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The $\Delta S = \Delta Q$ Rule

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1. The origins of the rule

The $\Delta S = \Delta Q$ rule was first proposed in 1958 by Feynman and Gell-Mann in order to explain the absence of certain weak transitions which would have led to hyperon decay modes other than those which had been observed. In the framework of their current-current theory of weak interactions, they found it necessary to limit the number of possible currents. In particular, for currents causing a change in strangeness (ΔS) between initial and final hadrons, the charge (ΔQ) was to have the same sign, hence the name $\Delta S = \Delta Q$. This current would therefore have the same quantum numbers as the $K^{\pm}$, in particular isospin $I = \frac{1}{2}$. It then followed that if the $\Delta S = \Delta Q$ rule was good, the $|\Delta I| = \frac{1}{2}$ rule, which is less restrictive, would also be valid.

All this phenomenology was put on a more solid theoretical base with the theory of Cabibbo who proposed in 1963 that all weak hadronic currents transformed like the charged elements of an octet representation of the group SU(3). A violation of the $\Delta S = \Delta Q$ rule would demonstrate the existence of weak currents belonging to higher multiplets.

The $\Delta S = \Delta Q$ rule has therefore attained a position of fundamental importance in weak interaction theory.

2. Experimental predictions of the rule

The most direct way of studying the $\Delta S = \Delta Q$ rule experimentally, is to look for the following strange particle decays, which are allowed:

$\Sigma^- \rightarrow n\ell^- \nu$

$K^+ \rightarrow \pi^+ \pi^- e^+ \nu$

$K^0 \rightarrow \pi^- \ell^+ \nu$

$\bar{K}^0 \rightarrow \pi^+ \ell^- \nu$

and the following decays which are forbidden by the rule:

$\Sigma^+ \rightarrow n\ell^+ \nu$

$K^+ \rightarrow \pi^+ \pi^+ e^- \nu$

$K^0 \rightarrow \pi^+ \ell^- \nu$

$\bar{K}^0 \rightarrow \pi^- \ell^+ \nu$

where ℓ is an electron or muon.

For the charged particle decays above, it is sufficient to find a single "forbidden" event to show that the rule is not exact.

For the K^0 decays, the intrinsic mixing of K^0 and $\bar{K}^0$ states makes the situation much more complicated and at first sight more difficult since one must study not only the number of decays of different types, but also their distribution in proper time from the K^0 or $\bar{K}^0$ production. In fact, it is just this problem which makes the K^0 case more interesting than the charged particle decays, for the following reasons:

(a) The possibility of interference between the amplitudes $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ makes K^0 decays more sensitive to a small violation of the rule.

(b) Using K^0, one can measure not only the amplitude $\Delta S = -\Delta Q$, but also its phase relative to the amplitude $\Delta S = \Delta Q$, which is a test of CP violation.

If one defines

$$x = \frac{\text{Amplitude } (\Delta S = -\Delta Q)}{\text{Amplitude } (\Delta S = \Delta Q)}$$

and if one starts with a pure K^o ($S = +1$) at the time $t = o$, then the relative probability of observing a decay into $\pi^{\pm} \ell^{\mp} \nu$ at time t is given by:

$$N^{\pm}(t, x) = |1+x|^2 e^{-\lambda_s t} + |1-x|^2 e^{-\lambda_L t}$$
$$\pm\ 2 \cos \delta t\ (1-|x|^2)\ e^{-\Lambda t} - 4\ \text{Im}\ (x) \sin \delta t\ e^{-\Lambda t}$$

where

$$\lambda_s = {}^1/\Gamma_s \approx 1.16 \times 10^{10} \text{ sec}^{-1}$$

$$\lambda_L = {}^1/\Gamma_L \approx 1.86 \times 10^{7} \text{ sec}^{-1}$$

$$\Lambda = (\lambda_s + \lambda_L)/2$$

$$\delta = M(K_L) - M(K_s) \approx +\ 0.46\ \lambda_s$$

This formula assumes CPT (but not CP) invariance. Smaller CP-violating terms, known to be of order $\epsilon \sim 10^{-3}$ compared with the leading terms, have been neglected. If the initial state is $\bar{K}^o$ instead of K^o, the same formula holds, but the sign of the last two terms is inverted.

There are several arbitrary choices of sign involved in this equation, which determine the sign of Im (x), namely,

(a) the sign of the exponential in the Schrödinger equation;

(b) the definition of K_L^o in terms of K^o and $\bar{K}^o$;

(c) the definition of δ;

(d) the actual sign of δ (not arbitary).

This has caused some confusion in combining different experimental results, especially in the early days of the rule when (d) was not known (which made the other choices irrelevant). After some hesitation, I believe that all investigators now use the same conventions, which lead to the signs as given in the formula above.

3. The expected violation

Any possible violation of the $\Delta S = \Delta Q$ rule is measured by the quantity x defined above. If the rule is exact, $x = 0$. If CP (and CPT) are conserved in the decay then Im $(x) = 0$, even if the rule is not valid.

Beyond these rather simple predictions, one might well ask what amount of violation would be expected on general theoretical grounds. There are, for example, many selection rules known to be approximate, such as isospin conservation in strong interactions which is violated to order $\alpha \approx 1\%$ because of electromagnetic effects. But a corresponding violation of the $\Delta S = \Delta Q$ rule would have to be due to second order weak interactions which are many orders of magnitude too weak to be observed in this way. Such considerations lead to the conclusion that the rule, if it is at all valid, should be quite exact.

Sachs[1] has suggested that the surprisingly small CP - violation observed in $K_L^o \rightarrow 2\pi$ decays might in fact be a manifestation of a large CP violation in a less important decay mode, namely the leptonic mode $K^o \rightarrow \pi \ell \nu$. He has pointed out that the 2π violation could be accounted for by assuming "maximal" CP - violation in leptonic decays with Im $(x) \approx \pm 1$, Re $(x) = 0$.

Finally, it should be noted that if x is not zero, it might also not be constant. In particular:

(a) The value of x could be different for muonic and electronic decays. This would be an interesting direct test of μ-e universality. Experimentally, however, muonic decays are more difficult to detect than electronic decays, and as we shall see, most experiments observe only electrons.

(b) If $x \neq 0$, there is no reason for it to be constant over the decay Dalitz plot. This brings up the question of $K_{\ell 3}^o$ form factors which are already difficult to measure with K_L^o beams; to measure the time dependence of these form factors does not appear within reach of current experimental techniques.

4. The experimental situation for charged decays

There are no new results on the $\Delta S = \Delta Q$ rule in charged decays, so the situation remains that there is no evidence for a violation here.

The current limit on the violation in $\Sigma^{\pm}$ decays is

$$\frac{\Gamma(\Sigma^{+} \rightarrow n e^{+} \nu)}{\Gamma(\Sigma^{-} \rightarrow n e^{-} \nu)} \leq 0.01$$

and

$$\frac{\Gamma(\Sigma^{+} \rightarrow n \mu^{+} \nu)}{\Gamma(\Sigma^{-} \rightarrow n \mu^{-} \nu)} \leq 0.05$$

with 90% confidence. The three possible "violating" events seen so far are consistent with the expected background. (2)

The current limit on the violation in K^{+} decays is

$$\frac{\Gamma(K^{+} \rightarrow \pi^{+} \pi^{+} e^{-} \nu)}{\Gamma(K^{+} \rightarrow \pi^{+} \pi^{-} e^{+} \nu)} \leq 0.02$$

with 90% confidence, with no "violating" events found, and 269 "non-violating" events observed. (3) However it must be pointed out that this is not as sensitive to the $\Delta S = \Delta Q$ rule as it might seem, since the $\pi^{+} \pi^{+}$ decay is anyway suppressed compared with the $\pi^{+} \pi^{-}$ decay because the $\pi^{+} \pi^{+}$ can only be in a state of $I = 2$.

A Saclay-Geneva group has recently completed and will shortly publish results on an improved K^{+}_{e4} experiment where about 2000 events are observed. The same group is also proposing a new experiment where they expect to have 20 000 K_{e4} decays.

One can therefore look forward to some real progress in measuring the $\Delta S = - \Delta Q$ amplitude for $K^{\pm}$ decays, although it does not seem that current techniques can be stretched much further for the $\Sigma^{\pm}$ case.

5. K^o leptonic decays 1960-1969

The first experiment to test the $\Delta S = \Delta Q$ rule in K^o decays was carried out by a Berkeley-Padova-Wisconsin group [(4)] in a small propane bubble chamber. Their results, based on 28 events, were published in 1962 when CP-violation had not been discovered and the $K_L - K_S$ mass difference was not known, so that their analysis was necessarily somewhat different from the more recent approach. Nevertheless, they do have one important thing in common with later experimenters in this field, namely a misguided belief that certain statistical techniques such as the likelihood ratio behave the same for small samples as for large samples and do not depend on the fact that parameters have been estimated from the data. This helped them to make the rather strong statement: "We therefore conclude that the $\Delta Q = \Delta S$ selection rule is not valid".

This statement naturally triggered a series of experiments on leptonic decays. A variety of techniques were used, namely heavy liquid, deuterium, and hydrogen bubble chambers as well as spark chambers. The results of this "second generation" of experiments were summarized by J. Cronin at the 1968 Vienna Conference [(5)] and are presented in Table I. Although the large violation suggested by the first experiment seemed to be excluded by the average of the others, this average was more than two standard deviations from zero and so suggested strongly that the rule was violated, but by a smaller amount. Still people did not take these results too seriously, partly because they were not very compatible with each other and partly because of a related experiment described below.

It turns out[(14)] that information on the $\Delta S = \Delta Q$ rule can also be obtained by measuring the charge asymmetry in leptonic decays of a long-lived K^o beam. The argument is quite complicated and the results, unfortunately, depend on the knowledge of several auxiliary parameters, the most important being the energy-dependent forward scattering amplitudes of both K and $\bar{K}$ on nuclei. In addition, the experiment cannot measure both the real and imaginary parts of x, but only the combination

TABLE 1

Test of $\Delta Q = \Delta S$ in $K^o_{\ell 3}$ decays (1969)

Group	Method	No. of $K_{\ell 3}$ events	Re (x)	Im (x)
Berkeley/Padova/ Wisconsin(4)	PropaneBC $K^+n \rightarrow K^o p$	28	$0.55 \begin{smallmatrix} + 0.08 \\ - 0.12 \end{smallmatrix}$	--
Paris(6)	Freon/Prop.BC $K^+n \rightarrow K^o p$	315	$0.035 \begin{smallmatrix} + 0.11 \\ - 0.30 \end{smallmatrix}$	$-0.21 \begin{smallmatrix} + 0.15 \\ - 0.11 \end{smallmatrix}$
Padua(7)	Freon/Prop.BC $K^+n \rightarrow K^o p$	152	$0.06 \begin{smallmatrix} + 0.18 \\ - 0.44 \end{smallmatrix}$	$-0.44 \begin{smallmatrix} + 0.32 \\ - 0.19 \end{smallmatrix}$
Columbia/Rutgers(8)	HBC $\bar{p}p$	109 μ, e	$-0.08 \begin{smallmatrix} + 0.16 \\ - 0.28 \end{smallmatrix}$	$+0.24 \begin{smallmatrix} + 0.40 \\ - 0.30 \end{smallmatrix}$
Pensylvania(9)	Spark Chamber $\pi^- p \rightarrow \Lambda^o K^o$	116	$0.17 \begin{smallmatrix} + 0.16 \\ - 0.35 \end{smallmatrix}$	0.0 ± 0.25
Brookh./Carnegie(10)	D_2 BC $K^+n \rightarrow K^o p$	335	0.17 ± 0.10	-0.20 ± 0.10
Berkeley(11)	HBC $K^- p \rightarrow \bar{K}^o n$	242 μ, e	$0.22 \begin{smallmatrix} + 0.07 \\ - 0.09 \end{smallmatrix}$	-0.08 ± 0.08
CERN/Paris(12)	HBC $\bar{p}p$	121 μ, e	$0.09 \begin{smallmatrix} + 0.13 \\ - 0.11 \end{smallmatrix}$	$+0.22 \begin{smallmatrix} + 0.29 \\ - 0.37 \end{smallmatrix}$
La Jolla(15)	Spark Chamber $K^+n \rightarrow K^o p$	686	$0.09 \begin{smallmatrix} + 0.14 \\ - 0.16 \end{smallmatrix}$	$-0.11 \begin{smallmatrix} + 0.10 \\ - 0.11 \end{smallmatrix}$
Average			$+0.11 \pm 0.04$	-0.08 ± 0.04
CERN/Columbia(13)	Counter exp. K^o_L beam	?	$\frac{1-\lvert x\rvert^2}{\lvert 1+x\rvert^2} = 0.96 \pm 0.05$	

$$X = \frac{1 - |x|^2}{|1 + x|^2} \approx 1 - 2\,\mathrm{Re}(x) \text{ if } x \approx 0.$$

However, the statistical accuracy obtainable in such an experiment is superior to anything that had been achieved at the time. This experiment did not indicate any violation(13) and did not agree well with previous experiments. (In this regard, note that Fig. 2 of Ref(14), presenting previous results, has inverted scales, and all the points are plotted incorrectly. These mistakes have also been transmitted to some later review articles).

6. K^o experimental difficulties

Experiments to test the $\Delta S = \Delta Q$ rule in K^o decays are probably among the most difficult in high energy physics, a fact too little appreciated in designing the "second-generation" experiments. In fact it is worthwile looking in some detail into the experimental problems posed, and seeing how the "third-generation" experiments (described in the next section) represent improvements over earlier attempts.

Let us consider a typical experiment as represented schematically in figure 1. One needs in general a K^o produced at a known point

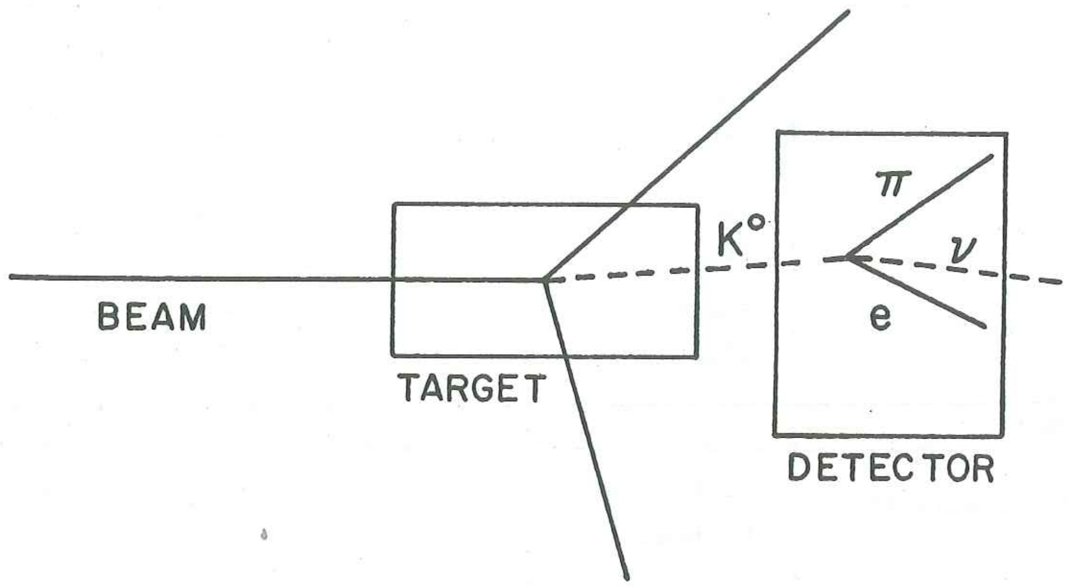

Fig. 1.

with a known momentum, preferably in a known reaction. The leptonic decay must then be detected with a known efficiency and must be separable from other (especially $\pi^+ \pi^-$) decay modes. Since the neutrino is never detected, both the decay pion and lepton must be measured and identified, and even then there are no kinematic constraints unless the K^o momentum is known from the production reaction.

In designing a particular experiment, the first problem is to get enough events. Any of several production reactions may be chosen, but they all have cross sections of about five to ten millibarns at most. Only about 1% of the K^o produced will decay leptonically in the first few K^o_s lifetimes, and usually not all of these will be detected in the apparatus, so one is always limited by a low event rate.

The second problem is that of identification of events and unbiased measurement of K^o decay length and momentum. Although the identification of the lepton charge and K^o strangeness for each event is not absolutely necessary, it enhances greatly the statistical value of the sample, and in any case one must be sure that they are in fact leptonic decays.

Probably the most delicate problem is that of elimination (or evaluation) of background from competing decay modes and production reactions. When searching for decays with small branching ratios, most of what is happening inside the apparatus is of no interest and some of it may look like the desired mode.

From a consideration of the above problems, we may establish a list of desired properties of a K^o experiment to test $\Delta S = \Delta Q$:

A. Rate

(1) an intense beam

(2) a long or dense target

(3) a large detector, very near to target

B. Identification

(4) a known production reaction (on H_2 or D_2)

(5) identified strangeness of K^o

(6) identified lepton charge

(7) well-measured decay products

(8) decay length and K^o momentum known

C. Elimination of background

(9) $K^o \rightarrow \pi^+ \pi^-$

(10) $K^o \rightarrow \pi^+ \pi^- \gamma$

(11) $K^o \rightarrow e^+ e^- \gamma \pi^o$ (Dalitz pairs)

(12) wrong production reaction

Many of these properties are mutually exclusive, so that it is always a question of balancing some good points against some not-so-good ones. For example, it is impossible to combine points 2, 3 and 4 in an electronics experiment since all of a large detector cannot be close to all of a long target. This is possible in a bubble chamber, where the target is the detector, but it is then not possible to have an intense beam. Similarly, lepton identification is good in a heavy liquid bubble chamber where electrons spiralize, but then the production reaction kinematics cannot be used because of the heavy nuclei. In a hydrogen bubble chamber the production reaction is usually well known and momentum measurement is good, but lepton identification is more difficult and contamination from $\pi^+ \pi^-$ decays is a problem. For electronics experiments that identify the electron with a Cerenkov counter, the Dalitz pair decays make a serious background, and the detection efficiency may be small and difficult to calculate accurately as a function of time.

7. Recent experimental results

Within the last year, five new experiments have yielded at least preliminary results on the $\Delta S = \Delta Q$ rule, with a considerable improvement over previous experiments, both from the point of view of statistics and clear identification of events.

A Padova-Wisconsin group[16], including many of the people involved in the very first $\Delta S = \Delta Q$ experiment[4], has done a new study in the Argonne high-field (47KG) heavy liquid bubble chamber. Their preliminary results based on 380 events show no evidence for a violation.

An Illinois-Northwestern group[17] has performed a spark chamber experiment, also at Argonne, using a π^- beam on a carbon target giving $K^o \Lambda^o$. The electron identification was by shower chambers, and both the K^o and Λ^o decay were observed in spark chambers in a large magnet. They also observe no violation, based on 400 events in the first 6 lifetimes.

A new hydrogen bubble chamber experiment is being performed by a CERN-Saclay-Oslo group[18], including people involved in two previous $\Delta S = \Delta Q$ experiments. They are using the reaction $K^+p \rightarrow K^o p \pi^+$ at around 1.5 GeV/c, which allows all tracks and points to be measured with great accuracy. The K^o decay products are usually slow and only electrons identifiable by ionization are used, although nearly all events are also unambiguous kinematically. Their preliminary results also show no evidence for a violation.

The same production reaction as in the above experiment is being used in a very high statistics wire spark chamber experiment by the CERN-Orsay-Vienna group. Although nothing has as yet appeared in print about this experiment, the group has announced a preliminary result with considerably smaller errors than previous experiments, and in agreement with the $\Delta S = \Delta Q$ rule.

And finally, a group from the California Institute of Technology has also performed a high statistics spark chamber experiment[19], this one using a π^- beam on two brass targets each followed by a lead gamma converter and veto counter to select $K^o \Lambda^o$ events. Both shower chambers and Cerenkov counters were used for electron identification. A curious feature of this experiment is that it does not attempt to measure the K^o momentum, taking for each event only the average value from a very wide (but known) spectrum. The decay distance is however measured and it seems that this is indeed sufficient to establish the usual test of the $\Delta S = \Delta Q$ rule, with only a small loss in statistical efficiency. They are able to make some nice internal checks on their data. In particular they can make their final fit ignoring the charge of the electron so that they are not sensitive to charge bias, and they can fit using only the charge asymmetry as a function of time so that they are not sensitive to a bias in overall detection efficiency. Their results are also consistent with no violation, although it is the only experiment to report a substantially negative real part of X, two standard deviations below zero.

The results from all these recent experiments are given in Table 2. Note that three of them are preliminary results, based on only part of the data, so that more improvement can be expected. In addition, another high statistics experiment has been performed at Rutherford Laboratory and is expected to give results soon.

8. Conclusions

Figure 2 shows all the results to date on the $\Delta S = \Delta Q$ rule in K^0 decays, plotted in the complex X plane. Point E is the average of all the older results given in Table 1, and shows that there was some evidence for a small violation at that time. It is clear that the addition of the more recent points changes the situation considerably. The points are:

A. Caltech, Ref (19)
B. CERN/Saclay/Oslo, Ref (18)
C. Padova/Wisconsin, Ref (16)
D. CERN/Orsay/Vienna, not published
F. Illinois/Northwestern, Ref (17)

and the shaded band is the result of Ref (13). The usual weighted average of all results to date (taking account of asymmetric errors, but not of correlations) gives:

$$Re(X) = 0.021 \begin{array}{l} + 0.021 \\ - 0.022 \end{array} \quad (S = 1.4)$$

$$Im(X) = 0.003 \pm 0.021 \quad (S = 1.2)$$

TABLE 2

Recent results on $\Delta Q = \Delta S$ in $K^o_{\ell 3}$ decays

Group	Method	No. of $K_{\ell 3}$ events	Re (x)	Im (x)
Padova/Wisconsin(16)	FreonBC $K^+n \to K^op$	380 (prelim)	$+0.1 \begin{smallmatrix} +0.10 \\ -0.08 \end{smallmatrix}$	$+0.1 \begin{smallmatrix} +0.07 \\ -0.12 \end{smallmatrix}$
Illinois/Northw. (17)	Spark Chambers $\pi^-p \to K^o\Lambda^o$	400	$+0.06 \pm 0.12$	-0.15 ± 0.12
CERN/Saclay/ Oslo(18)	H_2BC $K^+p \to K^op\pi^+$	142 (prelim)	$+0.06 \pm 0.10$	$+0.10 \begin{smallmatrix} +0.12 \\ -0.10 \end{smallmatrix}$
CERN/Vienna Orsay	Wire Spark Chamber $K^+p \to K^op\pi^+$	5000? (prelim)	$0.050^{+0.055}_{-0.065}$	0.01 ± 0.02
Caltech(19)	Spark Chamber $\pi^-p \to K^o\Lambda^o$	1079	-0.069 ± 0.036	$0.108 \begin{smallmatrix} +0.092 \\ -0.074 \end{smallmatrix}$

where the errors include the scale factors shown. Without scale factors, the X^2 probabilities for compatability among experiments are 17% for the real part and 27% for the imaginary part. The final conclusion is that there is no evidence for a violation of the $\Delta S = \Delta Q$ rule, but there is some evidence that the errors for at least some experiments are underestimated.

The above experimental result really pertains only to the electronic decays, since very few muonic decays have been observed in any experiment. The only separate muonic value is by the Berkeley group (11) which claims some evidence for a difference between muons and electrons, but their result does not seem significant.

A. Caltech, Ref. 19
B. CERN, Saclay, Oslo, Ref. 18
C. Padova, Wisconsin, Ref. 16
D. CERN, Orsay, Vienna, not publ.
E. Previous world average (1969)
F. Illinois, Northwestern, Ref. 17

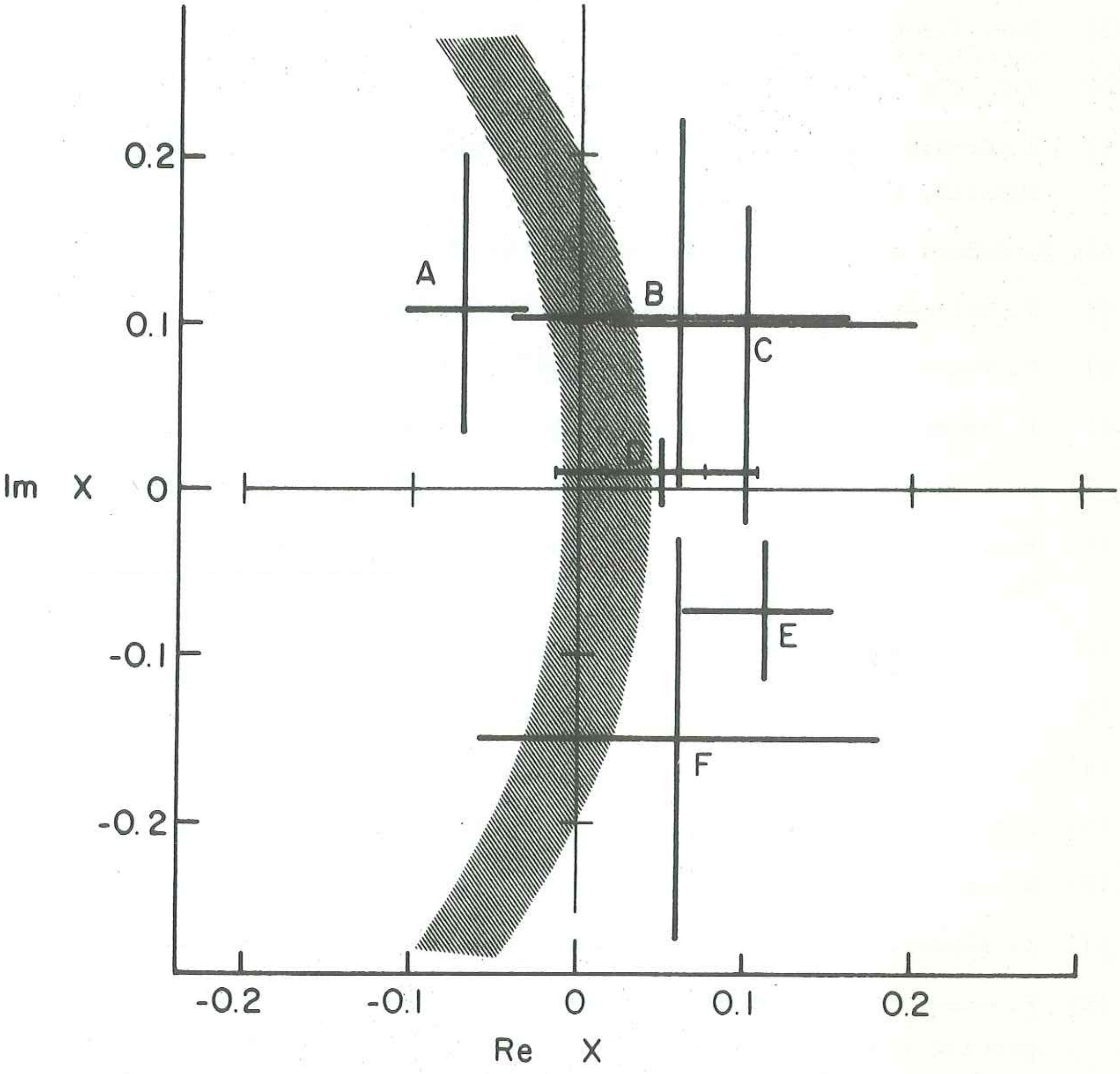

Fig. 2. Recent experimental results on K^0_{e3} decay

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