Double Cyclic Codes Over the Rings $\mathbb{Z}_2^\alpha \times \mathbb{Z}_2^\beta$ and $\mathbb{Z}_2^\alpha \times \mathbb{Z}_4^\beta$ *

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Abstract

Consider the rings R_1 and R_2, such that R_1 is an R_2-module, and $C \subset R_1^\alpha \times R_2^\beta$ an additive code. The code C is a double cyclic code if the set of coordinates can be partitioned into two subsets, the set of coordinates in R_1 and the set of coordinates in R_2, such that any cyclic shift of the coordinates of both subsets leaves invariant the code. The code can be identified as submodules of the $R_2[x]$-module $R_1[x]/(x^\alpha - 1) \times R_2[x]/(x^\beta - 1)$. We define two cases. First, when the code C is binary, that is $R_1 = R_2 = \mathbb{Z}_2$, which is called $\mathbb{Z}_2$-double cyclic. The second case is when $R_1 = \mathbb{Z}_2$ and $R_2 = \mathbb{Z}_4$, that is the code is a $\mathbb{Z}_2\mathbb{Z}_4$-additive code, and it is called $\mathbb{Z}_2\mathbb{Z}_4$-cyclic. In both cases, we determine the structure of these double cyclic codes giving their generator polynomials. We also determine the related polynomial representation of its duals in terms of the generator polynomials.

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