Paradox Lost: the Cost of a Virtual World

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Abstract

This paper touches on a number of seemingly disparate topics—Artificial Intelligence, Fuzzy Logic, String Theory, the search for extra-terrestrial intelligence, the Cantorian concept of infinite sets—in order to support the thesis that for a large part of the educated public in the Western world, the very concept of reality has been changing over the last few generations, and that the change is being accelerated by our increasing acceptance of the Virtual as a substitute for the traditional Real. This, as I hope to convince you, is a momentous shift in the our world view, and like so many profound but gradual shifts, has gone largely unnoticed. Whether the shift is ultimately a good thing or a bad, it ought not to go unscrutinized; this paper aims to bring it to public attention. (The paradox whose loss is referred to in the title is discussed at the end of the paper.)

Key words:

Artificial Intelligence (AI)— the virtualizing of the human mind

In another paper[1], I argued that the Turing Test was invalid as a means of establishing the claim that a computer might be said to think. I was somewhat handicapped in arguing against that claim because I regarded it as absurd, and it is hard to argue effectively against what one regards as merely silly—one must perceive one's opponents as giants, not mere windmills, if one is to charge at them with all one's force. So I had to discipline myself to treat Turing's arguments seriously, and argue as solemnly as if in a court of law against a position that I privately regarded as a house of cards, fit to be knocked down by a puff of wind or a scornful word from Alice. I will not repeat here the arguments I offered in that paper; it is readily available to anyone who wishes to see them. The only point about the Turing Test that I want to call attention to here is that it is close to being a conscious piece of self deception; it tells us, almost in so many words, that if a computer can fool us into believing that it is thinking, than we must grant that it *is* thinking; in this matter, we are apparently living in an anti-Berkeleyan world where to seem is to be — *percipi esse est*. Turing even implied, and some of his epigones have said explicitly, that we believe that other people think for the same reason; they seem to respond to questions much as we would, so we assume that they, like us, are thinking.

Since Turing, a scientific genius, was not speaking *ex tempore* but presenting, in print, a well-considered opinion, something more specific than general human fallibility seems called for as an explanation for his adoption of a view that seems nothing less than absurd[2]. My explanation is that Turing was lonely; first because it is in the nature of things that a genius must be lonely, but also and more particularly because he was a homosexual in a society that strongly rejected homosexuality. In seeing the computer as a potential friend he was by no means alone—many of the strange beliefs I will be discussing are at bottom searches for a buddy—and the computer is not the most unlikely entity in which lonely man has attempted to find a friend. But whatever the cause that led a genius of Turing's caliber to suppose a computer capable of thought, I see in it a failure of confidence in their own experience on the part of modern cultivated people. It is a failure that shows itself again in our increasing preoccupation, sometimes even obsession, with the world of the Virtual—that is, of things that do such a good job of *seeming* that mere *being* cannot compete. In explaining why I think this is so, I must begin by supporting my use of the term *absurd* for the contention that we have already in some measure achieved Artificial Intelligence in the strong sense originated by Turing, and carried forward by McCarthy and Minsky.

What justifies that term, I think, is that the computer is often credited by AI champions with thinking on the basis of just those characteristics that are most unlike real—that is, human — mental activity. For example:

- the computer is absolutely obedient — if it does do something other than what we have ordered it to do, we say that there's a bug, and modify the program or hardware until it is again perfectly obedient.
- it has a perfect memory — as with obedience, if it seems to have "forgotten" anything it has been told, we test its memory and replace the faulty module when we find it.
- it executes algorithms faultlessly and at great speed.
- it never gets bored, never rebels, never initiates any new action or topic, and is quite content with endlessly repetitive tasks (what we usually call, tellingly, 'mindless repetition').

These attributes are the very antithesis of human mental behavior— so much so that I suggest that the computer might best be regarded as the first machine man has ever made that does *not* think. It never behaves spontaneously or mysteriously, never appears to have its own ideas, never seems to have a mind of its own. (By "never behaves mysteriously" I mean that it never behaves in a way that, when closely examined, is incomprehensible; for observers too busy to follow its workings in detail, of course, it is *always* mysterious—but then, everything is mysterious to the inattentive.) If nevertheless the computer seems to be thinking, that is precisely because, being unable to "think for itself," it's the perfect repository for and executor of *human* thought. Virtually every other machine that man has created is likely to misbehave from its owner's point of view; the computer stands alone in being perfectly subservient to its human masters.

And it was Turing himself, in what was probably his finest piece of work, who so defined the computer as to show why it is so apt a vehicle for algorithm execution. He coined for the computer, or at least put into general circulation, the term *finite state machine*, meaning a machine that can assume only a fixed, known number of internal configurations or states, each of which can be infallibly distinguished from any other by a test that can be performed at any time, and each of which the machine can be caused to assume by some finite number of available operations. The computer, in short, is a uniquely constrained machine; unlike other useful artifacts, it has a completely determinate number of degrees of freedom (or 'free parameters'), and this inability to deviate into an undefined or unforeseeable state makes it a perfect executor of our algorithms, and the only possible vehicle for the immensely long programs that realize human intentions. And so good is the properly programmed computer at performing useful tasks that spectators may begin to murmur that the computer is thinking — and, amazingly, not only innocent spectators, but sometimes even the very programmers who have laboriously developed the programs whose effects they now marvel at.

Only slightly less amazing than those programmers who, after working hard for months and years to get an important program behaving correctly, wonder if the computer is thinking, we have the other experts who believe that they have exhibited computers thinking, but bemoan the fact that they receive little credit for doing so. AI champions and enthusiasts have often expressed their chagrin at the fact that their achievements, however surprising and counter-intuitive when they are first announced, are always soon downgraded in the public mind to the status of routine, even boring, computer-science progress. A typical complaint is this from Martha Pollack, professor at the AI Laboratory of the University of Michigan and executive editor of the *Journal of Artificial Intelligence Research*:

> It's a crazy position to be in. As soon as we solve a problem, instead of looking at the solution as AI, we come to view it as just another computer system.[3]

One sympathizes with Professor Pollack and her colleagues, but there is no help for them; they are no doubt familiar with Clarke's Law—*Any sufficiently advanced technology is indistinguishable from magic*—but may not have appreciated that "is indistinguishable from" is a symmetric relation, equally capable of yielding Halpern's Corollary—*Any sufficiently explained magic is indistinguishable from technology*.

It should be observed, too, that AI applications are founded on no theory of the nature of intelligence. We have gotten the computer to do whatever it does despite having no complete, or perhaps even rudimentary, theory of intelligence—and how could we have built a machine that thinks without thoroughly understanding thinking? To build a thinking machine, we would have to accomplish something not merely beyond our present powers, but perhaps impossible in principle: we must thoroughly comprehend our own minds, stand outside ourselves as observers, pick ourselves up by our own bootstraps. If we can do this, then indeed we will have taken a momentous step in the evolution of our species—but we have not yet done it. And if someday we do, will we need to—will we *want* to—build a machine that reflects this knowledge of what thinking really is? As some wag has noted, all we would achieve by building a thinking robot — an artificial person — is reproduction without sex, when the human race seems to want just the opposite.

Nothing more will be said here about the shortcomings of Artificial Intelligence; the point I hope to leave you with as I move to another topic is that in AI we have highly intelligent and accomplished people believing propositions that are so easily shown to be invalid that they fully deserve the label *absurd*, and that the phenomenon is one deserving of investigation.

Fuzzy Logic: the virtualizing of problems of precision

"Fuzzy Logic" (FL), a concept and term created in 1973 by Lotfi Zadeh, professor of computer science at the University of California at Berkeley, purports to be a new approach to dealing with problems about which we have incomplete information, and one that represents an advance in logic. It has proven a much more popular concept in Japan, for some reason, than in the U.S. or Europe, and you can buy Japanese appliances that, it is claimed, were designed with the aid of FL. Here in the U.S. it has its champions and its detractors—a Google search on *fuzzy logic* will retrieve a torrent of papers and arguments by both sides—but in the discussion that follows, I reject the arguments of both sides. Instead, I contend that FL is a highly useful development that is as widely misunderstood by its champions as by its critics. (In the course of discussing FL I make occasional allusions to mathematical or physical findings and results that may or may not be familiar—they are present only for the entertainment of the technically sophisticated; my argument does not depend on such technicalities, but only on common experience and common sense. Also, I have not bothered to distinguish here between FL, the theory, and Fuzzy Control, its principal application; for present purposes they may be regarded as one thing.)

Between the rigor and precision of the exact sciences and the confused messiness of most of life stands engineering, which attempts to apply the discoveries of the former to the problems of the latter. The process requires engineers to know a lot of science—at least of the *findings* of science—but also to have the ability to operate with less than perfect information, to apply that mysterious resource called common sense, to have an instinct for when to yield and when to press. And all of us, whatever our business cards say, are to some extent engineers; we all find ourselves in situations about which our information is in part exact, in part nebulous, and all have to use that mixed bag in dealing with practical situations.

But the stark contrast between one part of our knowledge and the other can be stultifying and paralyzing; given on the one hand exact numerical information, and on the other vague, imprecise, often subjective inputs and requirements, many people just freeze at the controls, and cannot get on with the job. The problem arises in working with kitchen recipes, for example: some people have to know *exactly* how much salt to add to the batter, and are baffled and incapacitated when their grandmothers tell them to add "a pinch" or—even worse—"enough." And when in the kitchen-like processing of industrial materials the same problem arises, and the people who have learned through experience say that at a certain point you have to "raise the temperature a bit" or "give it a little more juice," many scientifically-trained operators are helpless; "How *much* do I raise the temperature?" or "What do you mean by 'a little more'?" they cry plaintively, and they can't act until the question is answered.

This is the practical problem addressed, and at least partially solved, by FL. A system designed in accordance with FL principles allows a user to enter instructions in the loose form he's comfortable with, without worrying about their exact meaning. And somewhere in the bowels of the system—usually within the computer controlling it—a mysterious process takes place whereby the vagueness is resolved and transformed into precise control information: "raise the temperature by 3.2 degrees centigrade" or "add 200cc/second of reagent A." The process, seen from the outside, calls to mind the classic cartoon in which a theoretical physicist is going through a long and complicated chain of mathematical reasoning at the whiteboard, and, pointing to one particularly abstruse expression, says "At this point a miracle occurs."

In the case of an FL system the miracle is easily explained: somewhere in the innards of the system is a table (or algorithm—it doesn't matter which for our purposes[4])—that transforms the vague expressions into precise values; that says, for example, "'a little' means 200cc/second" and "'heat gently' means raise temperature by 3.2 degrees centigrade."

That table enables the human operator to do what he often cannot do without help: stop dithering and act in some reasonable, though possibly imperfect, way. And the table was created by some human being—a human being who may well be just as unable to take action without exact information as the user who is rescued by it, who may even be the very same person. But—and this is the deep secret of FL—*the table or algorithm which replaces vagueness with precision was created at another time, in another place, and under very different circumstances.*

The table's creator did not have to worry about the practical effect of equating "a little" to 3.2, because at the moment he established that equivalence there was no pressing problem before him, just a rather academic, non-urgent decision as to what "a little" might reasonably mean. And later, when there *was* a pressing problem to deal with, the user (who may even be, as noted, the very one who earlier created the table of equivalences) will be able to give the system a "vague" instruction without taking responsibility for the interpretation the system will put on it; that responsibility is someone else's.

By decoupling the act of defining "a little" from that of using the term in anger, and letting each act be performed by a different person—or one person on two distinctly different occasions, wearing a very different hat on each—FL breaks the impasse, and lets us get on with the job. This means, of course, that FL has nothing to do with logic, fuzzy or smooth; it is simply a psychological trick we play on ourselves to get past one of our inhibitions—a trick something like the placebo in clinical medicine. It is also like the legal entity that the United States calls a corporation and England a limited liability company—a device for protecting investors from any loss beyond their investment by means of the creation of an artificial legal person who bears the responsibility for its acts, as if it were a natural person. And it is also like the device whereby the members of a firing squad are spared at least some of the guilt feelings that might otherwise haunt them—the device of secretly inserting a blank cartridge into one of the rifles, so that each squad member can entertain the hope that his was the rifle that fired the blank.

One point here is worth repeating: the process whereby a fuzzy input is converted into a (sufficiently) precise one is either algorithmic or rule-of-thumb, either theory-based or empirical; we either have some rigorous basis for saying that "a little" means "3.2 ounces," or we're just making a "reasonable guess." Those who claim that the former is the case, and that FL rests on new insights in logic or scientific method, must be prepared to display that discovery. If they cannot display it, then FL has no pretensions to being an advance in logic; the benefit of a FL-based system can only be that it lets us get on with the job rather than get bogged down in gazing into our navels for a definition of "a little."

In my experience, FL apologists forever blur this critical distinction; they cannot display a rigorous basis for turning "a little" into "3.2 cc," but they want to believe that if the process of conversion is very complex, and involves a lot of calculation, somehow the missing rigor is supplied by that very process. But a sow's ear cannot be turned into a silk purse by processing, however elaborate; as programmers used to say, garbage in, garbage out. Defenses and explanations of the FL process, which somehow never come to grips with this point, mask the simple fact that at its foundation is a set of arbitrary and merely common-sensical decisions—something to conceal and be ashamed of only because the process is being touted as hard science and irrefutable logic.

None of the foregoing should be taken as an accusation that FL is useless. On the contrary, such inhibition-suppressing and action-releasing devices are enormously valuable, and the inventor of this one, Lotfi Zadeh, is to be thanked and congratulated. What he has given us, however, is not a contribution to logic or scientific method, but a tool with which we can overcome the Hamlet-like inhibitions that so often bind the best souls among us in dealing with recalcitrant reality—a tool that works, we may say, by 'virtualizing' the decision process.

Curled-up Dimensions: the virtualizing of spatial perception

Albert Einstein cannot be held responsible for the misuse made of some of his ideas and utterances. It's not his fault that many people think that the General Theory of Relativity, his revolutionary concept of how gravity shapes space, somehow means that nothing is really true, since everything is really relative to … well, something. Nor is it his fault that many people think that there is a fourth dimension that, if they keep their eyes wide open, they will be able to spot someday before it can get away. Modern physicists, however, are largely responsible for a kind of confusion that is widespread among the general, non-scientific population, and that originated with Einstein's great discoveries. That is the notion that science has discovered a fourth, fifth, or even an eleventh dimension that you can see, apparently, only if you have a PhD in physics. This misconception is a serious one, because it seems to confirm the non-scientists' gloomy feeling that what scientists are talking about is utterly beyond them, and that it's hopeless to try to understand anything they say—not the frame of mind that we want an enlightened citizenry to be in.

What the scientists are saying can indeed be hard to understand, but not because it's too technical, as the public thinks, but because it's too poetical. And not only is it more like a poem than a theorem or equation, it's also poor poetry—and poor poetry is the most corrupting of all types of discourse. Samuel Taylor Coleridge, discussing critics of poetry, said

> The question should be fairly stated, how far a man can be an adequate, or even a good (so far as he goes) though inadequate critic of poetry, who is not a poet, at least *in posse*. Can he be an adequate, can he be a good critic, though not commensurate? But there is yet another distinction. Suppose he is not only not a poet, but is a bad poet! What then? [5]

I would ask the same question, only replacing Coleridge's "critic of poetry" by "explainer of science." We ordinary folks perceive three dimensions, no more, no less. So do scientists; so did Einstein. But when it was found that time, no less than space, needed to be considered in understanding some physical phenomena, scientists ventured into the unfamiliar realm of figurative speech, the mode characteristic of poetry, and did it poorly.

To fix the location of a point in ordinary Euclidean space, as represented in a Cartesian graph, we specify numerically its distance along each of three axes from some agreed-upon fixed point called the origin. And because there are three dimensions, it takes three numbers to specify the location of a point in such a space. This fact established in everyone's mind a correlation between the number of dimensions and the number of magnitudes that had to be measured: three dimensions, three measured quantities. But then it was found that for some purposes a fourth quantity had to be specified: time. And the agreement between number of dimensions and number of quantities that needed to be specified was extended "poetically": if it now took four quantities to describe some phenomenon, then by a simple reversal of cause and effect, there must be, or could be considered to be, four dimensions.

Of course time is not a dimension in the proper—that is, the common—sense; it is one only by courtesy. Like the three spatial dimensions, it must be specified by a number representing its distance from some agreed-on starting point—a point physicists conventionally label $t_{0.}$ But unlike those three, its numerical specification does not measure a spatial distance—it is a dimension only figuratively, not literally. And this figure of speech, this primitive bit of poetry, has been a prolific cause of confusion, suggesting to generations of laymen that there is a fourth direction in space that is just as real as the three we know, but very hard to detect. Some consequences of this confusion may be merely ridiculous, such as the temptation experienced by more than one child I know to whirl about suddenly, hoping to catch sight of the fourth dimension before it can escape. And many is the portentous half-baked theory about what it would mean to "enter" the fourth dimension, and many the fantasies about the strange creatures we would find there. Perhaps the only good thing about this error of taking literally the term "fourth dimension" is the spur that it gave the writers of science fiction, horror stories, and wildly imaginative literature in general. But if that is a good consequence, it is the only one; for the rest, its effect has been wholly negative.

It was bad enough when those who lacked any scientific education thought there were four dimensions; all right, they thought, not being a scientist, I can see only three of them, but three out of four isn't too bad, is it? But now we learn that the theoretical physicists who are working in string theory, one of the current approaches to a unified theory of physics that will subsume both relativity and quantum mechanics, have determined that they need eleven dimensions if their theory is to work. These new dimensions, they tell us, are invisible because they are (a) unimaginably tiny, and (b) "curled up" inside other things. All of this, of course, is poetry—junk poetry. What the theorists mean is that in order to get their theory to hang together and yield the results they want, they need to postulate the existence of seven further quantities that are dimensions only in the same sense that time is a dimension.

In part, the notion of dimensions beyond the familiar three is due to a simple logical error: supposing that if we need four (or eleven) quantities, there must be four (or eleven) dimensions is like supposing that if for every A there's a B, then for every B there must be an A. By that reasoning, just as for every baby there must be a woman who bore it, so for every woman there must be a baby she bore. But this fallacy is too easily seen through; the seemingly endless proliferation of dimensions would not be taking place if there were no deeper root. And there is: it is our need to find analogies, models, comprehensible patterns that relate these increasingly abstract discoveries to something we already know. The last time theoretical physics made a discovery that lent itself to such treatment, it was Newton's, in the 18th century; we had all whirled objects around ourselves at the end of strings, or at least could readily imagine doing so, and if we called the string "gravity," we could entertain the hope that we had some sort of understanding of that force.

But when in the 19th century Maxwell produced the ideas and equations that gathered together electricity, light, and magnetism, we could no longer do even that much. We revealed our failure when we supposed that if all these phenomena were waves, as Maxwell told us, there had to be a medium through which waves might propagate, and invented the aether to fill that role. But then experimentation showed that there was no such thing, and we realized that our mental picture was nothing like the reality. And when in the 20th century Einstein told us what we had to think about space, time, signal propagation, and the relationship of mass to energy if we wanted to understand some otherwise inexplicable phenomena, we were utterly defeated. A few components of Einstein's vision could be illustrated with analogies from human experience: we were shown pictures of heavy ball bearings sinking into thin rubber sheets, and were told that this was how mass distorted space, and there were comparisons of clocks aboard speeding trains with other clocks that had remained in the station, and we grasped a little of the idea of time dilation. But the essentials of the General Theory of Relativity are so counter-intuitive, so at odds with ordinary human experience, that the notion of understanding it in the same sense in which we understand physical reality on the human level is hopeless.

It was something of a *reductio ad absurdum* when Einstein himself complained about quantum mechanics, the theory that relates many of the phenomena of nature that relativity does not address, that he couldn't accept it because he couldn't understand it—that is, couldn't make it *feel right* even though it was superb at enabling us to predict the behavior of the phenomena it dealt with. Einstein's reputation among other physicists suffered from his emotional rejection of the indeterminacy of quantum mechanics, but in fact in doing so Einstein was making yet another contribution to human thought. He was telling us that even he, the very archetype of the scientific genius, could not understand the latest developments of physics in the sense that we had once understood scientific discoveries, and the sense in which we have always wanted to understand them. And this lesson, if we had absorbed it, might have made us stop calling the new entities whose existence we had to postulate "dimensions", in a futile effort to make them seem homely, familiar things. All that such junk poetry does is to confuse us further, since by no stretch of the imagination can we come to grips with eleven dimensions, no matter how tiny or curled up.

The physicist Brian Greene tries to show us what such "curling up" means in his recent best-seller on modern theoretical physics, asking you, his reader, to contemplate a picture of a stretched-out garden hose seen from so far away that it looks like a mere one-dimensional line.[6] Then he asks you to come closer; when close enough, he points out, you will perceive that the hose has a second dimension: thickness. Just so, he contends, if we could come close enough to some inconceivably minute entities in space called "strings," we would see that they somehow enclosed dimensions far beyond the familiar three. But this analogy is nonsense: the observer who at first thought the hose one-dimensional made that mistake not because he thought that there is only one dimension, but simply because he was too far away to see the hose in sufficient detail. He was already aware that there are three dimensions, and as soon as he got close enough to the hose, he immediately and without the slightest difficulty granted it existence in (at least) two of those dimensions. His experience is radically different from that of being asked to perceive not just that some object when examined closely has more dimensions than it appeared to have when seen from far away, but that there exist more dimensions than he ever knew about or can observe or conceive.

In the one case, the observer simply made a trivial error of estimation because of the limitations of human eyesight; in the other, he is shown nothing—you don't get to "see" a curled up dimension no matter how close you get—but asked to acknowledge the existence of several additional dimensions, apparently on no better grounds than that he was initially wrong about the dimensionality of the garden hose, and therefore owes it to the physicist to atone for his mistake by conceding that strings have eleven dimensions.

The truth is that modern theoretical physics is almost entirely a branch of mathematics, and can only be understood as such. It differs from other branches of mathematics in that its practitioners try to draw from it physically observable consequences, like the bending of light rays by massive objects, but the experimentalists who attempt such observations are only following in the wake of the theoreticians, attempting to corroborate or refute the predictions of the theoreticians' equations; they are not primarily trying to make discoveries of their own. (They may in the course of checking a theory come up with some unforeseen phenomena that the theorists, in turn, will have to consider in their theorizing, but that is just a happy side-effect when it occurs.) Modern theoretical physics has sailed far, far beyond the coastal waters of common sense and common experience, where you could navigate by reference to natural landmarks and lighthouses; we are now sailing on the open ocean, far out of sight of land, and must lay our course solely by means of the stars, clocks and logic. No one will ever again understand the findings of science in the traditional sense of making them compatible with our deepest intuitions about space and time; insofar as they can be understood in any sense, they can be understood only by those who have the requisite mathematical training. And the efforts of popularizers to make them available to the public through the analogies and models and figures of speech that I've called "junk poetry" are not only doomed to failure, they can have no effect other than to increase the public's confusion, as witness the career of "dimension."

SETI & UFOs: virtualizing the search for knowledge

The search for extraterrestrial intelligence—SETI, as the knowing call it—is so important to some people that it was actually funded for some time by the U. S. Government. The official rationale for this subsidy was that if we can make contact with intelligent beings from elsewhere than Earth, we can benefit from their presumably highly advanced technology. The number of implausibilities and improbabilities silently glossed over in this claim is of astronomical magnitude.[7]

So far, not a single place has been found in the extraterrestrial universe where the conditions thought to be necessary for life are known to be present—liquid, surface water being only the first of many. But let's waive this objection; after all, we've examined only an infinitesimally small fraction even of just our own galaxy, let alone the universe, so no conclusions should be drawn from our failure so far. Let's suppose that we get an answer to some message we've sent out[8], thus finding not just life, but intelligent life, somewhere out there. And let's stipulate that the source of the reply is only a few light-years away, so that something approaching a conversation is possible. Now what? Presumably we know that we are in touch with intelligent beings because the signals we receive have some regularity to them; they will convey some mathematical sequence, such as the squares of the first *n* integers[9]. Good; now we know that just a few light-years out there is an intelligent entity that (a) can receive and send signals of the type that we have been transmitting, (b) shares with us at least arithmetic reasoning, and (c) is willing to make contact with us, at least intellectually.

Now how do we pass from this stage to that of building a common language with the extraterrestrials so that we can communicate about more than elementary number theory? It may be that human ingenuity will come up with some clever means of doing this, but none has been proposed yet, as far as I know, and if we are to spend many millions of dollars and much valuable talent on this project, at least some idea should be available as to how we would proceed if its first step were successful.

Supposing this requirement somehow met, the next questions to arise are these: why should we suppose that the science and technology possessed by this alien being or species are superior to ours? And why should that being or species be willing to pass its knowledge to us, gratis? And if they do have valuable knowledge, and are willing to give it to us, would we be wise to accept it? The dangers of such a windfall of unearned knowledge is just what Lessing had in mind when he said that if God offered him a choice between effortless possession of knowledge and a passion for discovering knowledge, he would choose the latter. Whether it's answers or advice or praise, what you get for nothing is notorious for being worth nothing—the intellectual equivalent of junk food. And of course there is always the possibility that the alien beings we manage to contact will be hostile, and capable of making their hostility felt. This is a consideration routinely dismissed by SETI enthusiasts as "paranoid," although the difficulty we earthlings have in getting along among ourselves suggests that the difficulties of getting along with alien life forms, even if they felt no animosity toward us, might be very great.

The fact that the U.S. Government has spent, and is still spending, millions of dollars on SETI without seeking serious answers to these questions, and the many others that are their corollaries and consequences, suggests that the funding of that project was, in official eyes, just a kind of hush money paid to quiet the complaints and retain the services of space enthusiasts.

To show that my point of view is not solely that of knuckle-dragging troglodytes and short-sighted Luddites, here is the opinion of one of the world's leading experts on biology:

> Even if something parallel to the origin of human intelligence should indeed have happened somewhere in the infinite universe, the chance that we would be able to communicate with it must be considered as zero. Yes, for all practical purposes, man is alone.[10]

And while the SETI project is certainly, like AI, another of the quests for non-human friends, it is something else as well: it is a form of Cargo Cult, in which the entire human race plays the role of a tribe of technologically undeveloped island-bound peoples, waiting for some beings in the sky to deliver marvels to them, just as some South Pacific islanders are reportedly still expecting American ships or planes to visit them as they did during World War 2, bringing goods of such variety and quantity as to establish a new quasi-religious cult among the recipients.

The pro-SETI party does not take criticism well; it is a party of enthusiasts and determined optimists who regard critics as sour, unimaginative fellows who are best ignored. Here is a specimen of pro-SETI cheer leading, chosen because it is a little more coherent than most such expressions:

> How easy it is to scoff at `desperate' seekers of aliens, as the science-fiction writer Brian Aldiss did in these pages two years ago *(Nature* 409, 1080; 2001). He touched some nerves: aliens are indeed as real as ghosts or numerous deities (in other words, they continue to be purely imaginary); their fictitious portrayals impede understanding; their air of being the product of scientific thinking is indeed spurious. But when all was said and done, his put-down, although erudite, was ultimately no more than a stimulating polemic.
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> Other prominent sceptics have been more soberingly analytical. Peter Ward and Donald Brownlee, in their book *Rare Earth* (Copernicus/Springer, *2000),* discussed various factors that, on the face of it, conspire to make the emergence of life around stars probable but the appearance of intelligent species almost impossible. "Is that it?"; one might ask when faced by the likelihood that we are, alas, the pinnacle of our Galaxy's intelligence. And according to a theory discussed at last week's meeting of the American Association for the Advancement of Science, if it wasn't for the chance mutation of the *FOXP2* gene 50 millennia ago, we wouldn't have the creativity to explore these ideas.
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> Yet more cold water has been poured on our hopes that we have intelligent company in a recent collection of answers to Enrico Fermi's pointed question about the lack of visitors: "Where is everybody?". In a sceptical book of that title (Copernicus/Springer, *2002),* Stephen Webb's explanations for the absence of alien visitors range from: "They are here and they call themselves Hungarians", attributed to Budapest-born physicist Leo Szilard, to: "Science is not inevitable"; science being one of many improbable developmental steps from primitive organisms to interstellar communication or travel.
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> A pox on such pan-Galactic pessimism. These arguments may give sober government agencies an excuse to stop funding searches for extraterrestrial intelligence (SETI), as the US Congress did over a decade ago. But the rest of us should look more favourably on such expressions of the fundamental yearnings of humanity. All credit to the likes of William Hewlett, David Packard and Paul Allen, whose funds have allowed the SETI projects to establish new technologies — albeit still pitifully insensitive if we are to detect the equivalent of *Friends* leaking through some planetary ionosphere 1,000 light years away. All credit too to the tens of researchers who devote themselves to the dispiriting quest for such electromagnetic detritus. The rest of us should drag ourselves away from revivals of *ET, Close Encounters* and *Taken* long enough to scan the latest ambitions of the SETI institute, outlined in *SETI2020* (http://www.seti.org), and send it a donation for the hunt for the real Thing.[11]

Fascinating to learn that it was the writer of science fiction who rejected SETI as being science fiction, while the presumably level-headed and critical editor(s) of *Nature*, one of the two foremost scientific journals of the English-speaking world, can only say "A pox on such pan-Galactic pessimism." And what a devastating charge to bring against government agencies, calling them "sober"!

But for some, the problem of making contact with intelligent extraterrestrial beings has been solved already: we need not search for them, because they have come to us, riding in the vehicles we call Unidentified Flying Objects (UFOs). The existence of UFOs is of course not in doubt; one can hardly look up at the sky without noting all kinds of things up there, or seeming to be up there, that one cannot identify. Even after eliminating the floaters in their own eyes that so many mistake for distant objects, there are plenty of UFOs. By the same token, the land and the sea are full of unknown objects—objects unknown, at least, to untrained observers—but Unidentified Creeping Objects and Unidentified Swimming Objects do not evoke much interest, because one can hardly suppose them to be modern visitors from outer space.

The question, of course, is what reason we have to suppose that the unknown objects seen up in the sky are such visitors. My answer is that we have no rational reason to do so. There have not been many fully recorded sightings of UFOs (I will henceforth use "UFO" to mean something supposed to be an extraterrestrial vehicle), and those few are unimpressive, to put it mildly. Why is there not a single clear and undoubtedly authentic photograph of a UFO? There are millions of cameras in North America and Europe alone, and some observers have had their cameras with them at the time of a sighting, and managed to snap a picture or two—but not one such picture offers a clear image. The evidence for the existence of UFOs consists entirely of eye witness reports, and anyone with experience in the criminal justice system or in clinical psychology knows that eye witness reports are generally unreliable, even when the witnesses are intelligent, honest, and unprejudiced. So we need not enquire whether reporters of sightings are people hungry for attention, or members of the Chamber of Commerce of Roswell, New Mexico—even in the absence of such corrupting factors, these reports deserve very little credence.

But the objection that has greatest force, I think, is one founded on the behavior of the supposedly intelligent beings operating the UFOs: why have they neither completely hidden themselves from us, nor made open contact with us? They came to earth for some reason, presumably, and they have tremendous technical capabilities. Surely they are capable of concealing themselves from us if they wish to, and equally they are capable of landing and demanding of the nearest human (if they are not afraid of clichés) "take me to your leader!" But they have done neither; instead they have let their vehicles be seen by a very few human observers, but never closely enough nor long enough to allow unambiguous identification of those vehicles, much less themselves, to be made. Why? What can be their motives for coming all this way, only to behave in so pointless a fashion?

My conclusion is that such behavior is inconsistent with the intelligence that would be required to build an inter-stellar space craft, and that therefore alien-carrying UFOs are non-existent. And the broader SETI movement, though not necessarily linked to the UFO cult, is nevertheless another troubling transposition of our quest for friends, support, and technical progress to the virtual realm, where the conflicts and complications of the actual can be waved away.

Cantorian Sets: virtualizing the concept of the infinite

> *One concept corrupts and confuses the others. I am not speaking of the Evil whose limited sphere is ethics; I am speaking of the Infinite.*
> —Jorge Luis Borges, "Avatars of the Tortoise," *Otras Inquisiciones*

In an earlier section I said that many of the names mathematicians assign to the objects they generate were "punk poetry"—that is, weak metaphors whose figurative nature tends to be quickly forgotten, and which turn out to be seriously misleading even to many intelligent and educated laymen. In that section, I was specifically concerned with the use made by advanced mathematical physicists of the term "dimension" to label the extra parameters they found necessary to make their theories (e.g., string theory) coherent. Here I'm concerned with the use of the term "infinite" in describing sets that conform to the criterion proposed by Georg Cantor (1845-1918)—namely, that their elements can be put into one-to-one correspondence with their own proper subsets. Cantor's position, now accepted by the vast majority of working mathematicians, is that the same general method used to compare finite sets for size is available for infinite sets as well, which can likewise be ranked by size. And in doing so, he and his disciples tell us, he makes rigorous and precise our formerly nebulous idea of the infinite.

What Cantor did vs. what he thought he'd done

I think Cantor achieved something important, but was mistaken about the nature of that achievement. What he did, I suggest, was to deal with the problem of measuring infinite sets as Alexander solved the problem of untying the Gordian Knot—he rejected the problem as stated, and transformed it into another, more tractable one. By inventing transfinite numbers, he assigned infinite sets a cardinality; this made it possible to treat infinite sets just as if they were finite sets, and compare them in size by pairing off their members against other infinite sets—and it worked. The move produces useful mathematical results—but one observes that the results have nothing to do with the traditional infinite, which cannot be so treated, as every philosopher and mathematician from Aristotle to Gauss and Poincaré agreed. What Cantor did was not to refine or clarify our concept of the traditional infinite, but to invent a wholly new object, the Cantorian Set. His achievement is much like that of the geometers who explored the consequences of letting traditional parallel lines converge or diverge; they taught us nothing about Euclidean space, which remains a realm where parallels remain the same distance apart forever—what they did was to create wholly new kinds of "space," which have their own independent value. As Columbus sought a better route to the Indies, but instead discovered the New World, Cantor sought to capture the infinite, and instead invented a new kind of number.

Such cases of important discoveries or inventions that are misunderstood by their discoverers, or so misnamed as to be misunderstood by others, are quite common. Mathematicians and physicists regularly commandeer common terms, use them to label more or less related abstract constructs that are more amenable to formal manipulation, and then tell users of those terms in their traditional senses that they are misusing them. The discussion of "dimension" above offers one example; here is another, especially blatant one offered both in a book by a modern writer on science, and in a widely respected reference work:

> Democritus of the fifth century B.C. held the true belief that all matter is made of particles so tiny as to be invisible: atoms. Although Democritus' works are lost, it is unlikely he had anything we would consider valid evidence. His was a philosophical insight that turned out to be right. (The serendipity is less striking for the fact that the atoms of twentieth-century physics are not indivisible as Democritus thought.)[12]

In the first decade of the 19th century, John Dalton, supposing he had found the fundamental, non-decomposable unit of physical reality (which later would be shown to be 'splittable' into even more primitive entities) assigned the name 'atom' to it. He thereby created a misnomer, and his error is seen by Poundstone as casting discredit on Democritus—his 'serendipity' is tarnished. And *The Columbia Encyclopedia*, 6th edition, s.v. 'atom', falls into the same anachronism, telling readers that "…the atom is not at all indivisible, as the ancient philosophers thought, but can undergo a number of possible changes."

In much the same way Cantor applies the term "infinite" to an invention of his own, and then criticizes earlier thinkers for their misuse of the term. What makes it evident that the 'infinite' of Cantor's vision is not that of all earlier Western thought is the corollary that some infinities are bigger than other infinities—indeed, that there is an infinite hierarchy of progressively bigger infinities. The notion that one infinite set can be larger than another implies that infinity is a quantity or magnitude, and as such capable of being measured, when what we have called the infinite up to the time of Cantor is not a magnitude (if it is anything for which we have a name, it is a process: "…and another one, and another one…"), and it makes no sense to call one example of it bigger than another. What do we mean by bigger? When dealing with finite sets, we call set A bigger than set B if every element of A can be paired off with an element of B until there are no more elements in B, but at least one still in A. This cannot be done, of course, with infinite sets, so the traditional notion of bigger or smaller has no application. Consider that the pairing-off process has no significance in itself; it is only when that process *terminates* in the exhaustion of (at least) one of the two sets involved that a conclusion can be reached as to the relative sizes of the two. But since inexhaustibility is the very definition of a (traditional) infinite set, that moment of truth cannot arrive. The reason why one cannot reach the end of an infinite set is not that the end is so far away, but that the end is non-existent.

To forestall misunderstandings, note that the idea of comparing infinite sets in magnitude is to be rejected not because it would take an infinite amount of time to perform—such a process is clearly a *gedankenexperiment* taking place in the timeless logical realm, and immune to objections drawn from the temporal realm—but because it is *logically* impossible; there is no way to exhaust the supposedly smaller set, and so no way to find an unmatched element in the supposedly larger one. I know, too, that criticism of this and other instances of misleading mathematical terminology can easily be misconstrued as an attack on the professional practices of working mathematicians; I intend no such thing. The results achieved by Cantor, and those built on them, are so important to modern mathematics that any criticism of them is liable to be greeted by mathematicians as biologists greet critiques of Darwinian evolution. But my critique is not a demand that we abolish the Cantorian set, only a plea that we stop misinterpreting it. I have no problem with the use by mathematicians of Cantor's concrete results; my quarrel is only with the misuse, as I see it, of some traditional terms to describe such results—an abuse that may hold no dangers for mathematicians themselves, who understand each other, but often make for serious misunderstandings on the part of the laity.[13]

The rhetoric of mathematics

When mathematicians create a new mathematical object, they generally label it with a name that reflects some resemblance it seems to have to a familiar object or quality. Open a mathematical encyclopedia or dictionary, and you will find many such names, all sounding familiar to laymen for the good reason that they are all common English words, but all of which have meanings in mathematics that are severely technical, and to which their ''popular'' names give no clue. Examples: *ring, group, manifold, class, graph, curl, domain, flag, ideal, kernel, lattice, map, matrix, model, pontoon, obelisk, path, pencil, ray, field, space, hull, martingale*—the number could be greatly increased if there were any need to do so. Not one of these mathematical objects can even begin to be understood on the basis of their names, but that doesn't matter in most cases; the objects themselves are the concern only of mathematicians, and they are not misled. But there are a few such names whose capacity to mislead does matter, sometimes very much. I argued earlier that "dimension" was one such; I make the same argument with respect to the post-Cantorian use of "infinity." It must be remembered that Cantor himself insisted on the significance of his concept of infinity beyond the mathematical world; he contended that it had profound consequences for philosophy and even for theology, and spent many of his later years more involved in those studies than in mathematics[14].

What Cantor actually conceived is a new kind of mathematical object with some interesting and novel properties. It has proved to be a useful concept, opening up many new possibilities for mathematicians, and potentially for scientists. What is being questioned here is not the admissibility of the concept, but its identification with the much older concept of the infinite. It is my contention that Cantorian sets have no more to do with the traditional infinite than the mathematical object called a *field* has to do with a pasture, or the mathematical *ring* with a hula hoop. There is nothing illegitimate about the Cantorian set, nothing wrong with positing a kind of set whose members are in one-to-one correspondence with those of its own proper subset (or of a number that is somehow the cardinal of an infinite set)—but that something is not the infinite. The giveaway here is the idea that such sets, while all infinite, can be shown to be greater and lesser than others; insofar as these entities are assigned the attribute of magnitude, it is clear that they are not what all Western thinkers from the pre-Socratics to Cantor himself thought of as infinite. Cantor has not refined the traditional concept of the infinite, he has changed the subject. He had every right to do so, and I have no desire of driving mathematicians out of the paradise that he created for them; I would only observe that Cantor was even more creative than he is usually given credit for—he did not merely refine the existing concept of the infinite, but created a completely new creature in the mathematical world.

Foundations: castles in the air

> *It is, indeed, the common fate of human reason to complete its speculative structures as speedily as may be, and only afterwards to enquire whether the foundations are reliable. All sorts of excuses will then be appealed to, in order to assure ourselves of their solidity, or rather indeed to enable us to dispense altogether with so late and so dangerous an enquiry.* —Kant, *Critique of Pure Reason.*

Throughout much of the nineteenth and early twentieth centuries the greatest mathematicians and logicians, like Frege and Peano and Russell, who interested themselves in the foundations of mathematics were divided into warring camps: some called themselves *Platonists*, and believed that mathematical objects existed absolutely, and were discovered, not invented, by man; others called themselves *empiricists*, or believers in *psychologism*, or *intuitionism*, or *formalism*, or *logicism*[15]. What is most remarkable about the conflict among these various schools of thought, some of which persists to this day, is how little it mattered to working mathematicians. Even those, like Brouwer and Kronecker, who differed vehemently with most of their colleagues on supposedly basic ideas, continued working for the most part with the same criteria of what constituted a proof. Almost everyone agreed, in practice, on what was and was not an important result, and most mathematicians, if they gave the mathematical philosophers any thought at all, dismissed them as mystics and dreamers—or simply as superannuated elder statesmen of the profession who couldn't do real mathematics anymore, and were reduced to arguing about philosophy.

This problem of the infinite, if it is a problem, appears in the very foundations of mathematics, but despite that—or perhaps because of that—it is hardly important to mathematicians; they are very little interested in or affected by "foundations"—it hardly matters what mathematicians believe, because their beliefs have almost nothing to do with how they work. It may not matter to mathematics, then, whether mathematicians talk as if one infinite set could be greater than another, any more than it mattered for many centuries that astronomers believed that the sun revolved around the earth—their observations and results were limited not by their beliefs about what we now call the solar system, but by their instruments and computing powers. But even if it did matter, and mathematics were partially disabled by current views on infinity, there is something still deeper and more ominous to be feared here. The current acceptance of the concept of greater and lesser infinities reflects a loss of nerve on our part, a loss of faith in our own experience as something that has to be satisfied by any proposed hypothesis.

Paradox lost: the cost of changing the locus of reality

I have claimed that the confusions and misunderstandings that have been reviewed here can lead to real trouble, well beyond the mere confusion on the part of laymen about what mathematicians and scientists have really discovered. What I have in mind is the loss of that vitally necessary intellectual tool, the paradox. The history of Western philosophy is in large part the history of our confrontation with and resolution of paradoxes (I mean the serious logical paradoxes — the antinomies — not the merely verbal ones that are really just puns), the seemingly faultless reasoning that leads to the obviously false conclusion. When we are confronted by a paradox we know that there is an error somewhere in the process that led us inexorably to that conclusion, and analyzing that chain of reasoning at least sometimes lets us uncover that error, and thereby learn something that usually applies much more widely than just to the example at hand. Rarely if ever does the error turn out to lie in any of the explicit steps of reasoning that led us to the false conclusion; it is virtually always due to some overlooked ambiguity in our terms, some assumption built into one of them that we were unconscious of. And the resolution of that ambiguity, the uncovering of that faulty assumption, often leads in turn to a clarification that benefits us in realms far beyond that in which we started.

The best-known example of such a paradox is that of Achilles and the Tortoise, which has led to great refinements in our concept of the infinite and the infinitesimal, even though it is not, even today, solved in a way that satisfies us simply and directly—that is, there is no explanation of the paradox that is couched in the same verbal terms in which it was posed. We are told about infinite series that converge to a value of 1, but this does little more than repeat what we knew already, that the conclusion is false—what we want is to be shown exactly where the verbal statement of the situation is wrong. That paradox of Zeno's made us think, and is still making us think more than two thousand years after being propounded, because it posed a perfectly clear contradiction: we could put our finger on no flaw in Zeno's reasoning, but it led to a conclusion we knew absolutely to be false—Achilles *does* catch up with the tortoise, always. And the flat contradiction between the apparently inexorable conclusion and our experience-based certainty that the conclusion is false is what gives the paradox its value, makes it something from which we can learn. But what shall we do when the conclusion to which a chain of reasoning leads is highly surprising, even incredible, but not as unarguably counterfactual as the proposition that Achilles will never catch the tortoise? We are liable, nowadays, to accept the conclusion to which pure reasoning seems to have led, however bizarre, rather than reject it and look for the misstep that led us there. I think that if the paradox of Achilles and the Tortoise were propounded for the first time today, many would passively accept its conclusion, and argue that if we seemed to be able to catch tortoises, that was only an illusion we entertained in order to protect our egos.

That is at least one of the causes, I think, of the various absurd notions that have been reviewed here. When a scientist tells us that we should credit machines with at least the potential for thought because a machine might someday hold a conversation with us that cannot be distinguished from one with an average person; or that an advance in logic has been made that enables us to extract precise numeric values from vague verbiage; or that the number of parameters currently fashionable physical theories require implies the same number of dimensions in actual space; or that highly intelligent extraterrestrial beings not only exist, but can be communicated with, and have much of value to teach us (and perhaps are even visiting us now, albeit with no clear idea in mind); or that one can treat the infinite as if it were finite, and compare two infinities to see which is larger, many of us tend to react with a kind of dumb fatalism. We aren't sure we really believe these things, but then so many of the things we always believed seem to have been proven false, and so many bizarre ideas seem to have been shown to be true, that we cannot stir ourselves to resist these impostures even if we cannot totally accept them. In doing so, we deprive ourselves of one of rationality's greatest tools, that deliberately contrived case of the paradox called *reductio ad absurdam*; to employ it, we must first agree that certain things are absurd, and nowadays who dares dismiss any proposition as absurd? And one of the principal causes of this loss is that the tools invented for their own use by mathematicians and scientists drag with them highly dubious ideas that go on to infect the general population, although the mathematicians and scientists themselves, like Typhoid Marys, suffer no ill effects from the pathogen they spread.

Throughout most of Western history, the notion that there are smaller and larger infinities, for example, would have been taken as one of those paradoxical results that mean that we have erred somewhere, and even as late as the early 20th century many eminent mathematicians so took it. But today our intellectual self-confidence fails us in such straits. When we are faced with highly surprising results that do not, like footraces between men and tortoises, fall within our actual or at least imaginable experience, we find ourselves unable to declare them paradoxical, and hence to use them to find faults in our methods or assumptions; we have been beaten down by too many other results that seemed unacceptable, but proved to be unassailable. We have been forced to accept quantum mechanics, of which it has been said that if it begins to seem comprehensible to you, you should see a therapist immediately; we have sadly accepted that Einstein was wrong, and that God *does* play dice with the world. And so the burden of proof has shifted. No longer do newly conjectured theoretical objects have to establish their right of entry into the house of mathematics and science; all are admitted unless critics can prove they should not be, and the only grounds for excluding any newcomer is that it is inconsistent with those already admitted. (And one wonders when that last barrier will crumble; how long can it be before some advanced thinker proclaims that science should follow the Walt Whitman Rule: "Do I contradict myself? Very well then I contradict myself, I am large, I contain multitudes.)"

We have until recently demanded that any hypothesis not only *work*—that is, yield results that are consistent with the relevant observables—but *explain*—that is, show how those results fit in with our intuitive ideas and general knowledge of the world. But the two great achievements of physics of the last century, the Special and General Theories of Relativity, and Quantum Mechanics, have imposed themselves on us by their superb satisfaction of the first requirement alone: they work astonishingly well, so well that we commonly waive the other requirement, that of making the world more intelligible and coherent. We thereby gain many a valuable tool for predicting and controlling nature, even at the price of feeling less and less sure that we ourselves are rational creatures whose experience is the ground for our belief and actions, and properly so. This, I think, is a profound and profoundly disturbing development; when human beings give up their demand for coherence and intelligibility, and content themselves with what merely works, we have entered a new epoch in human thought—an epoch that is in many ways very like a past we thought we had transcended. Pure reason, followed blindly, seems to be leading us back to an age of magic and superstition.

Notes

(Endnotes)

1 "The Trouble With the Turing Test," *The New Atlantis* (Winter 2006), pages 42-63.

2 Professor Maurice Wilkes (*Memoirs of a Computer Pioneer* [MIT Press, 1985], p. 197) writes that Turing told him in a letter that he was "not very pleased with [his paper]"; unfortunately, Turing did not make it clear just what aspect of the paper he was dissatisfied with.

3 Quoted in Mark Baard, "AI Founder Blasts Modern Research," *Wired News* (May 13, 2003), pages 1-2, at page 2. (www.wired.com/news/technology/0,1294,58714,00.html).

4 It need not be literally a table; it may be a function that generates a value *ab initio* in return for any argument, rather than a pre-established table that lists selected argument/value pairs on which look-ups can be performed. But if it is a function, it is irrelevant whether it involves quantum chromodynamics, string theory, and the Riemann Zeta function, or is just a simple one-for-one substitution rule; in either case it's a deterministic algorithm that yields the same results for the same inputs, every time. In effect, such a function generates just the relevant part of a table, on demand. As such, it can be thought of as a table with no loss of generality or rigor.

5 Coleridge, *Anima Poetae*, pp. 127f.

6 Brian Greene, *The Elegant Universe* (NY: Random House Vintage Books, 1999), pp. 186-187.

7 For an authoritative statement of the views of SETI enthusiasts, see Jill Tarter, "Ongoing Debate over Cosmic Neighbors," *Science* (3 January 2003), pages 46-47. More recently, SETI has enjoyed a rebirth as "Optical SETI," whose followers search for communications from extraterrestrials not in the form of radio-frequency messages, but in that of laser pulses. See Monte Ross, "The New Search for E.T.," *IEEE Spectrum* (November 2006), pages 30-37.

8 In order to get such an answer, we had better make sure that the messages we send out are coherent; as noted in *Science* (28 May 1999), page 1457, at least one such message was not.

9 This also assumes that postmodernist mathematicians like Reuben Hersh are wrong, and that mathematical truth is the same everywhere. What a shame if we were indeed getting signals from intelligent extraterrestrial beings, but were dismissing them as random noise because the truths the aliens were trying to communicate to us were so different from our truths as to be unintelligible to us.

10 Ernst Mayr, "Are Humans Alone?," *What Evolution Is* (NY: Basic Books, 2001), page 263.

11 Editorial titled "In support of xeno-optimism: Despite recent gloom, there are worse things on which to spend personal wealth than a hunt for intelligent extraterrestrials." *Nature* (20 February 2003), page 769.

12 William Poundstone, *Labyrinths of Reason* (Doubleday Anchor Press, 1988), page 118.

13 A story reporting the apparent solution of the Poincaré conjecture (Dennis Overbye, "An Elusive Proof and Its Elusive Prover," *The New York Times* [August 15, 2006], page D1)starts with a sketch of a rabbit, and reads "That rabbit is actually a sphere."

14 For a full account of Cantor's philosophical and theological beliefs, and the energy with which he promoted them, see Joseph Warren Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Princeton University Press, 1977, and Michael Hallett, *Cantorian set theory and limitation of size*. Oxford: Clarendon Press, 1984. It is tempting to quote extensively from these two studies, both of them authoritative and sympathetic to Cantor, but I will content myself with just one quotation, from the Hallett volume (page 63):

> The upshot [of Cantor's principle of finitism] is that in the finite case, ordinality completely and directly determines cardinality: the two kinds of number coincide. Indeed, one might go so far as to say that cardinality in the finite case is just an expression of the fact that counting leads to unique results. I suggest that the baasis of Cantor's theory of power is the thesis, following priniciple (b) and the homogeneity of the ordinals, that the same method can also be applied in the infinite realm. Thus the suggestion here is that here also size can be determined by *counting*, that is by counting all elements against a fixed stock of transfinite ordinal numbers.
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> There is a certain sense in which Cantor took 'counting' in the infinite realm literally. … Of course infinite sets cannot literally be counted. But the boldness of Cantor's theory is that he proceeded *as if they can*. In other words he deliberately suppresses here a fundamental intuitive distinction between the finite and infinite.

And Hallett goes on this highly critical vein for some pages. I find his words expecially gratifying because before reading them I had independently come to very much the same conclusions about the audacity, to put it no more strongly, of Cantor's fundamental method of assimilating the infinite to the finite.

15 For extended discussion of these various philosophical positions, see Evert W. Beth, *The Foundations of Mathematics*, North-Holland Publishing Co., 1959; rev. ed. 1964; Harper & Row Torchbooks, 1966. Beth, who is both a historian of the subject and a contributor to it, is not without his own doubts as to our ability to rank infinite sets by size: he writes (rev.ed., page 379) "The theory of cardinal numbers already provided a conceptual apparatus which enable us to "*count*" infinite sets and to show, for instance, that there are "*more*" sets of natural numbers than there are natural numbers." The quotation marks and italics are Beth's.