2.2 η-η' mixing: overview

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The subject of η-η' mixing is now becoming interesting in view of the present and forthcoming experiments at COSY (Jülich), DAPHNE (Frascati), ELSA (Bonn), MAMI (Mainz), VEPP-2000 (BINP, Novosibirsk), CEBAF (JLAB), BEPCII/BESIII (Beijing) and B-factories (BABAR, Belle and Belle II) where many different processes involving η and/or η' mesons are/will be measured abundantly and precisely as compared to earlier experiments.

Relevant topics concerning η-η' mixing are the mixing parameters, that is, the pseudoscalar decay constants associated with η and η' and the related mixing angles in the octet-singlet and quark-flavour bases, the possibility of a gluonic content in the η' wave function, and the different sets of observables ($V \to P\gamma$ decays, with $V = \rho, \omega, \phi$ and $P = \eta, \eta'$, $J/\psi \to VP$ decays, and η and η' transition form factors, among the most precise sets) where these parameters can be extracted from.

Concerning the mixing parameters, a brief introductory summary is the following. There are two kinds of mixing, that of mass eigenstates and that of decay constants. The mixing of mass eigenstates consists of a rotation matrix described in terms of single mixing angle, θ_P in the octet-singlet basis and ϕ_P in the quark-flavour basis, that connects the mathematical states, η_8 and η_0 or η_q and η_s, depending on the basis, to the physical states η and η'. Both mixing angles are related through $\theta_P = \phi_P - \arctan\sqrt{2}$. In this mixing scheme three assumptions are implicit: i) there is no mixing with other pseudoscalars (π^0, η_c, radial excitations, glueballs...); ii) the mixing angle is real (supported by the fact that $\Gamma_{\eta,\eta'} \ll m_{\eta,\eta'}$); and iii) there is no energy dependence. The mixing of decay constants is characterized by $\langle 0|A_\mu^{a(i)}|\eta^{(\prime)}(p)\rangle = i\sqrt{2}F_{\eta^{(\prime)}}^{a(i)}p_\mu$, with $a = 8, 0 (i = q, s)$ and $A_\mu^{a(i)}$ the corresponding axial-vector current. The four independent decay constants can be parameterised in terms of either $F_{8,0}$, the octet and singlet decay constants, and two mixing angles $\theta_{8,0}$, in the octet-singlet basis, or $F_{q,s}$, the light-quark and strange decay constants, and the mixing angles $\phi_{q,s}$, in the quark-flavour basis, respectively. Are all these mixing angles related? To answer this question, one must resort to Large-N_c Chiral Perturbation Theory [1], where the effects of the pseudoscalar singlet η_0 are treated perturbatively in a simultaneous expansion in p^2, m_q and $1/N_c$. In this framework, one can see: i) that a one mixing angle scheme can only be used at leading order in this expansion, where $\theta_8 = \theta_0 = \theta_P$ (or $\phi_q = \phi_s = \phi_P$) and the decay constants are equal; ii) that at next-to-leading order the two mixing angles scheme must be used, thus making a difference between θ_8 and θ_0 and with respect to θ_P (or similarly between ϕ_q and ϕ_s and with respect to ϕ_P) and where the decay constants are all different among themselves; and iii) that the mixing structure of the decays constants and the fields is exactly the same. For a compendium of formulae see Refs. [2, 3, 4, 5]. At the same time, one can also see that $\sin(\theta_8 - \theta_0) \propto (F_K^2 - F_\pi^2)$, a $SU(3)$-breaking effect expected to be of the order of 20% ($F_K/F_\pi \simeq 1.2$), and $\sin(\phi_q - \phi_s) \propto \Lambda_1$, an OZI-rule breaking parameter expected to be small. In the FKS scheme [6], this Λ_1 parameter is assumed to be negligible, a hypothesis that is tested experimentally since the two mixing angles are seen to be compatible [5]. If one forces this equality, $\phi_q = \phi_s = \phi_P$, which is not based in theory, the result of the fit is $F_q/F_\pi = 1.10 \pm 0.03$, $F_s/F_\pi = 1.66 \pm 0.06$, and $\phi_P = (40.6 \pm 0.9)^\circ$ [5]. Therefore, a recommendation for experimental collaborations would be to use for the time being (until the achieved accuracy permits to distinguish between ϕ_q and ϕ_s) the quark-flavour basis in their analyses. To finish, just to mention that the decay constants F_η and $F_{\eta'}$ do not exist similarly to F_π or F_K but instead the four different decays constants mentioned before, in one basis or the other, must be used for the η-η' system. The interested reader can use Ref. [6] as a reference text for a complete introduction to these topics and a detailed list of publications and analyses prior to year 2000.

Concerning the possible gluonic content in the η' wave function, two complete and precise sets of experimental data haven taken into account to explore this possibility: the $V \to P\gamma$ decays, with $V = \rho, \omega, \phi$ and $P = \eta, \eta'$, and the $J/\psi \to VP$ decays. In the first case, using a very general model for $VP\gamma$ transitions [7], one gets $\phi_P = (41.4 \pm 1.3)^\circ$ and $Z^2_{\eta'} = 0.04 \pm 0.09$, or, equivalently, $|\phi_{\eta' G}| = (12 \pm 13)^\circ$ (the parameter $Z_{\eta'}$ weights the amount of gluonium in the wave function and $\phi_{\eta' G} = -\arcsin Z_{\eta'}$), that is, absence of gluonium in the η' [8]. This result is in contradiction with the experimental analysis performed by the KLOE Collaboration, where, using several ratios of $V \to P\gamma$ decays, described by the same model as before, in addition to the ratio $\eta'/\pi^0 \to \gamma\gamma$, they found $\phi_P = (40.4 \pm 0.6)^\circ$ and $Z^2_{\eta'} = 0.12 \pm 0.04$ [9], thus confirming their first analysis with the results $\phi_P = (39.7 \pm 0.7)^\circ$ and $Z^2_{\eta'} = 0.14 \pm 0.04$ [10]. The reason for the discrepancy between the first phenomenological analysis mentioned above and the former two experimental analyses is the inclusion in the latter of the ratio $\eta'/\pi^0 \to \gamma\gamma$ in the fits. This sole observable makes the difference. However, we believe that the way KLOE characterises this ratio, as a function of F_q, F_s, ϕ_P, and, simultaneously, $Z_{\eta'}$ is a contradiction in terms, since Chiral Perturbation Theory assumes that η and η' are quark-antiquark bound states. In the case of $J/\psi \to VP$ decays, the values obtained were $\phi_P = (44.6 \pm 4.4)^\circ$ and $Z^2_{\eta'} = 0.29^{+0.18}_{-0.26}$ [11], thus drawing a conclusion less definitive but in accord with the $V \to P\gamma$ phenomenological analysis. Anyway, more refined experimental data will contribute decisively to clarify this issue. For completion, when the gluonic content of the η' is not allowed, $Z_{\eta'} = 0$, the fitted value of the η-η' mixing angle in the quark-flavour basis is found to be $\phi_P = (41.5 \pm 1.2)^\circ$, from $V \to P\gamma$ decays [8], and $\phi_P = (40.7 \pm 2.3)^\circ$, from $J/\psi \to VP$ decays [11], respectively. Other relevant analyses on this topic are Refs. [12, 13].

Finally, a more recent and novel approach for the extraction of the η-η' mixing parameters is the analysis of the η and η' transition form factors in the space-like region at low and intermediate energies in a model-independent way through the use of rational approximants (see P. Masjuan's contribution to these proceedings for more details). Using the normalization of the form factors as obtained from the experimental $\eta^{(\prime)} \to \gamma\gamma$ decay widths as well as the fitted result for the asymptotic value of the η form factor, one gets $F_q/F_\pi = 1.06 \pm 0.01$, $F_s/F_\pi = 1.56 \pm 0.24$, and $\phi_P = (40.3 \pm 1.8)^\circ$ [14], in nice agreement with previous results, a bit less precise but very promising for the near future if more space- and time-like experimental data for these form factors are released together with a more precise measurement of the decay widths.

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