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The Firmament of Odysseus: Musings on the Precession of Equinoxes

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Dedicated to Viola, Arianna, Matilde, and Alice

This paper introduces the precession of the equinoxes in an elementary way and presents a formula for calculating the angle between the star Polaris and the Celestial North Pole over the years.

1 Introduction.

Let's go back in time 28 centuries when Homer described Odysseus sailing the sea after bidding farewell to Calypso. In Samuel Butler's translation ([1], Book V), we find the following words:

> *He never closed his eyes, but kept them fixed on the Pleiads, on late-setting Boötes, and on the Bear - which men also call the wain, and which turns round and round where it is, facing Orion, and alone never dipping into the stream of Oceanus - for Calypso had told him to keep this to his left.*

Why does Homer omit any reference to Polaris, the North Star, when it ranks among the 30 brightest stars illuminating the entire celestial canvas? Remarkably, Homer does depict prominent constellations like the Bear, and Boötes, the Guardian of the Bear, along with the Pleiades and Orion. Nevertheless, the conspicuous absence of Polaris in his narratives remains a puzzling enigma, given its celestial significance and brilliance.

The reason for that lack of enthusiasm is that the star we now call Polaris was not the North Star back then. It was a bright star, yes, but practically isolated, just like today, and unlike today, it described a wide circle in the sky, descending from about 55° degrees to 25° degrees above the horizon, as we will see later. Some stars never set, as the Bear "never dip into the stream of Ocean". Navigators were skilled enough to determine the position of the North by observing a few constellations. They located the hub of the celestial sphere, even if for long periods it was a virtual, obscure point, but so useful in those times as it is now with the North Star being right next to it.

If you have the patience to follow me, I will try to explain the phenomenon that underlies the significant difference between Odysseus' firmament and ours. It is known as the *"precession of the equinoxes"* and it helps us understand other facts, such as the presence, in a current star chart, of the zodiac sign of Aries on the constellation of Pisces, or the sign of Leo on the stars of Cancer. In short, the zodiac signs correspond to constellations shifted backward, a fact that did not occur in ancient times when astrology was born, which speaks volumes about the validity of horoscopes.

2 Precession.

The Earth rotates around the line connecting the poles, known as the axis of rotation, in a counterclockwise direction for an observer at the North Pole. This rotation is the reason for the alternation of day and night. Additionally, it undergoes a "revolution" around the Sun, also in a counterclockwise direction relative to the same observer, in an almost circular orbit completed in one year. The axis of rotation is tilted at approximately 23° degrees with respect to the perpendicular to the orbit, and this inclination is the cause of seasons, just as the inclination of the Euganean Hills, near Padua, allows cacti to grow on Mount Ceva, which is unusual at the latitude of 45° degrees N.

The tilt of 23° is rather stable over time. A study by Jacques Laskar and his colleagues, as referenced in [2], reveals that this stability is quite unusual among planets, and it is attributed to the presence of the Moon, which is unusually massive compared to the planet. Considering that small variations can cause ice ages, we realize how fortunate we are to be here discussing the Earth's axis of rotation. But I digress from my objective: the precession of the equinoxes.

Equinoxes are the two days of the year when the night and day are of equal duration, more precisely they are the moments in time when the Sun crosses the Earth's equator. One occurs in spring, and the other in autumn. There are two other significant days in the year for seasonal changes: the solstices. The summer solstice marks the day with the longest duration of daylight, while the winter solstice marks the day with the shortest duration. More precisely the solstices are the moments when the Sun reaches its most northerly or southerly excursion.

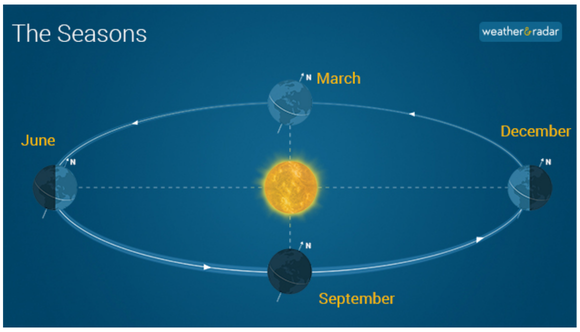

Figure 1: Equinoxes and solstices

The great scientist Hipparchus of Nicaea, comparing the positions of certain stars during the nights of seasonal changes in the 2nd century BCE with those observed in the 5th century BCE by Meton, noticed that the "celestial sphere" had slightly rotated.

The Earth's axis of rotation does not have a fixed direction in space but rotates around the line perpendicular to its orbit, describing a cone. This movement is called precession and occurs in a clockwise direction for an observer at the North Pole. A complete precession takes place in approximately 26 000 years. Let us read Lucio Russo [4], pages 315–316.

> *The main result of Hipparchus mentioned by Ptolemy is the discovery of the precession of the equinoxes. The precession is so slow that any available observational data would have given Hipparchus displacements of only a few degrees. Nevertheless, the astronomer, who is known to have been very rigorous in his use of experimental data, dared to extrapolate from a tiny arc the existence of a circular uniform motion with a period of* 26 000 *years. If his astronomy was "dynamical", any old top might have given him the idea of sifting through observational data for the existence and periodicity of precession.*

Figure 2 illustrates the precession, which describes the cone in a clockwise direction, as well as the counterclockwise rotation of the Earth, which has a much shorter period: one day.

When we refer to the "celestial sphere", we are talking about the positions of the stars regardless of their distance from Earth. In other words, if we fix a point on the Earth's surface defined by latitude and longitude, we consider the half-line extending from the center of the Earth through that point and assign the same latitude and longitude to any star that lies on that half-line *at a given moment*. Notice that the celestial latitude and longitude rotate rigidly with the Earth and their values change continuously for any given star.

Similarly, the planets and the Sun also have positions on the celestial sphere, which can overlap with the positions of the stars. The stars are

Figure 2: Counterclockwise rotation and clockwise precession

grouped into constellations, which are sets of stars in a relatively small area of the celestial sphere, but they can vary greatly in distance from Earth.

Since ancient times, constellations have been defined by their distinctive shapes, which have sparked the imagination of our ancestors, who saw mythological figures within them. Among the constellations, the zodiacal ones hold particular importance. These are the constellations that overlap with the Sun's apparent annual motion, which is actually due to the Earth's revolution.

Precession causes the positions of the stars to vary slightly on the celestial sphere after one year, although this change is extremely slow. The position of the Sun also differs slightly. The Sun passes through all 12 constellations of the zodiac on the same day of the year, let's say the spring equinox, over the course of 26 000 years. Therefore, the zodiac shifts by one constellation every 26 000/12 years, which is approximately 2166 years. Going back in time by such a period brings us to the time when the zodiac signs were associated with the constellations.

Today, for example, we refer to the sign of Aries while the Sun passes in front of Pisces.

Figure 3: Alignments Earth-Sun-Zodiac constellation

Figure 3 shows the Earth on the left, the Sun, and the signs of Pisces and Aries. The top alignment represents the astronomical alignment of Pisces, which occurs approximately from March 12th to April 18th. However, the zodiac sign from March 21st to April 19th is Aries. This alignment is based on ancient times and forms the basis of horoscopes, one of the many manifestations of superstition that changes its forms but has never abandoned its influence on humanity.

Naturally, the most noticeable movement of stars in the celestial sphere is due to the rotation of the Earth. For an observer at the North Pole, all the stars circle around the celestial North Pole, which is directly above the observer and located vertically. No star rises or sets in this case. Additionally, only the stars of the northern celestial hemisphere are visible from the North Pole.

At the equator, an observer instead has the celestial equator on the vertical, the celestial North Pole on the northern horizon and the South Pole on the southern horizon. Both celestial hemispheres are visible, and all the stars rise in the east and set in the west.

We have discussed the situations at a latitude of 0°, the equator, and at a latitude of 90° N, the North Pole. At an intermediate latitude, let's say 40° N, the celestial North Pole is 40 degrees above the northern horizon, as shown in Figure 4.

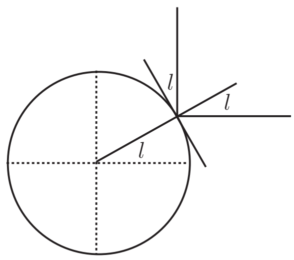

Figure 4: Latitude

All the stars that are located within 40° of the celestial North Pole are always above the horizon. In the words of Homer, they "never dip into the stream of Ocean". The other stars rise in the east and set in the west, just like the Sun. I chose 40° N because it is near the latitude where a portion of Odysseus' journey takes place.

3 Deducing the trigonometric formula.

The celestial North Pole will be at its closest point to the North Star around 2100 (more precisely in Feb 2102, see [3]). Since, at that time, it will be within half a degree, we can approximate the latitude of Polaris in 2100 as 90° N. That is consistent with the approximation we made by choosing the round value 23° for the inclination of the Earth's axis. The following formula gives the angle $f(n)$ between the celestial North Pole and the position of

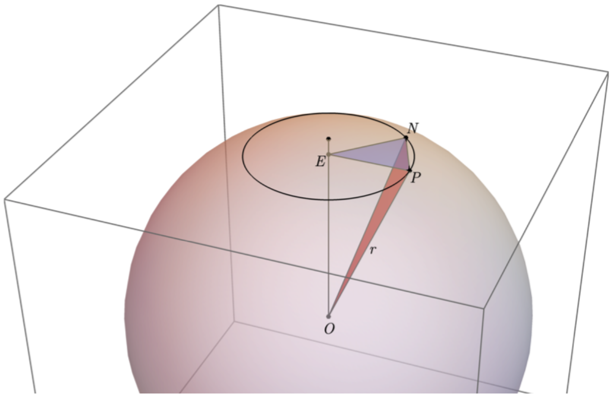

Figure 5: Deducing the trigonometric formula

Polaris in the year $n = 1, 2, \ldots$:

$$\sin \frac{f(n)}{2} = \sin 23^\circ \times \sin \frac{|2100 - n|\, 180^\circ}{26\,000} .$$

In Figure 5, we observe the celestial sphere with a non-important radius $r > 0$. The point P represents Polaris, while the center of the sphere O is the Earth. The celestial North Pole in the year n is N while E is the center of the circumference traced by N in the precession. The length of half of the segment PN is determined by $r \sin(f(n)/2)$. This length is calculated using the triangle OPN. On the other hand, the length is equal to r times the right-hand side of the formula mentioned above, if it is computed on triangle EPN.

There has never been a year 0 because years are numbered similarly to centuries (even though we don't say we are in the 2024th year), and before the year 1 AD, there is the year 1 BC. However, the formula also holds for $n = 0$, which corresponds to the year 1 BC, and for any $n < 0$, representing the year $-n + 1$ BC. In this paper, the Odyssey "corresponds" to $n = -800$ (approximate time of the narration of mythological events that occurred several centuries earlier). Some values derived from the formula are as follows:

$$f(-800) = 15.4^\circ, \quad f(0) = 11.2^\circ, \quad f(500) = 8.6^\circ,$$
$$f(1000) = 5.9^\circ, \quad f(1500) = 3.2^\circ, \quad f(1800) = 1.6^\circ,$$
$$f(2024) = 0.4^\circ, \quad f(2100) = 0^\circ, \quad f(15\,100) = 46^\circ.$$

Homer therefore observed the star we call Polaris following a circular orbit around the celestial North Pole with a radius of approximately 15°. Thus, we have found the values mentioned in the Introduction: Polaris descended from $40^\circ + 15^\circ = 55^\circ$ to $40^\circ - 15^\circ = 25^\circ$ on the horizon.

To conclude with a question: Will Polaris ever set on the horizon of the reader's town?

Acknowledgements

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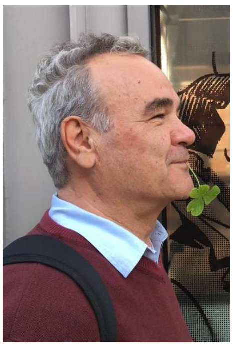

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