of triangles, so that change from one to another may be driven in part by productivity change. We undertake this exercise in Section 4.

4. Drivers of ROA Change

In this Section we convert an atemporal duPont triangle to an intertemporal duPont triangle change. We then show how change in the rate of capacity utilization and productivity change affect ROA change.

The ratio of comparison period to base period duPont triangles is

$$(\pi/A)^1/(\pi/A)^o = (\pi/R)^1/(\pi/R)^o \times (R/A)^1/(R/A)^o. \qquad (7)$$

We consider three different strategies for incorporating change in the rate of capacity utilization and productivity change into expression (7). All three strategies are based on Gold's expression

$$Y/X = Y^c/X \times Y/Y^c, \qquad (8)$$

in which Y and X are output and input quantity indexes and Y^c is a full capacity output quantity index derived from any one of the six direct and indirect capacity output vectors defined in Section 3. The three quantity indexes can be either theoretical technology-based indexes or empirical price-based indexes. Expression (8) states that actual productivity change Y/X is the product of potential productivity change Y^c/X and change in capacity utilization Y/Y^c. Gold provides a detailed discussion of the relationship, and of the relative merits of Y/X and the less volatile Y^c/X as productivity indexes.

One strategy is to introduce $Y/X = Y^c/X \times Y/Y^c$ directly into the profit margin change leg of expression (7), generating a model in which CU change influences Y/X, which influences π/R change, which drives ROA change. In this strategy the analysis proceeds in two steps. In the first step we use any of the six optimization problems to create a full capacity output vector y^c. In the second step we use y^c to derive $CU = Y/Y^c$ and to derive (and perhaps decompose) the Y^c/X component of Y/X. This generates the expression

$$(\pi/A)^1/(\pi/A)^o = (\pi/R)^1/(\pi/R)^o \times (R/A)^1/(R/A)^o$$

$$\uparrow\!\!\lrcorner\ Y/X = Y^c/X \times Y/Y^c. \qquad (9)$$

A second strategy starts with a duPont triangle that incorporates capacity utilization. It converts the triangle to a triangle change and continues by introducing Y/X into the profit margin change leg of the triangle. This strategy generates a model in which Y/X influences ROA change through the π/R change leg, and CU change influences ROA change independently, but CU change does not influence Y/X. Using generic expression (1) and writing $R^c = p^T y^c$ generates the expression

$$(\pi/A)^1/(\pi/A)^o = \underset{\substack{\uparrow \\ Y/X.}}{(\pi/R)^1/(\pi/R)^o} \times CU^1/CU^o \times (R^c/A)^1/(R^c/A)^o \qquad (10)$$

A third strategy starts with a duPont triangle that incorporates capacity utilization, converts it to a triangle change, and continues by introducing $Y/X = Y^c/X \times Y/Y^c$ into the profit margin change leg of the triangle change. This strategy generates a model in which Y/X influences ROA change through the π/R change leg, and CU change influences ROA change twice, once through its impact on productivity change in the profit margin change leg, and again independently. Again using expression (1) to illustrate, we have[8]

$$(\pi/A)^1/(\pi/A)^o = \underset{\substack{\uparrow \\ Y/X = Y^c/X \times Y/Y^c.}}{(\pi/R)^1/(\pi/R)^o} \times CU^1/CU^o \times (R^c/A)^1/(R^c/A)^o \qquad (11)$$

These three strategies raise an issue. What is the most likely relationship linking CU change, productivity change and ROA change? The first strategy is preferred if CU change influences productivity change, which influences ROA change, but CU change has no independent influence on ROA change. The second strategy is preferred if CU change influences ROA change independently and has no influence on productivity change. The third strategy encompasses the first two, and is preferred if CU change influences ROA change independently, and again through its impact on productivity change, which influences ROA change. All three choices face the challenge, originally encountered by Gold, of showing analytically how Y/X influences the profit margin change leg of the duPont triangle. The driving relationships in expressions (9) – (11) are hypotheses rather than analytical demonstrations. We meet this challenge in Section 5.

5. Incorporating Productivity Change into a duPont Triangle Change

In this Section we introduce price change, and we show how productivity change and price change drive margin change, and thus ROA change. We have already shown that it is straightforward to incorporate change in the rate of capacity utilization into a duPont triangle change expression, and we write, using y^c as the solution vector to any of the six direct and indirect optimization problems in Section 3,

$$\frac{(\boldsymbol{\pi}/\mathbf{A})^{\mathbf{1}}}{(\boldsymbol{\pi}/\mathbf{A})^{\mathbf{o}}} = \frac{(\boldsymbol{\pi}/\mathbf{R})^{\mathbf{1}}}{(\boldsymbol{\pi}/\mathbf{R})^{\mathbf{o}}} \times \frac{(\mathbf{p}^{\mathbf{1T}}\mathbf{y}^{\mathbf{1}})/(\mathbf{p}^{\mathbf{1T}}\mathbf{y}^{\mathbf{1c}})}{(\mathbf{p}^{\mathbf{oT}}\mathbf{y}^{\mathbf{o}})/(\mathbf{p}^{\mathbf{oT}}\mathbf{y}^{\mathbf{oc}})} \times \frac{(\mathbf{p}^{\mathbf{1T}}\mathbf{y}^{\mathbf{1c}})/\mathbf{A}^{\mathbf{1}}}{(\mathbf{p}^{\mathbf{oT}}\mathbf{y}^{\mathbf{oc}})/\mathbf{A}^{\mathbf{o}}}, \qquad (12)$$

which attributes ROA change to profit margin change, change in the rate of capacity utilization, and change in potential asset turnover. Change in the rate of capacity utilization exerts an independent influence on ROA change, but neither productivity change nor price change appears in expression (12).

We now consider how price change and productivity change influence ROA change. The key is to acknowledge that change in the profit margin derives from price change and quantity change, and we write

$$\begin{aligned} \frac{\pi^1/R^1}{\pi^o/R^o} &= \left[\frac{\pi^1/R^1}{\pi_o^1/R_o^1}\right] \times \left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] \\ &= \left[\frac{\pi_1^o/R_1^o}{\pi^o/R^o}\right] \times \left[\frac{\pi^1/R^1}{\pi_1^o/R_1^o}\right], \end{aligned} \qquad (13)$$

where $\pi_o^1 = p^{oT}y^1 - w^{oT}x^1$ and $R_o^1 = p^{oT}y^1$ in the first equality are comparison period profit and revenue evaluated at base period prices, and π_1^o and R_1^o in the second equality are base period profit and revenue evaluated at comparison period prices. We focus on the first equality, in which the first term on the right side is that part of the margin change that can be attributed solely to price change, since it compares nominal and real comparison period margins. The second term on the right side is that part of the margin change attributable solely to quantity change, since it compares the real comparison period margin with the nominal base period margin. We return to the second equality in Section 5.2.[9]

We develop two strategies for decomposing the margin change component of ROA change. In the first we express the quantity effect in terms of the theoretical productivity index proposed by Caves *et al*. (1982). In the second we express the quantity effect in terms of empirical Laspeyres, Paasche and Fisher quantity indexes. Both strategies decompose the quantity effect, but in different ways. Problems with the first strategy include (i) the CCD productivity index is not in Y/X form; (ii) decomposing the quantity effect in terms of a CCD productivity index requires cost allocation, so that $w^Tx = c^Ty$, with $c \in R_{++}^M$ a vector of unit costs of producing each output; (iii) it is not possible to express the price effect in terms of a CCD price recovery index that has a meaningful economic interpretation; and (iv) the CCD productivity index must be estimated, which requires degrees of freedom. The second strategy requires information on output and input prices. Of course drawbacks of one strategy are strengths of the other.

5.1 The Theoretical CCD Productivity Index Strategy

We focus on the quantity effect $\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right]$ in the first equality in expression (13). Assuming that cost allocation is feasible, we can write[10]

$$\begin{aligned}\pi^o &= p^{oT}y^o - w^{oT}x^o \\ &= p^{oT}y^o - c^{oT}y^o \\ &= (p^o - c^o)^T y^o, \qquad (14)\end{aligned}$$

where $w^{oT}x^o = c^{oT}y^o$, c^o being a vector of base period unit costs of producing each output. Writing base period profit in this way enables us to rewrite the base period profit margin as

$$\begin{aligned}\frac{\pi^o}{R^o} &= [(p^o - c^o)^T y^o]/R^o \\ &= [(p^o - c^o)/R^o]^T y^o] \\ &= \rho^{oT} y^o, \qquad (15)\end{aligned}$$

where $\rho^o = (p^o - c^o)/R^o$. Similarly, we can rewrite the real comparison period profit margin as

$$\begin{aligned}\frac{\pi_o^1}{R_o^1} &= [(p^o - c_o^1)^T y^1]/R_o^1 \\ &= [(p^o - c_o^1)/R_o^1]^T y^1 \\ &= \rho_o^{1T} y^1, \qquad (16)\end{aligned}$$

where $c_o^{1T}y^1 = w^{oT}x^1$ and $\rho_o^1 = (p^o - c_o^1)/R_o^1$. Consequently the quantity effect can be rewritten as

$$\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] = \frac{\rho_o^{1T}y^1}{\rho^{oT}y^o}. \qquad (17)$$

The next step is to interpret expression (17), which we do with the assistance of Figure 3, in which T^o and T^1 are base period and comparison period production frontiers analogous to $P^{Fo}(x^o)$ and $P^{F1}(x^1)$. We have

$$\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] = \left[\frac{\rho_o^{1T}y^1/\rho_o^{1T}y^C}{\rho^{oT}y^o/\rho_o^{1T}y^A}\right] \times \left[\frac{\rho^{oT}y^B}{\rho^{oT}y^A}\right] \times \left[\frac{\rho_o^{1T}y^C}{\rho^{oT}y^B}\right], \qquad (18)$$

where $y^A = y^o/D_o^o(x^o,y^o)$, $y^B = y^o/D_o^1(x^o,y^o)$ and $y^C = y^1/D_o^1(x^1,y^1)$. We can rewrite expression (18) as

$$\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] = \left[\frac{D_o^1(x^1,y^1)}{D_o^1(x^o,y^o)}\right] \times \left[\frac{\rho_o^{1T}y^C}{\rho^{oT}y^B}\right]$$

$$= \left[\frac{D_o^1(x^1,y^1)}{D_o^o(x^o,y^o)}\right] \times \left[\frac{D_o^o(x^o,y^o)}{D_o^1(x^o,y^o)}\right] \times \left[\frac{\rho_o^{1T}y^C}{\rho^{oT}y^B}\right], \qquad (19)$$

where $\left[\frac{D_o^1(x^1,y^1)}{D_o^1(x^o,y^o)}\right] = \left[\frac{D_o^1(x^1,y^1)}{D_o^o(x^o,y^o)}\right] \times \left[\frac{D_o^o(x^o,y^o)}{D_o^1(x^o,y^o)}\right]$ is an output-oriented comparison period CCD productivity index. We know from Caves *et al.* (1982) that the two components $D_o^1(x^1,y^1)/D_o^o(x^o,y^o)$ and $D_o^o(x^o,y^o)/D_o^1(x^o,y^o)$ measure technical efficiency change and technical change respectively, as is apparent from Figure 3. Consequently[11]

$$\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] = M_{CCD}^1(y^o,y^1,x^o,x^1) \times \left[\frac{\rho_o^{1T}y^C}{\rho^{oT}y^B}\right]. \qquad (20)$$

The term $[\rho_o^{1T}y^C/\rho^{oT}y^B]$ measures the productivity impact of size change that is absent from $M_{CCD}^1(y^o,y^1,x^o,x^1)$, and corresponds to the movement along T^1 from (x^o,y^B) to (x^1,y^C) in Figure 3. Thus the quantity effect $\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right]$ is a measure of productivity change, because it includes the impact of size change along with the impacts of technical efficiency change and technical change.[12]

Substituting expression (20) into expression (12) yields a decomposition of ROA change incorporating (and decomposing and augmenting) a theoretical CCD productivity index

$$\frac{\pi^1/A^1}{\pi^o/A^o} = \left[\frac{\pi^1/R^1}{\pi_o^1/R_o^1}\right] \times \left[\frac{D_o^1(x^1,y^1)}{D_o^o(x^o,y^o)}\right] \times \left[\frac{D_o^o(x^o,y^o)}{D_o^1(x^o,y^o)}\right] \times \left[\frac{\rho_o^{1T}y^C}{\rho^{oT}y^B}\right]$$

$$\times \left[\frac{(p^{1T}y^1)/(p^{1T}y^{1c})}{(p^{oT}y^o)/(p^{oT}y^{oc})}\right] \times \left[\frac{R^{1c}/A^1}{R^{oc}/A^o}\right], \qquad (21)$$

where $R^{tc} = p^{tT}y^{tc}$, t=o,1. Expression (21) attributes ROA change to price recovery change, three components of productivity change, change in capacity utilization and change in potential asset turnover. Although capacity utilization change influences ROA change, it does so without influencing productivity change.

Starting with the first equality in expression (13) leads to a decomposition of ROA change in expression (21) built on a comparison period CCD productivity index and a size change term measured along comparison period technology. Starting with the second equality in expression (13) and following the same procedures generates a decomposition of ROA change built on a base period CCD productivity index and a size change term measured along base period technology. Omitting intermediate steps, this decomposition is

$$\frac{\pi^1/A^1}{\pi^o/A^o} = \left[\frac{\pi_1^o/R_1^o}{\pi^o/R^o}\right] \times \left[\frac{D_o^1(x^1,y^1)}{D_o^o(x^o,y^o)}\right] \times \left[\frac{D_o^o(x^1,y^1)}{D_o^1(x^1,y^1)}\right] \times \left[\frac{\rho^{1T}y^D}{\rho_1^{oT}y^A}\right]$$

$$\times \left[\frac{(p^{1T}y^1)/(p^{1T}y^{1c})}{(p^{oT}y^o)/(p^{oT}y^{oc})}\right] \times \left[\frac{R^{1c}/A^1}{R^{oc}/A^o}\right], \qquad (22)$$

in which $\rho_1^o = [(p^1 - c_1^o)/R_1^o]^T y^o$, $R_1^o = p^1y^o$ and y^D is located on T^o in Figure 3. Expression (22) decomposes ROA change into price recovery change, a base period CCD productivity index, a size change term measured along base period technology, change in capacity utilization and change in potential asset turnover. The capacity and turnover terms are the same as in expression (21). It is straightforward to calculate the geometric mean of expressions (21) and (22) to create an ROA change decomposition based on a geometric mean price recovery effect, a geometric mean CCD productivity index, and a geometric mean size change effect.

Kendrick & Grossman (1980) have argued, and demonstrated empirically, that productivity change is *positively* related to change in the rate of capacity utilization at the aggregate level. Many subsequent writers concur. Our objective is to introduce capacity utilization change as a driver of productivity change in expression (21).

The key to relating the two is contained in Gold's expression $Y/X = Y^c/X \times Y/Y^c$, which states that Y/X depends on change in CU, which is nice because a lot of empirical evidence supports the linkage, and the sign of the impact of CU change on Y/X is indeterminate, which is also nice because it makes pro-cyclicality a testable hypothesis. Suppose, as seems reasonable but not certain, that $Y^c/X \gtreqless 1 \gtreqless Y/Y^c$, so that potential productivity and capacity utilization move in opposite directions. Then productivity is pro-cyclical if $(Y/Y^c) \gtreqless 1 \Rightarrow [(Y^c/X) \times (Y/Y^c)] \gtreqless 1$ and counter-cyclical if $(Y/Y^c) \gtreqless 1 \Rightarrow [(Y^c/X) \times (Y/Y^c)] \lesseqgtr 1$. Alternatively, if causation moves in the opposite direction, productivity is pro-cyclical if $(Y^c/X) \gtreqless 1 \Rightarrow [(Y^c/X) \times (Y/Y^c)] \lesseqgtr 1$ and counter-cyclical if $(Y^c/X) \gtreqless 1 \Rightarrow [(Y^c/X) \times (Y/Y^c)] \gtreqless 1$. In words, productivity change is pro-cyclical if CU adjusts more than proportionately to change in potential productivity.[13]

Referring to Figure 2, in each period $p^{tT}y^t/p^{tT}y^{at} = p^{tT}y^t/p^{tT}y^{GJt} \div p^{tT}y^{at}/p^{tT}y^{GJt}$, t=o,1, which states that wasted capacity (technical inefficiency) can be expressed as the ratio of gross excess capacity to net excess capacity. Change in wasted capacity coincides with the technical efficiency change component of the CCD productivity index. Following De Borger & Kerstens (2000), we replace the technical efficiency change component of the CCD productivity index with the ratio of gross excess capacity to net excess capacity to obtain

$$\left[\frac{D_o^1(x^1,y^1)}{D_o^1(x^o,y^o)}\right] = \left[\frac{D_o^1(x_f^1,y^1)}{D_o^o(x_f^o,y^o)}\right] \times \left[\frac{D_o^1(x^1,y^1)/D_o^1(x_f^1,y^1)}{D_o^o(x^o,y^o)/D_o^o(x_f^o,y^o)}\right] \times \left[\frac{D_o^o(x^o,y^o)}{D_o^1(x^o,y^o)}\right], \qquad (23)$$

which states that a CCD productivity index can be expressed as the product of technical efficiency change relative to $P^{Fo}(\bar{x}_f^o)$ and $P^{F1}(\bar{x}_f^1)$, change in the net rate of capacity utilization, and technical change. Substituting expression (23) into expression (19) yields

$$\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] = \left[\frac{D_o^1(x_f^1,y^1)}{D_o^o(x_f^o,y^o)}\right] \times \left[\frac{D_o^1(x^1,y^1)/D_o^1(x_f^1,y^1)}{D_o^o(x^o,y^o)/D_o^o(x_f^o,y^o)}\right] \times \left[\frac{D_o^o(x^o,y^o)}{D_o^1(x^o,y^o)}\right] \times \left[\frac{\rho_o^{1T}y^C}{\rho_o^{oT}y^B}\right], \quad (24)$$

which decomposes actual productivity change into change in net capacity utilization and potential productivity change (the CCD productivity index analogue to Y^c/X). Gold's expression (8) is embedded in expressions (23) and (24), in theoretical index number form. Finally, substituting expression (24) into expression (21) yields the complete CCD decomposition of ROA change

$$\frac{\pi^1/A^1}{\pi^o/A^o} = \left[\frac{\pi^1/R^1}{\pi_o^1/R_o^1}\right] \times \left[\frac{D_o^1\left(x_f^1,y^1\right)}{D_o^o\left(x_f^o,y^o\right)}\right] \times \left[\frac{D_o^1(x^1,y^1)/D_o^1\left(x_f^1,y^1\right)}{D_o^o(x^o,y^o)/D_o^o\left(x_f^o,y^o\right)}\right] \times \left[\frac{D_o^o(x^o,y^o)}{D_o^1(x^o,y^o)}\right]$$

$$\times \left[\frac{\rho_o^{1T}y^C}{\rho_o^{oT}y^B}\right] \times \left[\frac{(p^{1T}y^1)/(p^{1T}y^{1c})}{(p^{oT}y^o)/(p^{oT}y^{oc})}\right] \times \left[\frac{R^{1c}/A^1}{R^{oc}/A^o}\right], \qquad (25)$$

which attributes ROA change to price change, potential productivity change, change in the rate of capacity utilization, and change in potential asset turnover. Change in capacity utilization plays a dual role, as an independent driver of ROA change, and as a driver of potential productivity change, which in turn drives ROA change. A similar decomposition can be derived from expression (22), and the geometric mean of expression (25) and the decomposition based on expression (22) can be calculated.[14]

5.2 The Empirical Index Number Strategy

Expressions (23) – (25) use a pair of augmented CCD productivity indexes to interpret the quantity effect as a productivity effect, on the assumption that cost allocation is feasible. Although these expressions do provide an augmented CCD productivity index interpretation of the quantity effect, they do not provide an analogous interpretation of the price recovery effect. This requires empirical quantity-based and price-based indexes.

A few mathematical manipulations enable us to write the price recovery effect in the first equality of expression (13) as

$$\left[\frac{\pi^1/R^1}{\pi_o^1/R_o^1}\right] = \frac{\pi^1}{R^1 - (\frac{P_P}{W_P})w^{1T}x^1}\ , \tag{26}$$

in which P_P/W_P is a quantity-based Paasche price recovery index, with $\left[\frac{\pi^1/R^1}{\pi_o^1/R_o^1}\right] \gtreqless 1 \Leftrightarrow P_P/W_P \gtreqless 1$. Expression (26) contains comparison period and base period prices, but only comparison period quantities, and shows the contribution of P_P/W_P to profit margin change.

We follow the same strategy to write the quantity effect in the first equality of expression (13) as

$$\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] = \frac{\pi_o^1}{R_o^1 - (\frac{Y_L}{X_L})w^{oT}x^1}\ , \tag{27}$$

in which Y_L/X_L is a price-based Laspeyres productivity index, with $\left[\frac{\pi_o^1/R_o^1}{\pi^o/R^o}\right] \gtreqless 1 \Leftrightarrow Y_L/X_L \gtreqless 1$. Expression (27) contains comparison period and base period quantities, but only base period prices, and shows the contribution of Y_L/X_L to profit margin change.

Substituting expressions (26) and (27) into expression (12) yields a decomposition of ROA change based on empirical price and quantity indexes

$$\frac{\pi^1/A^1}{\pi^o/A^o} = \frac{\pi^1}{R^1 - (\frac{P_P}{W_P})w^{1T}x^1} \times \frac{\pi_o^1}{R_o^1 - (\frac{Y_L}{X_L})w^{oT}x^1)}$$

$$\times \left[\frac{(p^{1T}y^1)/(p^{1T}y^{1c})}{(p^{oT}y^o)/(p^{oT}y^{oc})}\right] \times \left[\frac{R^{1c}/A^1}{R^{oc}/A^o}\right], \tag{28}$$

which attributes ROA change to price change, productivity change, change in capacity utilization and change in potential asset turnover. The difference between expressions (25) and (28) is that the augmented CCD productivity index decomposes by economic driver, while the Paasche price recovery index and the

Laspeyres productivity index decompose by variable. In both expressions price recovery change, productivity change and change in the rate of capacity utilization exert independent influences on ROA change. Change in the rate of capacity utilization is a driver of productivity change in expression (25), but not in expression (28). We explore this relationship next.

The key to relating productivity change to capacity utilization change is, as in Section 5.1, Gold's expression $Y/X = Y^c/X \times Y/Y^c$. If the quantity indexes Y, Y^c and X are empirical indexes we can write

$$\frac{p^Ty^1/p^Ty^o}{w^Tx^1/w^Tx^o} = \frac{p^Ty^{c1}/p^Ty^{co}}{w^Tx^1/w^Tx^o} \times \frac{p^Ty^1/p^Ty^o}{p^Ty^{c1}/p^Ty^{co}}$$

$$= \frac{p^Ty^{c1}/p^Ty^{co}}{w^Tx^1/w^Tx^o} \times \frac{p^Ty^1/p^Ty^{c1}}{p^Ty^o/p^Ty^{co}}\,, \tag{29}$$

where p and w can be base period or comparison period price vectors. The first term on the right side of expression (29) is Y^c/X and the second is Y/Y^c. The second equality rewrites and clarifies the capacity utilization change term. Expression (29) is interpreted exactly as Gold's expression (8), in empirical index number form. Substituting a Laspeyres version of expression (29) into expression (27) yields

$$\left[\frac{\pi_0^1/R_0^1}{\pi^o/R^o}\right] = \frac{\pi_0^1}{R_0^1 - \left[\left(\frac{Y_L}{Y_L^c}\right)\left(\frac{Y_L^c}{X_L}\right)\right]w^{oT}x^1}, \tag{30}$$

which expresses actual productivity change in terms of change in capacity utilization and potential productivity change. Substituting expression (30) into expression (28) yields

$$\frac{\pi^1/A^1}{\pi^o/A^o} = \frac{\pi^1}{R^1 - (\frac{P_P}{W_P})w^{1T}x^1} \times \frac{\pi_0^1}{R_0^1 - \left[\left(\frac{Y_L}{Y_L^c}\right)\left(\frac{Y_L^c}{X_L}\right)\right]w^{oT}x^1}$$

$$\times \left[\frac{(p^{1T}y^1)/(p^{1T}y^{1c})}{(p^{oT}y^o)/(p^{oT}y^{oc})}\right] \times \left[\frac{R^{1c}/A^1}{R^{oc}/A^o}\right], \tag{31}$$

which attributes ROA change to price recovery change, productivity change, change in the rate of capacity utilization and change in potential asset turnover. Change in the rate of capacity utilization appears twice, as an independent driver of ROA change, and as a driver of productivity change.

The first equality in expression (13) generates a Paasche price recovery effect and a Laspeyres quantity index that eventually make their way into the ROA change decomposition in expression (31). We now return to the second line, in which the first term is a price recovery effect and the second term is a quantity effect. It is easy to manipulate the two effects to generate

$$\left[\frac{\pi_1^o/R_1^o}{\pi^o/R^o}\right] = \frac{\pi_1^o}{R_1^o - (\frac{P_L}{W_L})w^{1T}x^o}, \tag{32}$$

which is a Laspeyres price recovery effect in which P_L/W_L is a Laspeyres price recovery index, with $\left[\frac{\pi_1^o/R_1^o}{\pi^o/R^o}\right] \gtreqless 1 \Leftrightarrow P_L/W_L \gtreqless 1$.[15] Similarly,

$$\left[\frac{\Pi^1/R^1}{\pi_1^o/R_1^o}\right] = \frac{\pi^1}{R^1 - (\frac{Y_P}{X_P})w^{1T}x^1}, \tag{33}$$

which is a Paasche productivity effect in which Y_P/X_P is a Paasche productivity index, with $\left[\frac{\Pi^1/R^1}{\pi_1^o/R_1^o}\right] \gtreqless 1 \Leftrightarrow Y_P/X_P \gtreqless 1$. Noting that $\frac{Y_P}{X_P} = \left(\frac{Y_P}{Y_P^c}\right)\left(\frac{Y_P^c}{X_P}\right)$, substituting this expression into expression (33), and replacing the price and quantity effects in expression (31) with those in expressions (32) and (33) generates

$$\frac{\pi^1/A^1}{\pi^o/A^o} = \frac{\pi_1^o}{R_1^o - (\frac{P_L}{W_L})w^{1T}x^o} \times \frac{\pi^1}{R^1 - \left(\frac{Y_P}{Y_P^C}\right)\left(\frac{Y_P^C}{X_P}\right)w^{1T}x^1}$$

$$\times \left[\frac{(p^{1T}y^1)/(p^{1T}y^{1c})}{(p^{oT}y^o)/(p^{oT}y^{oc})}\right] \times \left[\frac{R^{1c}/A^1}{R^{oc}/A^o}\right], \tag{34}$$

which is an alternative decomposition of ROA change based on a Laspeyres price recovery recovery index and a Paasche productivity index with capacity utilization appearing twice.

Taking the geometric mean of expressions (26) and (32) generates a Fisher price recovery effect, and taking the geometric mean of expressions (27) and (33) generates a Fisher productivity effect. It does not, however appear possible to express the Fisher price recovery effect in terms of P_F/W_F or the Fisher productivity effect in terms of Y_F/X_F.

The quantity vectors needed to implement the ROA change decompositions in expressions (31) and (34) (and also in expression (25) in Section 5.1) are either observed (y^1,y^o,x^1,x^o) or solutions to optimization problems specified above (y^{c1},y^{co}). All that is required is to specify a functional form for the index numbers in expressions (31) and (34) (and specify base period or comparison period technology and conditioning variables for the distance functions in expression (25) in Section 5.1). The two decompositions are interpreted in exactly the same way; the only difference is that one uses distance functions and the other uses prices to decompose productivity change and to measure change in capacity utilization.

6. External Capacity Constraints

Thus far we have treated capacity utilization as a short run phenomenon created by a fixed input constraint $x_f \leq \bar{x}_f$ or by a weaker fixed input expenditure constraint $C_f \leq \bar{C}_f$. These capacity constraints are internal to the firm. However firms also face external capacity constraints that have financial consequences. Mining firms are constrained by health, safety and environmental regulations, by weather conditions, by a lack of social infrastructure (e.g., housing and schools), and also by inadequate transport infrastructure that inhibits their ability to move minerals to ports to satisfy demand in a timely fashion.[16] Fishers are constrained by a variety of fishery management policies intended to limit catch in a fishery in pursuit of maximum economic yield. Input-oriented policies constrain fisher fixed input use, or “effort,” and output-oriented policies impose total allowable catch (TAC) limits on the fishery, often combined with individual transferrable quota (ITQ) allocation among fishers.[17] In both industries external capacity constraints may make at least some internal capacity constraints redundant for at least some firms at least some of the time.[18]

Figure 4, a simultaneously simplified and augmented version of Figure 2, illustrates the potential impact of external capacity constraints. Two internal frontiers, $P^F(x)$ and $P^F(\bar{x}_f)$, remain, and the third internal frontier, $P^F(\bar{x}_f, x_v^{v\pi})$ remains as well, but for expositional simplicity is replaced by a new external frontier $P^F(Z)$. The three internal frontiers are interpreted as before. The external frontier $P^F(Z)$ represents the collective impacts of industry management practices and regulations, supply chain bottlenecks and other production-limiting capacity constraints unrelated to $\bar{x}_f$ or $\bar{C}_f$.

Using the output maximization framework of Gold and Johansen, output vector y has wasted capacity p^Ty/p^Ty^a and excess capacity p^Ty^a/p^Ty^{GJ}. It also has over-capacity p^Ty^{GJ}/p^Ty^E. In mining overcapacity may be due to the transport infrastructure constraint, and in fishing it may be due to the imposition of TAC and

ITQ. The interpretation is similar in the revenue maximization and variable profit maximization frameworks, although y^E would not be a revenue maximizing or profit maximizing output mix given output price vector p. Since $P(Z) \subset P(\bar{x}_f)$, the internal capacity constraints associated with the output maximization and revenue maximization frameworks are rendered redundant by Z. The external capacity constraints have eliminated overcapacity by reducing capacity, thereby increasing capacity utilization from p^Ty/p^Ty^{GJ} to p^Ty/p^Ty^E. $P^F(Z)$ is not a neutral contraction of $P^F(\bar{x}_f)$, and may constrain some outputs proportionally more than others. $P^F(Z)$ may also constrain some firms more than others, inducing exit by relatively weak forms that creates a more efficient industry structure.

7. Summary and Conclusions

Change in the financial health of a business depends on trends in its price recovery, its productivity, its rate of capacity utilization, and in the external capacity constraints it faces. We have developed a pair of analytical frameworks with which to examine the relationship between change in financial health and its four drivers. We measure financial health with return on assets, and both analytical frameworks begin with the duPont triangle. The first framework exploits a theoretical productivity index, and the second is based on empirical price and quantity index numbers. Both frameworks provide valuable information to management concerning the likely sources of changes in its financial performance. The two frameworks have offsetting strengths. The first does not require price information, and decomposes the productivity effect into three economic drivers of productivity change, technical efficiency change, technical change, and size change. The second framework decomposes both the productivity effect and the price recovery effect into the contributions of individual quantity changes and price changes. The second does not require cost allocation, and it is calculated rather than estimated, so it does not face a degrees of freedom constraint. Both frameworks include change in capacity utilization twice, once as an independent driver of ROA change and again as a driver of productivity change. We also show how external capacity constraints influence capacity output.

Figures

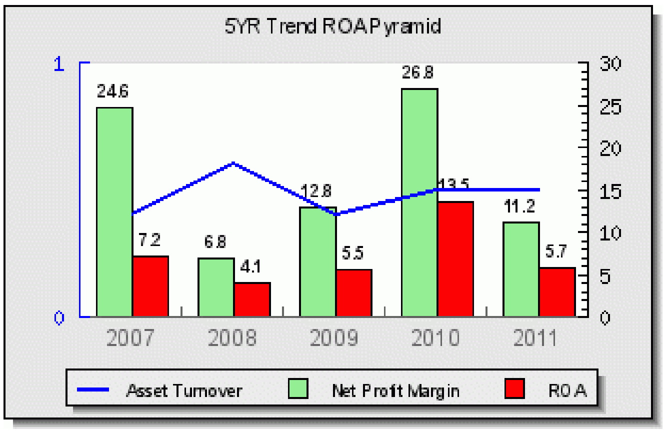

Figure 1 The duPont Triangle at Rio Tinto

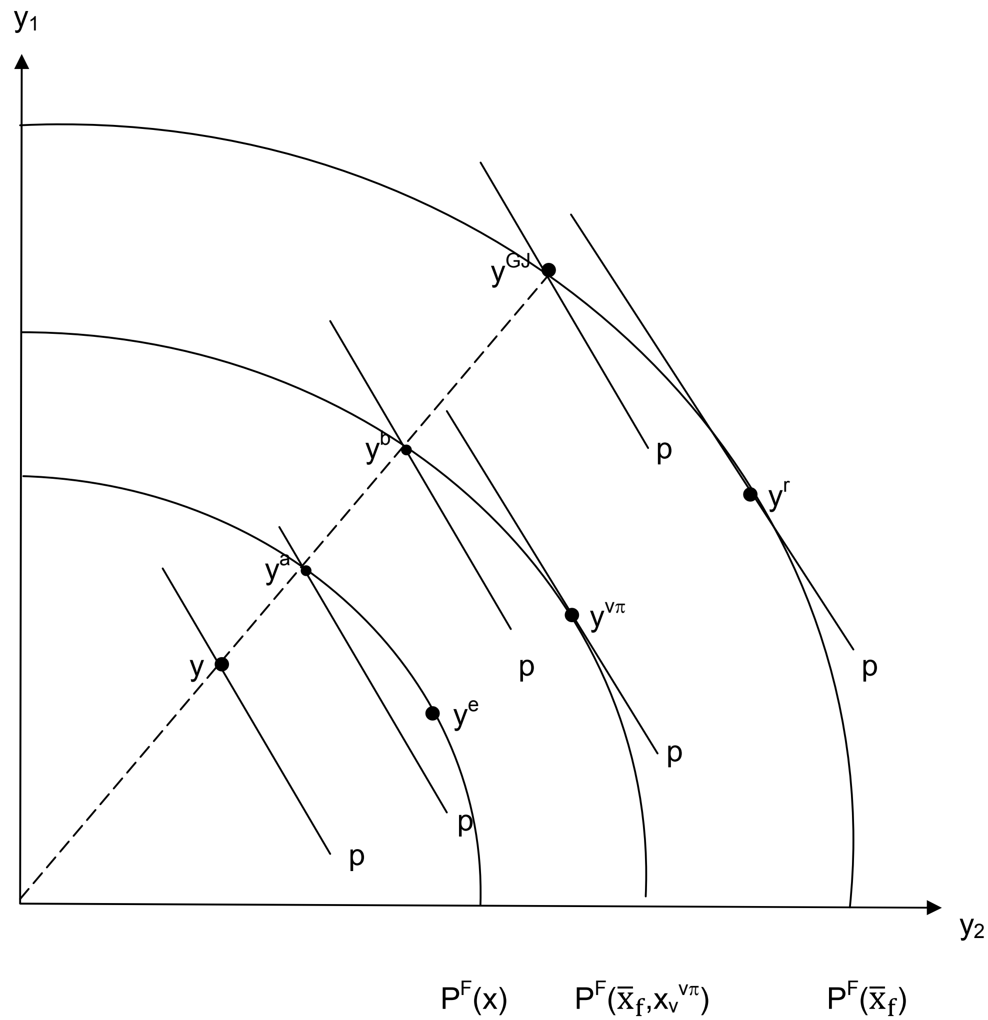

Figure 2 Capacity and its Rate of Utilization

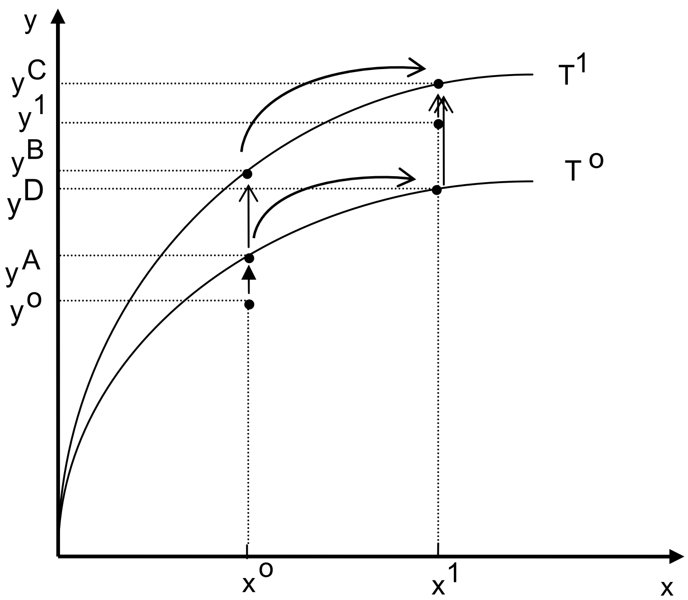

Figure 3 Output-Oriented Productivity Effect Decomposition

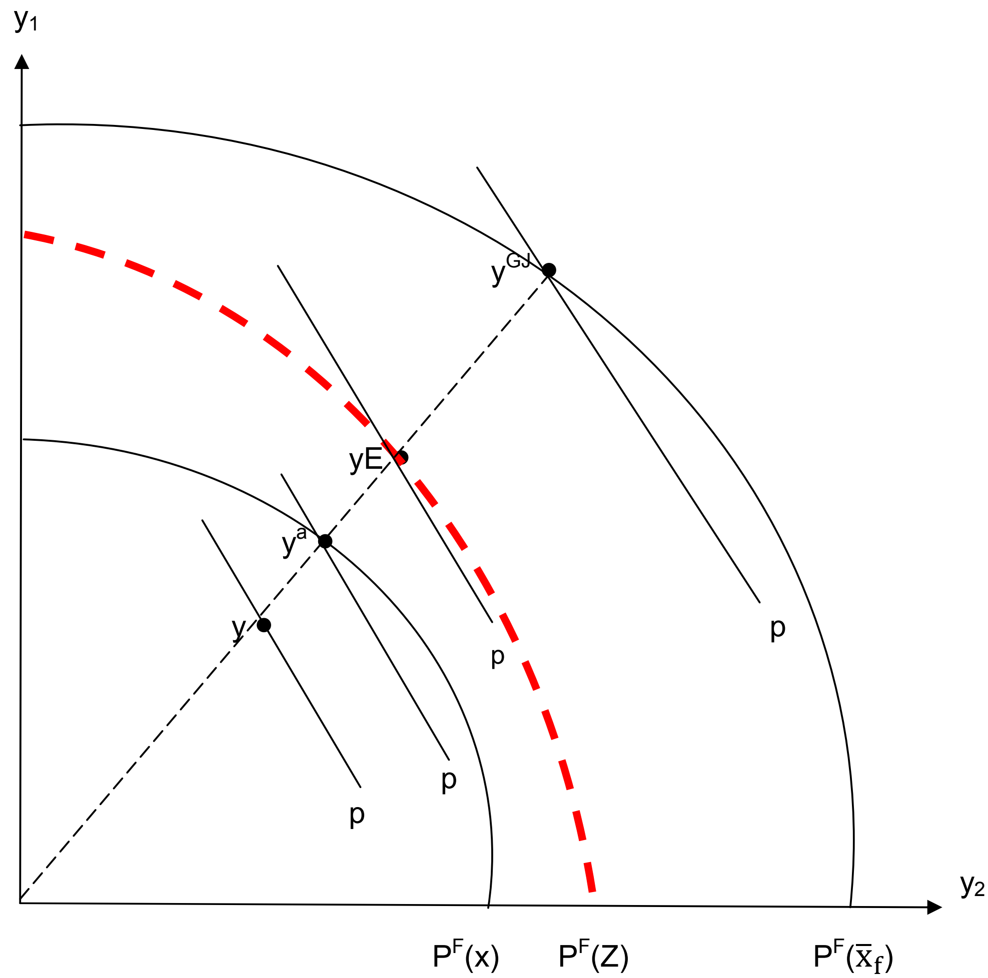

Figure 4 Internal and External Capacity Constraints

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[1] Source: http://au.advfn.com.

[2] Chandler (1962) and Johnson (1975, 1978) detail the development and use of the ROA triangle at duPont and GM.

[3] Gold and Johansen proposed virtually identical physical definitions of y^c, and their definition of y^c was given a managerial slant akin to the use of standard volume at GM. Gold emphasized "practically sustainable capacity," determined by "the customary number of shifts and the normally acceptable length of work day and work week," and with allowance made for breakdowns, repairs and maintenance. Johansen conditioned his definition on the assumption that the firm is "operating under normal conditions with respect to number of shifts, hours of work etc."

[4] The United Nations Food and Agriculture Organization (FAO) (2000) has endorsed the physical measure of capacity utilization proposed by Gold and Johansen, in part due to the shortage of reliable information on output and variable input prices.

[5] Segerson & Squires justify a revenue maximization objective on the grounds that in the short run *all* inputs are quasi-fixed, so that $x = x_f$. Their CU analysis is based on a dual shadow price approach.

[6] If M=1 the solution to the variable profit maximization problem is very similar to the solution to the short run average cost minimization problem proposed by Klein (1960) and Berndt & Morrison (1981) and widely used in the fisheries literature. Sources of the difference are (i) price $\neq$ minimum short run average cost and (ii) minimum short run average cost $\neq$ minimum short run average variable cost. An overlooked definition of full capacity output was proposed by de Leeuw (1962), who defined capacity output as that level of output at which short run marginal cost exceeds minimum short run average cost by some percent, the logic being that at that output level marginal cost is well above minimum average cost, signalling upward pressure on output price.

[7] The theory of cost indirect and return indirect production was developed by Shephard (1974). Empirical applications are regrettably rare. A fixed cost indirect capacity utilization measure was proposed by Färe *et al.* (2000).

[8] Schultze (1963) summarizes the theory behind and evidence for the argument that changes in capacity utilization influence productivity change and profit margin change.

[9] It does not appear possible to implement decomposition (13) into pure price and quantity effects using Edgeworth-Marshall arithmetic mean price and quantity vectors $(\bar{p}, \bar{w})$ and $(\bar{y}, \bar{x})$ because this introduces three pairs of price vectors (p^o,w^o), (p^1,w^1) and $(\bar{p},\bar{w})$, and three pairs of quantity vectors (y^o,x^o), (y^1,x^1) and $(\bar{y}, \bar{x})$.

[10] Cost allocation is a contentious issue. Allocating operating cost is feasible, although the allocation may not be optimal, but allocating overhead cost is difficult; Shubik (2011) calls it an open problem in economic theory and accounting. Estache & Grifell-Tatjé (2011) compromise by ignoring overhead cost, or general expenses, and allocating operating cost to three activities in a sample of Mali water utilities.

[11] Although the CCD productivity index is not in Y/X form, we can calculate $M_{CCD}(y,x)$ and $M^c_{CCD}(y^c,x)$ and define change in capacity utilization residually as $M_{CCD}(y,x)/M^c_{CCD}(y^c,x)$.

[12] Expression (20) augments the CCD productivity index with what we call a size change term, in an effort to introduce a size-related driver of productivity change that might capture

the joint impacts of economies of scale and diversification. Our effort has several antecedents; Färe *et al.* (1994), Ray & Desli (1997) and Grifell-Tatjé & Lovell (1999) all augment the CCD productivity index, which ignores the potential impact of size change on productivity change, with a size change term, although these terms differ.

[13] The indexes Y, Y^c and X, and therefore Y/X, Y^c/X and Y/Y^c, must equal unity in the base period. Thus, for example, CU grows or shrinks from an initial value of unity. However we observe or solve for the underlying output quantity vectors. This allows us to calculate $CU_m^t = y_m^t/y_m^{ct}$, m=1,…,M, t=o,1, for each output individually, or we can calculate an aggregate price-dependent measure $CU^t = R^t/R^{ct} = p^{tT}y^t/p^{tT}y^{ct}$.

[14] We base our decompositions on a CCD productivity index. We prefer to decompose the Malmquist productivity index proposed by Bjurek (1996), in part because it is in Y/X form. This index decomposes as

$$\frac{D_O(x,y^1)/D_O(x,y^o)}{D_I(y,x^1)/D_I(y,x^o)} = \frac{D_O(x,y^{c1})/D_O(x,y^{co})}{D_I(y,x^1)/D_I(y,x^o)} \times \frac{D_O(x,y^1)/D_O(x,y^{c1})}{D_O(x,y^o)/D_O(x,y^{co})},$$

where $D_I(y,x)$ is an input distance function. The first term on the right side is Y^c/X and the second is Y/Y^c. Unfortunately it does not appear possible to link this productivity index with the quantity effect $\left[\frac{\pi_O^1/R_O^1}{\pi^o/R^o}\right]$.

[15] Frankel (1963) recommends use of Paasche quantity indexes (and, to satisfy the product test, Laspeyres price indexes) because, being based on comparison period weights, they are better suited to a company's current needs than are the more popular Laspeyres quantity indexes.

[16] Mining Australia reports that floods in 2011 reduced Queensland's coal exports by 20%. http://www.miningaustralia.com.au/news/qld-flood-damage-confirmed. Pincus & Ergas (2008) analyze Australian mining supply infrastructure bottlenecks, due in part to diffuse and uncoordinated ownership of port terminals, tracks and rolling stock. They cite a study commissioned by the Queensland government that estimated that revenues in excess of a billion AUD per year were being sacrificed to inefficiencies in a single coal supply chain.

[17] Squires *et al.* (2010) provide evidence on the capacity-reducing and distributional impacts of TAC and ITQ in the British Columbia halibut fishery.

[18] Overcapacity in a fishery results from lack of ownership, which creates a tragedy of the commons; external capacity constraints such as TAC and ITQ are intended to create property rights and alter fisher incentives. Overcapacity in mining results from the opposite problem, diffuse and uncoordinated ownership of links in the transport infrastructure.