ANALITIC EXPRESSIONS FOR THE $K\pi$ S-MATRIX

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RESUMEN

Hemos obtenido una expresión analítica para los defasajes del sistema $K\pi$ en las ondas S y P $(T = \frac{1}{2})$.

A formalism has recently been proposed (1) (2) in order to study the consequences of causality on the scattering matrix of two elementary particles. This formalism has been successfully applied to πN scattering in reference (2) where simple analytic expressions for several phase-shift are given. In this note we present a similar treatment of $k\pi$ scattering in the state with isospin $\frac{1}{2}$. As a result we find analytic expressions for the S and P phase-shifts.

$$S_1(\omega) = \frac{h_1^-(KR)}{h_1^+(KR)} \exp(2i R\omega) \prod_1^n \frac{\omega - \beta_j}{\omega - \beta_j^*} \lambda \qquad (1)$$

where $h_1^{\pm}$ are the well known Hankel spherical functions, ω and K are the energy and momentum of one of the particles in the CM system, β_j is a complex energy such that $Im\beta_j > 0$, $|\lambda| = 1$ and R is the interaction range. Formula (1) is only suitable for elastic scattering. This is not a problem because we will consider an interval in which the inelasticity is small. It should be stressed that β_j has to be interpreted as a resonance if it lies close to the physical region. R is to be interpreted as the radius of a sphere outside of which the interaction is so small that it can be neglected.

In the following we determine the phase-shift of the S and P waves $(T = \frac{1}{2})$ of the $k\pi$ scattering in the total energy interval (720 MeV, 1420 MeV). We take as experimental data the results of reference (3), where the system $(k\pi)$ is studied from the annihilation $\bar{p}p \to k\bar{k}\pi$ at rest. We will take expressions of type (1) with only one factor in the product. This proves to be enough.

1- P wave. This wave is easily explained in terms of the resonance $k^*(891)$. The best fit obtained corresponds to the following values:

R = 1.94 fermi $\qquad$ argλ = -3.103

Reβ = .320 GeV. $\qquad$ Im β = .028 GeV.

The phase-shift is :

$$\delta_1(\omega) = \text{arctg } KR + R(\omega - K) + \text{arctg}\left(\frac{.014}{.320 - \omega}\right) - 1.551 \qquad (2)$$

with R = 1.94 fm.

In figure 1 we show this curve as well as the experimental results. The χ^2 is calculated over 29 points and turns out to be $\chi^2 = 13$. It is very sensible to variations of the parameters. The S-matrix has a singularity close to the real axis. It represents a resonance of mass 891 MeV. The width calculated as the energy difference between the points in which $\delta = 45^{\circ}$ and $\delta = 135^{\circ}$ is 47 MeV. It is clearly the K^{*}.

2- S wave. In reference (3) it is proved that experimental values can be approximated by a scattering length a = .3 fermi. This agrees with the theoretic predictions (4). It turns out that the phase-shift is rather smooth. For this reason the χ^2 value is not very sensitive to variation of the parameters, in contrast to what happens with the P wave. It is then difficult to ascribe a physical meaning to these parameters. Nevertheless the analytic expression of the phase-shift is useful.

We have found the best fit for the values

$R = .19$ fermi	$\arg\lambda = 2.647$
$\mathrm{Re}\,\beta = .018$ GeV.	$\mathrm{Im}\,\beta = .419$ GeV.

The singularity has not physical meaning.

The phase-shift is :

$$\delta_0 = R(\omega - k) + \mathrm{arctg}\,\frac{.419}{.018 - \omega} + 1.323 \qquad (3)$$

with R = .19 fm. This curve appears in Fig.2.

Finally we have also obtained a good fit to $\delta o(\omega)$ ($\chi^2 < 1$) using the same R value as in the P wave. It is

$$\delta o\,(\omega) = R(\omega - k) + \mathrm{arctg}\left(\frac{.229}{.144 - \omega}\right) + 1.008 \qquad (4)$$

with R = 1.94 fm.

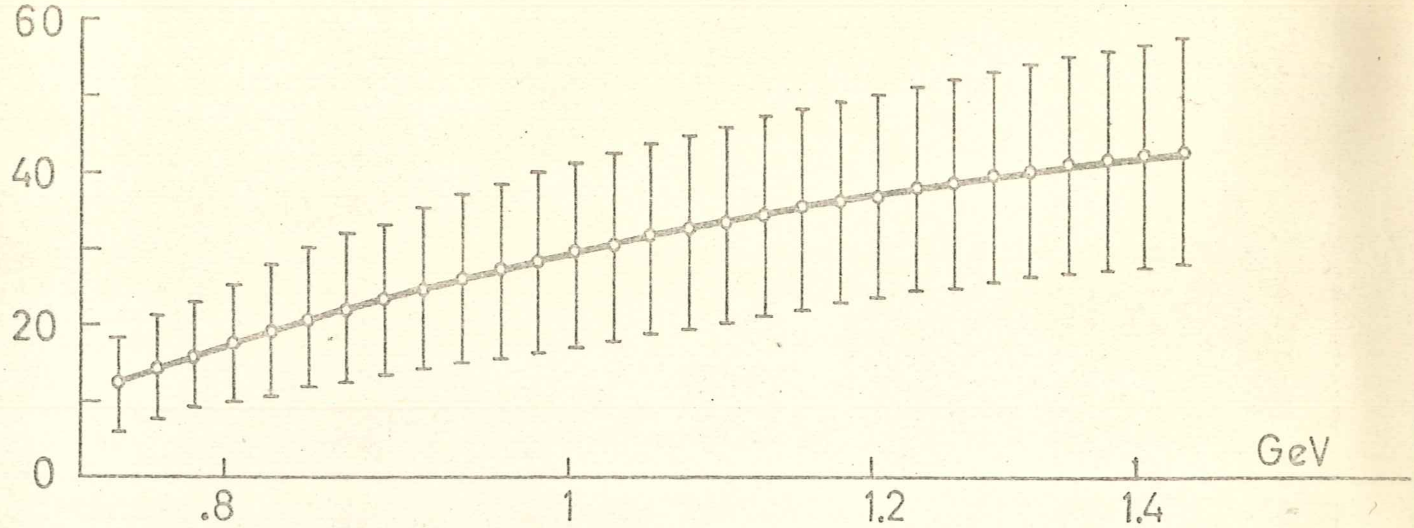

FIG.-2

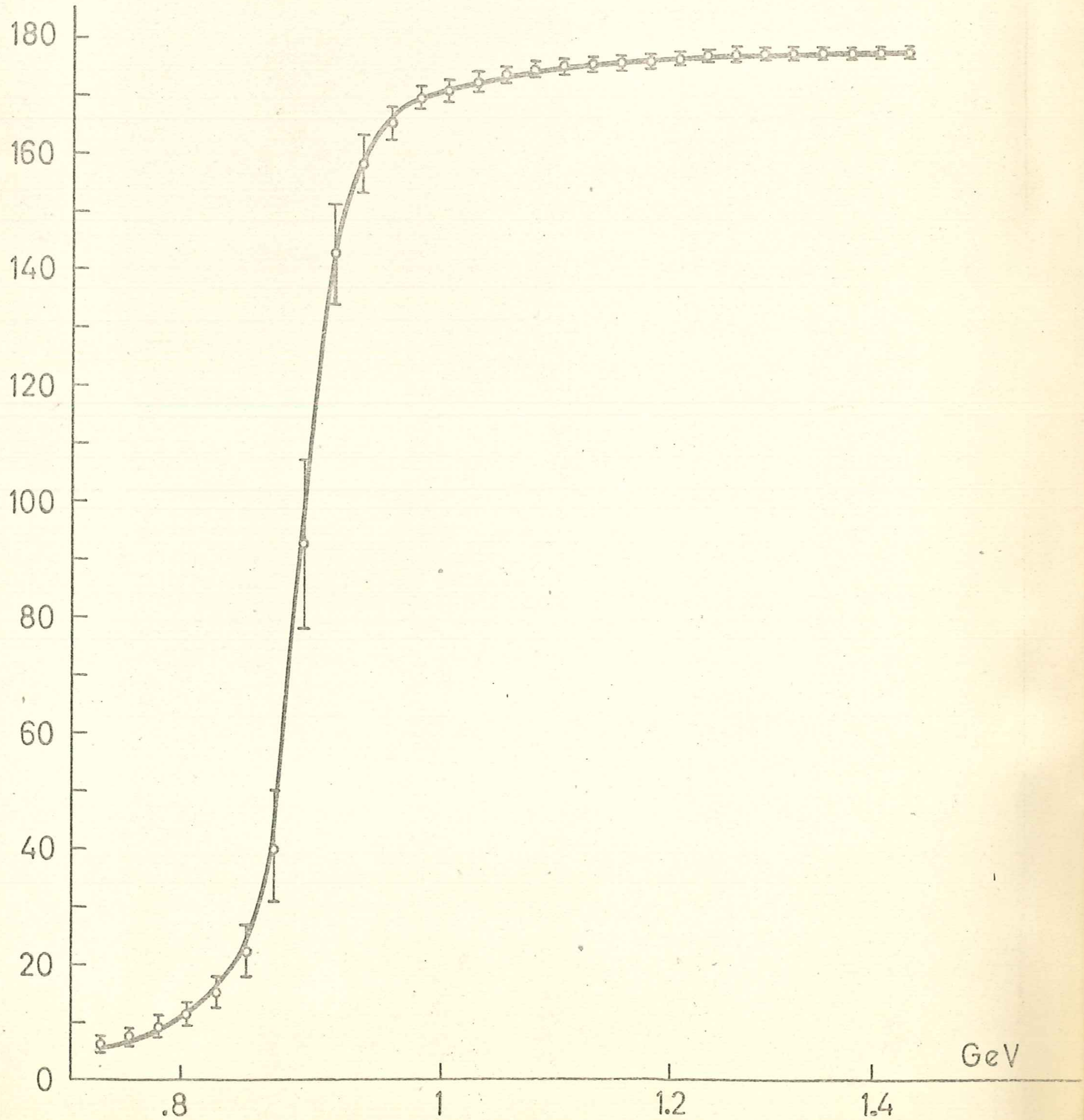

FIG.-1

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