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2.1 GeV/nucleon ^{12}C , ^{16}O AND M GROUP ($6 \leq Z \leq 9$) COSMIC
HEAVY IONS IN NUCLEAR EMULSIONS**

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RESUMEN.—Se han obtenido los recorridos libres medios de interacción en emulsiones nucleares de los iones carbono y oxígeno de energía 2,1 GeV/nucleón suministrados por el Bevatrón del L. B. L. de Berkeley (California, USA), y del grupo M ($6 \leq Z \leq 9$) de la radiación cósmica utilizando varios apilamientos de emulsiones nucleares. Los resultados obtenidos ($56,7 \pm 3,8$, $53,6 \pm 2,7$ y $53,6 \pm 7,7$ g/cm² para los iones ^{12}C , ^{16}O y grupo M, respectivamente) se comparan con los previstos por un modelo geométrico simple.

SUMMARY.—The interaction mean free path lengths of 2.1 GeV/nucleon Carbon and Oxygen heavy ions accelerated at the Bevatron (L. B. L.) and M group ($6 \leq Z \leq 9$) cosmic heavy ions have been obtained by exposures of several nuclear emulsion stacks. These results (56.7 ± 3.8 , 53.6 ± 2.7 and 53.6 ± 7.7 g/cm² for ^{12}C , ^{16}O and M group, respectively) are compared with a simple geometrical model.

INTRODUCTION

Until recent times the only possibility to get informations on the mean free path lengths of energetic heavy ions ($Z > 2$) was the study of the Cosmic Radiation, in spite of the difficulties and limitations due to the charge and energy indetermination. Another possibility is now open with accelerated heavy ions of known charge and energy with a high statistics, having energies comparable with those of the Cosmic Radiation.

EXPERIMENTAL

In June 1972, two stacks of Ilford K5 and G5 nuclear emulsions were exposed to ^{12}C and ^{16}O beams of 2.1 GeV/nucleon. The beam intensity, measured by two scintillation counters placed in front of the stacks, was of 10^5 particles/cm² approximately. The size of the nuclear emulsions is $3.5 \times 10.5 \times 0.06$ cm³, and the beam normally incident to one of the 3.5×0.06 cm² sides. The scanning was carried

out in 7 plates for carbon and 6 for oxygen. The area scanned consisted in a section normal to the beam equidistant from the 3.5×10.5 cm² surfaces, and at 2 mm from the beam entrance side, to avoid distortion effects. The height of the section scanned was 50 μm and its width covered a zone ranging between the maximum flux and that where it drop below 100 tracks/mm². The tracks found were followed along the plates until they interact or leave them. We consider an interaction any process which resulted in a formation of a star (with a minimum of one heavily ionising particle emitted). So that the interaction mean free paths measured are the inelastic ones. In table I we show the scanning results.

TABLE I

Scanning results

	Number of tracks followed	Number of interactions found	Total length followed (m)
^{12}C	458	203	30.0
^{16}O	913	424	59.4

A stack of nuclear emulsions Ilford K5 and K2 diluted of a total volume of ~ 3 litres has been exposed to the Cosmic Radiation during a 12 hours stratospheric balloon flight at 44° geomagnetic latitude and 35 km altitude. The scanning has been performed along the edges of the K5-

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and K2 nuclear emulsions, and the tracks of heavy ions found were followed throughout the stack. The charge discrimination was made using photometric measurements on each track (1), giving us a total of 206 tracks mainly belonging to the M group.

RESULTS

In table II the mean free paths values obtained for the accelerated and cosmic heavy ions are shown.

A Monte Carlo method has been used to find the best conditions to obtain a value of the mean free path λ of the same order of magnitude of the experimentally found, but with an error $< 5\%$. In these conditions we should follow a minimum of 700 tracks, which should find approximately 400 interactions.

the table II, and are in good agreement with our experimental results.

Considering insignificant the contribution of fluorine to the M group ($\leq 4\%$) we can calculate a mean value of the interaction mean free path $\langle \lambda_M \rangle$ at 2.1 GeV/nucleon for the M group using

$$\langle \lambda_M \rangle = \left(\frac{\sum_{i=1,3} \lambda_i N_i}{\sum_{i=1,3} N_i} \right) \quad [4]$$

λ_i being the corresponding interaction path length of each ion, and

N_i the relative abundancies taken as 44:21:31 from Ginzburg and Syrovatskii (12).

In table II we show the values obtained by us, as well as the average value obtained by several authors (2-8). We have also calculated, using expressions [1],

TABLE II

Mean free path lengths for energetic M group ions in nuclear emulsion

		Mean free path lengths (g/cm ²)		
		overlap model	non-overlap model	experimental values
Accelerated 2.1 GeV/ nucleon heavy ions	¹² C	59.3	58.2	56.7 ± 3.8
	¹⁴ N	56.3	55.9	55.1 ± 2.3 (9)
	¹⁶ O	54.0	54.0	53.6 ± 2.7
	M group (calculated)	55.9	55.5	55.5 ± 3.1
M group Cosmic Ray heavy ions	Our value			53.6 ± 7.7
	Mean values from references 2 to 8			54.0 ± 3.8

As comparison we have calculated the mean free paths using the relation

$$\lambda = 1 / \sum_i n_i \sigma_i \quad [1]$$

n_i being the number of nuclei of class i in the nuclear emulsion per volume unit, and

σ_i the corresponding cross section, given by

$$\sigma_i = \pi (R_i + R - 2 \Delta R)^2 \quad [2]$$

in the so-called «overlap» model of Bradt and Peters (10), or by

$$\sigma_i = \pi (R_i + R)^2 \quad [3]$$

for the «non-overlap» model of Daniel and Durgaprasad (11). R_i and R are the nuclear radius of the incident ion and the target nucleus, respectively; $\Delta R = 0.85 \cdot 10^{-13}$ cm is the overlapping factor found by Bradt and Peters. The nuclear radii are obtained by $R = r_0 A^{1/3}$, being A the mass number and r_0 the nucleon radius, taken as $1.45 \cdot 10^{-13}$ cm for the overlap and $1.17 \cdot 10^{-13}$ cm for the non-overlap models. The values obtained by these methods are shown in

[2] and [3], the theoretical values of the interaction mean free path lengths for the M group, using in [2] and [3] a mean value of these which take into account the three ion contributions, using the expression $\langle R \rangle = r_0 \langle A \rangle^{1/3}$ being $\langle A \rangle = (\sum_{i=1,3} M_i A_i) / (\sum_{i=1,3} M_i)$, where M_i are the number of tracks followed (see first column of table I for ¹²C and ¹⁶O, and 508 taken from reference (9) for ¹⁴N).

DISCUSSION

The values obtained for the interaction mean free path lengths of 2.1 GeV/nucleon ¹²C, ¹⁴N and ¹⁶O and the M group of the Cosmic Radiation are in good agreement. Furthermore, the values predicted by the geometrical cross section model are very close to the experimental ones.

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NOTAS

A NOTE ON THE SCALAR K_{l3} FORM FACTOR

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RESUMEN.—Se estudia el factor de forma $f_0(s)$ de K_{l3} utilizando simetría quiral. Se encuentra que la pendiente es positiva en la región $[0, (m_K + m_\pi)^2]$ y mayor que 0,12 en $s = 0$.

SUMMARY.— $f_0(s)$ is examined in the context of chiral symmetry. It is found that its slope is positive in the region $[0, (m_K + m_\pi)^2]$, and larger than 0.12 at $s = 0$.

The analysis realized in a recent article on 0^+ properties (1) gives also information on K_{l3} form factors. A value of +.016 is obtained from the slope λ_0 at s equal to zero, of the scalar form factor (*). Experimental measurements of the X-2 collaboration have found negative values for this parameter (2). It has been argued that a possible explanation would arise from the consideration of zeros of the scalar form factor. A simple «ansatz» like $(a+bs)/(s-m^2)$ could produce the convenient effect. This possibility has been discussed in detail by K. Kang (3). Another interesting suggestion was pointed out by H. Banerjee (4). His argument relies upon the use of the Fubini-

Furlan soft pion extrapolation methods. One can summarize it as follows. If one assumes a parabolic form for $f_0(s)$ in the region $\{0, (m_K + m_\pi)^2\}$ one can realize both negative λ_0 slope, in agreement with X-2 results, and the Callan-Treiman relation by employing a set of reasonable values, albeit ad hoc, for the parameters c , $\lambda = f_K/f_\pi$, $\xi = f_K/f_\pi$. Banerjee has taken $c = -1.35$, $\xi = .5$, $\sqrt{2}f_+(0) = 1$ and the kappa parameters given by Loveplace (5). $f_0(s)$ has a turnover behaviour changing slope from negative in the physical region to positive.

The model of reference (1) determines a unique set of values for c , λ , ξ , $\sqrt{2}f_+(0)$,... and the slope λ_0 as functions of the kappa parameters. Furthermore the slope of $f_0(s)$ cannot change of sign. The above quoted experimental result and Banerjee's explanation suggest us to consider less restrictive models and look

(*) The notation is the same as in reference (1). The scalar form factor $f_0(s)$ is defined as

$$d^{n^* K^*}(s)/(m_K^2 - m_\pi^2) = f_+(0) \left(1 + \frac{\lambda_0 s}{m_\pi^2} + 0(s^2) \right)$$

for the predicted λ_o values. The most attractive possibility would be to loosen the «quadratic smoothness» condition for the vertex $\langle \pi | D | K \rangle$ allowing the residuus to be a second degree polynomial («quartic smoothness»). That means $f_o(s)$ has a form

$$\sqrt{2}f_o(s) = \frac{\sqrt{2}Gf_k}{m_k^2 - m_\pi^2} \frac{s}{s - m_k^2} + \sqrt{2}f_o(0) + \frac{\sqrt{2}Q}{m_k^2 - m_\pi^2} s \quad [1]$$

Formulas for $f_+(0)$ and c of reference (1) are not modified. Formula (3-18) turns out to be

$$\begin{aligned} -4f_{\pi^0}\xi\lambda G = & -Y^{1/2} - (m_k^2\xi^2 + m_k^2\lambda^2 - m_\pi^2) + \\ & + (m_k^2 - m_\pi^2)(\lambda^2 + 1 - \xi^2) + m_k^2(\xi^2 + \lambda^2 - 1) + \\ & + 2\sqrt{2}Q\lambda(-m_k^2 + m_\pi^2 + m_k^2) \quad [2] \end{aligned}$$

It is important to realize that the Glashow-Weinberg selfconsistency bound $m_k < |m_k f_k - m_\pi f_\pi| / |f_k|$ holds. This relation gives allowed bands for λ and ξ therefore constraints, via eq. [2], the possible Q values. For instance λ is restricted to the zone ($\approx 1.16, 1.23$) for $m_k = 1.1$ GeV; the zone is ($\approx 1.12, 1.23$) for $m_k = .9$ GeV.

Typical values of Q and λ_o have been collected in tables I and II. In second and third columns $G\xi$ is chosen to be negative [as in reference (1)]; fourth and fifth columns are for $G\xi > 0$. Table I makes reference to $m_k = .9$ GeV and $\Gamma = .2$ GeV [reference (5)] whereas in table II $m_k = 1.1$ GeV, $\Gamma = .45$ GeV. In fact allowed Q values give always positive λ_o values. This result is general whatever the kappa parameters be. Numerically λ_o is larger than $+ .012$.

Models giving negative λ_o appear to be difficult to realize whereas «normal» positive values are a common feature to many models.

TABLE I

Q, λ_o values as function of $\lambda = f_k/f_\pi$
 $m_k = .9$ GeV and width .200 GeV

$\lambda = \frac{f_k}{f_\pi}$	Q	λ_o	Q	λ_o
1.16	.018	.018	.357	.027
1.18	.016	.016	.322	.024
1.20	.032	.016	.300	.022
1.23	.067	.016	.263	.021

TABLE II

Q, λ_o values as function of $\lambda = f_k/f_\pi$
 $m_k = 1.1$ GeV and width .450 GeV

$\lambda = \frac{f_k}{f_\pi}$	Q	λ_o	Q	λ_o
1.12	-.040	.015	.458	.036
1.16	-.041	.012	.385	.029
1.20	-.001	.013	.335	.025
1.23	+.042	.014	.290	.023

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