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**Multidimensional poverty measurement: Making the identification of the poor count**

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**Abstract**: The success of any poverty eradication program crucially depends on its ability to identify who is poor and who is not. In this paper, we show that the state-of-the-art methodology that is used to identify the poor in multidimensional contexts – the dual cutoff method suggested by Alkire and Foster – is insensitive to many of the subtle considerations that should be incorporated when making such delicate decisions. The simplicity of the counting approach that underlies the dual cutoff method precludes the possibility of generating 'poor-identification rules' that are sensitive to interactions between the different dimensions of poverty. To go beyond the apples-and-oranges aggregation procedures characterizing the dual cutoff method, we suggest a much broader identification approach that contains the latter as a particular case. Our empirical findings using 48 Demographic and Health Surveys across the developing world suggest that the percentage of households that are inconsistenly identified as 'poor' according to the dual cutoff and some of the methods suggested in this paper is around 30% – a result with enormous implications for the identification of the potential beneficiaries of poverty eradication programs worldwide.

**Keywords**: Multidimensional poverty measurement, Identification, Dual cutoff method, Counting approach, Consistency condition

**JEL Classification**: I3; I32; D63; O1

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## 1. Introduction

Who is poor and who is not poor? This is the fundamental question that must be addressed before any poverty eradication program can be implemented. While the answer to this question is relatively simple when poverty is measured in the space of income distributions (the relative position of individuals' income vis-à-vis the poverty line determines who is poor and who is not), matters can become much more complicated when poverty status is determined using several dimensions at the same time. For a long time, poverty has been analyzed on the basis of income distributions alone (e.g.: Sen 1976), but in recent years it has been acknowledged that both monetary and non-monetary attributes are essential to conceptualize and measure individuals' welfare levels (see, for instance, Bourguignon and Chakravarty 2003, Alkire and Foster 2011). In response, international institutions like the European Commission and the United Nations are currently implementing the multidimensional approach to complement official unidimensional income or consumption poverty measures,[3] and many scholars and policy-makers are engaging in an intense debate on what kind of poverty headline indicator should be used to guide poverty eradication strategies in the post-2015 global development agenda. The main concern of this paper is that, in the context of multidimensional poverty measurement, the problem of identifying who is poor and who is not has been unsatisfactorily addressed by the different approaches suggested in the literature so far. Alternative perspectives should be incorporated if one aims to generate sensible measures that accurately identify the individuals that should be targeted by anti-poverty programs. To the extent that the success of micro level anti-poverty programs depends on targeting the

[3] Following the definition adopted by the Europe 2020 strategy, Eurostat publishes since 2009 the values of the multidimensional AROPE index (people at-risk-of-poverty rate or social exclusion), and since 2010 the United Nations' Human Development Report (HDR) annually publishes the values of the so-called 'Multidimensional Poverty Index' for over a hundred countries all over the world (see Alkire and Santos 2010).

right individuals and that current international cooperation, development, and aid programs are guided by the macro level results derived from the corresponding measures, the issues analyzed in this paper have practical and financial implications for the design of effective poverty eradication strategies.

Assuming one is able to define dimension-specific poverty thresholds to determine whether individuals are deprived or not in the corresponding dimensions (that is: when one works in the deprivation space[4] ), there are currently three well-known approaches for the identification of the poor in a multi-attribute framework. According to the 'union approach', an individual is said to be multidimensionally poor if there is at least one dimension in which the person is deprived. At the other extreme, the 'intersection approach' states that an individual is 'poor' if s/he is deprived in all dimensions simultaneously. Respectively, these approaches are likely to over-estimate and under-estimate the set of individuals that should be considered as 'poor', particularly when the number of dimensions considered is large. While the union approach might include individuals that are only deprived in one relatively unimportant dimension among many, the intersection approach might fail to identify those individuals that are experiencing extensive but not universal deprivation. A natural alternative suggested by Alkire and Foster (2011) (which is inspired by the work of Atkinson 2003) is to use an intermediate cutoff level that lies somewhere between the two extremes. According to the so-called 'intermediate approach', an individual is poor if the number of dimensions in which s/he is deprived is above a given poverty threshold – denoted as $k$ – that

[4] Whenever the different dimensions are commensurable, some scholars suggest working in the attainment space (that is: aggregate individuals' attainments into a unidimensional welfare indicator and identify them as 'poor' whenever their aggregate well-being level falls below a given poverty threshold). This is the route advocated by Ravallion (2011) and implicitly used by Duclos et al (2006). As argued by Alkire and Foster (2011) and many others, a key conceptual drawback of viewing multidimensional poverty through a unidimensional lens is the loss of information on the dimension-specific shortfalls. In addition, the problem of identification of the poor becomes trivial in the unidimensional setting, so it will not be considered in this paper.

is exogenously chosen by the analyst (note that both the union and intersection approaches are particular cases of the intermediate approach). Since this counting methodology uses deprivation thresholds *within* dimensions and an overall poverty threshold $k$ *across* dimensions, it has been denoted as the 'dual cutoff' identification method – also referred to in the literature as the 'counting approach', the 'AF identification method', or the 'AF method'.

Given its flexibility to accomodate many reasonable alternatives lying between the – admittedly extreme – 'union' and 'intersection' perspectives, the dual cutoff approach is the state-of-the-art methodology currently employed by researchers, policy-makers and institutions around the world to identify the poor in multidimensional settings. To illustrate: the AF method is currently being implemented by the governments of Bhutan, Brazil, Chile, China, Colombia, El Salvador, Malaysia, Mexico or the Philippines to complement their income poverty measures, with many other countries to follow soon, and the United Nations Development Program (UNDP) has since 2010 annually published the worldwide distribution of the Multidimensional Poverty Index – which is based on the AF method. The book 'Multidimensional Poverty Measurement and Analysis', published in 2015 by Oxford University Press, describes in detail the AF method and its applications and will further contribute to settle and reinforce the global diffusion of the approach.

While there is much to praise in the dual cutoff method, it is also important to highlight some of its limitations, especially because its use is becoming predominant. Among the several factors that have contributed to its widespread acceptance and implementation,[5] in this paper we focus on two of them: (i) The apparent simplicity and intuitiveness that characterize the method, and (ii) the possibility of identifying the contribution of each dimension

[5] Among others, these factors include the ability of the AF method to accomodate the ordinal data that commonly arise in multidimensional settings and its plasticity and flexibility in adapting to alternative contexts where different variables are available.

to overall poverty levels. Regarding the former, the counting approach that underlies the AF identification method is a straightforward procedure that, roughly speaking, simply adds up the number of deprivations across dimensions to decide whether the individuals experiencing them should be considered poor or not. While the counting approach is reflective of the current state of the literature, such 'apples and oranges' aggregation exercises are a crude way of proceeding that sidesteps many of the subtle and complex considerations that have to be incorporated when deciding what combinations of deprivations should be included in the identification of the poor. The main point of this paper is that the purported simplicity and intuitiveness of the dual cutoff method comes at a high price because it severily limits the potential ways in which individuals can be identified as being 'poor' and leaves aside reasonable criteria one might want to incorporate. As discussed below, such alternative identification criteria are likely to arise if composite poverty indices are hierarchically structured – as is increasingly common in multidimensional settings (see sections 2 and 3 for the formal definitions). An implication of our results is that the set of poor individuals targeted by the dual cutoff method and the other criteria proposed in this paper do not necessarily coincide – an issue that might over- or under-represent certain sectors of the population as potential beneficiaries of poverty eradication programs worldwide.

Another attractive feature of the AF method is the alleged possibility of knowing the contribution of each dimension to overall poverty levels once the identification step is over (see Alkire and Foster 2011: 481-482). According to this model, it is possible to conclude that deprivations in variable $V_i$ have contributed to overall multidimensional poverty levels by, e.g., $v_i\%$ – thereby giving an apparently clear and appealing message to researchers or policy-makers aiming to identify the single most important dimension that contributes to poverty so as to eradicate it in the most effective way. We argue that this dimension-decomposability approach might give a misleading picture of the ways in which multidimensional poverty is articulated because it disregards the *joint* patterns of deprivation that individuals must experience in order to classify them as poor. We suggest complementing the potentially misleading dimension-decomposability property by another decomposability property – referred to as 'profile decomposability' – that is naturally derived from the identification method suggested in this paper. Profile decomposability is superior to its dimension-wise counterpart in informing about the structure of multidimensional poverty and in conveying clearer and more focused messages to those working toward its eradication. The rest of the article is organized as follows. The next section introduces notation and formally describes the problem. Section 3 discusses the proposed solutions. Section 4 presents two empirical applications illustrating our results and Section 5 provides some concluding remarks. The proofs are relegated to the appendix.

## 2. Notation and Definitions

We introduce some notation that are used in the rest of the paper. Let $N$ be the set of individuals[6] and $D$ the set of dimensions under consideration (with $n := |N| \geq 1, d := |D| \geq 2$). For any natural number $G \leq \lfloor |D| / 2 \rfloor$, let $\Pi_{D,G}$ denote the set of partitions of $D$ into $G$ exhaustive and mutually exclusive groups $D_1, \ldots, D_G$ (i.e.: $D_i \cap D_j = \emptyset \forall i \neq j$ and $D = \bigcup_{i=1}^{i=G} D_i$) where each group has at least two members (i.e.: $m_i := |D_i| \geq 2 \forall i$). A generic element of $\Pi_{D,G}$ is denoted as $(D_1, \ldots, D_G)$. We denote by $X_d$ the set of $d-$dimensional vectors whose elements can either be 0 or 1, that is $X_d := \{0, 1\}^d$. For any subset $S \subset D$, we define $\mathbf{1}_S$ as the $d-$dimensional vector in $X_d$ whose $i-$th element is equal to 1 if $i \in S$ and 0 otherwise. To illustrate: if $d = 5$, one can define the partition of $D = \{1, \ldots, 5\}$ into

[6] The word 'individuals' refers to the basic unit of analysis – even if such unit involves households or other aggregates.

$D_1 = \{1,2\}$ and $D_2 = \{3,4,5\}$. Then $(D_1, D_2) \in \Pi_{D,2}$, $\mathbf{1}_{D_1} = (11000)$ and $\mathbf{1}_{D_2} = (00111)$. $\mathbb{R}^q, \mathbb{R}^q_+$ are the $q-$dimensional Euclidean space and its nonnegative counterpart respectively. Let $\mathbf{a} = (a_1, \ldots, a_d)$ be a $d-$dimensional vector of positive numbers summing up to 1, whose $j^{th}$ coordinate $a_j$ is interpreted as the normalized weight associated with dimension $j$. The set of all possible $d-$dimensional weigthing schemes summing up to 1 is called the *$d-$dimensional simplex*, and will be denoted by $\Delta_d$ (i.e.: $\Delta_d = \big\{(a_1, \ldots, a_d) \in \mathbb{R}^d_+ | \sum_i a_i = 1\big\}$). $[0,1]$ is the closed interval of real numbers between 0 and 1.

The achievement of individual $i$ in attribute $j$ will be denoted by $y_{ij}$. The results in this paper are independent of the measurement scale of our attributes: They can either be ordinal or cardinal. Therefore, the range of values of $y_{ij}$, denoted as $I_j$, can either be the set of non-negative real numbers $\mathbb{R}_+$ (an almost universal assumption in both unidimensional and multidimensional cardinal poverty measurement) or a discrete subset of it. The vector $\mathbf{y}_i = (y_{i1}, \ldots, y_{id}) \in I_1 \times \cdots \times I_d$ contains individual $i$'s achievements across dimensions and is called the *achievement vector*. In this context, an *achievement matrix* $M$ is a $n \times d$ matrix containing the achievement vectors of $n$ individuals in the different rows. The set of all $n \times d$ achievement matrices is denoted as $\mathcal{M}_{n \times d}$. The set of all achievement matrices is defined as $\mathcal{M} = \bigcup_{n \in \mathbb{N}} \bigcup_{d \in \mathbb{N}} \mathcal{M}_{n \times d}$.

For each attribute $j$ we consider a poverty threshold $z_j$ representing a minimum attainment in that attribute that is needed for subsistence – which in this paper we consider as exogenously given. Whenever $y_{ij} \leq z_j$, we say that individual $i$ is *deprived* in attribute $j$. The vector of dimension-specific poverty thresholds is denoted by $z = (z_1, \ldots, z_d) \in I_1 \times \cdots \times I_d$. In this context, an *identification function* $\rho : (I_1 \times \cdots \times I_d) \times (I_1 \times \cdots \times I_d) \to \{0,1\}$ is a non-trivial mapping from individual $i$'s achievement vector $\mathbf{y}_i$ and the poverty thresholds vector $z$ to an indicator variable in such a way that $\rho(\mathbf{y}_i, z) = 1$ if person $i$ is poor and

$\rho(\mathbf{y}_i, z) = 0$ if person $i$ is not poor. For analytical clarity, it will be convenient to write the identification function $\rho$ as the composite $\rho = \rho^b \circ \rho^w$, with

$$\rho^w : (I_1 \times \cdots \times I_d) \times (I_1 \times \cdots \times I_d) \to X_d \quad (1)$$

and

$$\rho^b : X_d \to \{0, 1\} . \quad (2)$$

The function $\rho^w$ converts the achievement vector $\mathbf{y}_i$ and the vector of poverty thresholds $z$ into a $d-$dimensional vector of 0s and 1s indicating whether individual $i$ is deprived or not in the different dimensions taken into account (where 1 denotes deprivation and 0 non-deprivation). The set $X_d$ contains all possible combinations of deprivations/non-deprivations across $d$ dimensions, and we refer to it as the *set of deprivation profiles*. Its members are denoted as $\mathbf{x} = (x_1, \ldots, x_d)$, with $x_j \in \{0, 1\}$ indicating the deprivation status in dimension $j$. Therefore, the profile $(0, ..., 0)$ corresponds to someone who is not deprived in any dimension and $(1, ..., 1)$ to someone who is deprived in all dimensions. Clearly, $|X_d| = 2^d$. By construction, $\rho^w$ only considers the deprivation status of individuals *within* dimensions according to the criterion introduced in the previous paragraph. On the other hand, the function $\rho^b$ identifies who is poor and who is not on the basis of individuals' list of deprivations *between* dimensions. Therefore $\rho^w$ and $\rho^b$ are referred to as *within*- and *between*-dimension identification functions, respectively. In this paper, we consider $\rho^w$ as exogenously given,[7] and we focus on the different ways in which $\rho^b$ can be defined. Given the set of deprivation profiles $X_d$ and any between-dimension identification function $\rho^b : X_d \to \{0, 1\}$, we derive

[7] Implicitly, this assumes that we are working in the space of deprivations (i.e.: taking into account the dimension-specific gaps between attainments and the corresponding poverty threshold – see footnote #4). The alternative approach advocated by Ravallion (2011) of working in the space of attainments is not followed in this paper because the collapse of multivariate distributions into unidimensional ones trivially simplifies the problem of identification of the poor.

the partition $X_d = P_d \sqcup R_d$, where

$$P_d := \left\{\mathbf{x} \in X_d | \rho^b(\mathbf{x}) = 1\right\} = \left(\rho^b\right)^{-1}(1) \tag{3}$$

and

$$R_d := \left\{\mathbf{x} \in X_d | \rho^b(\mathbf{x}) = 0\right\} = \left(\rho^b\right)^{-1}(0) = X_d \backslash P_d. \tag{4}$$

Whenever an individual experiences a combination of deprivations like those included in $P_d$ (resp. $R_d$), that individual is identified as poor (resp. non-poor) according to $\rho^b$. For this reason, we refer to $P_d$ (resp. $R_d$) as a *set of poor profiles* (resp. *non-poor profiles*). Since there is a one-to-one correspondence between "sets of poor profiles" and "sets of between-dimensions identification functions" (see equation (3)), we use both sets of objects interchangeably when no confusion arises. For any $\mathbf{x} \in X_d$, let $N_\mathbf{x} \subseteq N$ denote the set of individuals experiencing deprivations as described in $\mathbf{x}$. Clearly, $\bigcup_{\mathbf{x} \in X_d} N_\mathbf{x} = N$. The number of elements in $N_\mathbf{x}$ is denoted as $n_\mathbf{x}$. For any set of poor profiles $P_d \subset X_d$ let $Q(P_d) := \{i \in N | \rho^w(\mathbf{y}_i, z) \in P_d\} = \bigcup_{\mathbf{x} \in P_d} N_\mathbf{x}$ be the set of individuals considered poor according to $P_d$. The number of '$P_d$−poor' individuals is defined as $q := |Q(P_d)| = \sum_{\mathbf{x} \in P_d} n_\mathbf{x}$.

The elements of $X_d$ can be partially ordered by $\preceq$, the partial order[8] generated by vector dominance in $X_d \times X_d$. That is: For any $\mathbf{x}, \mathbf{y} \in X_d$, $\mathbf{x} \preceq \mathbf{y}$ if and only if $x_i \leq y_i$ for all $i \in \{1, \ldots, d\}$. When this happens, we say that $\mathbf{y}$ vector-dominates $\mathbf{x}$. Observe that when a given deprivation profile $\mathbf{x}$ is vector-dominated by another deprivation profile $\mathbf{y}$ (i.e.: when $\mathbf{x} \preceq \mathbf{y}$), we might reasonably say that the state of affairs represented by the former is better than the one represented by the latter. Let $Z$ be any subset of $X_d$. On the one hand, the *up-set* of $Z$ (denoted as $Z^\uparrow$) is defined as $Z^\uparrow := \{\mathbf{x} \in X_d | \exists \mathbf{z} \in Z \text{ s.t. } \mathbf{z} \preceq \mathbf{x}\}$

[8] A *partial order* over a set $S$ is a binary relation $\preceq$ which, for any $a, b, c \in S$, satisfies the following conditions: (i) $a \preceq a$ (Reflexivity); (ii) If $a \preceq b$ and $b \preceq a$ then $a = b$ (Antisymmetry); (iii) If $a \preceq b$ and $b \preceq c$ then $a \preceq c$ (Transitivity).

(i.e.: it is the set of deprivation profiles vector-dominating at least one member of $Z$). On the other hand, the *set of undominating elements of* $Z$ (denoted as $U(Z)$) is defined as $U(Z) := \{\mathbf{x} \in Z | \nexists \mathbf{y} \in Z \backslash \{\mathbf{x}\} \text{ s.t. } \mathbf{y} \preceq \mathbf{x}\}$ (i.e.: it is the set of elements in $Z$ that do not vector-dominate any other element in $Z$). By construction, if $\mathbf{x} \in U(P_d)$ and $\mathbf{y} \in X_d$ is such that $\mathbf{y} \preceq \mathbf{x}$, then $\mathbf{y} \in R_d$. In words: for a given set of poor profiles $P_d$, the members of $U(P_d)$ are the elements representing the least deprived situation among the poor.

To clarify ideas, it is useful to graph the *Hasse diagram* corresponding to the set $X_d$ (whose elements are the nodes of the diagram) and the partial order $\preceq$ (represented by the edges between nodes). The different deprivation profiles (i.e.: the nodes) are ordered in rows depending on the number of deprivations they contain: The first row includes the profile with no deprivations, the second one the profiles with at most one deprivation, and so on. In these diagrams, it is useful to distinguish whether the different nodes belong to $P_d$ or $R_d$. In Figure 1 we show two examples of Hasse diagrams for the case $d = 4$ that will be useful to illustrate other sections of the paper. In the first one (Fig. 1a), the set of poor profiles is $P_4^1 = \{1100, 1010, 1001, 0110, 0101, 0011, 1110, 1101, 1011, 0111, 1111\}$ and in the second one (Fig. 1b) $P_4^2 = \{1100, 1011, 1110, 1101, 1111\}$. Observe that $U(P_4^1) = \{1100, 1010, 1001, 0110, 0101, 0011\}, U(P_4^2) = \{1100, 1011\}$.

[[[Figure 1a,b]]]

After the seminal contribution of Sen (1976), the measurement of poverty is commonly divided in two different but interconnected steps: the 'identification step' we have been discussing so far and the 'aggregation step' (*i.e.*: summarizing information about 'the poor' into a single number). A multidimensional poverty index is formally defined as a non-trivial function that converts an element $M$ from the space of achievement matrices $\mathcal{M}$ and a vector of deprivation thresholds $z$ (with as many elements as the number of columns in $M$) into a real number $f(M;z)$ indicating the extent of poverty in the corresponding distribution. This paper does not delve into the issue of how to perform the aggregation step (for a detailed discussion on that topic see Permanyer 2014), but we present an axiom that will be needed in section 3.2 when discussing different ways in which multidimensional poverty indices can be broken down into constituent parts to facilitate their interpretation. Assume the population we are taking into account, $N := \{1, \ldots, n\}$, is partitioned into $p$ subgroups of size $n_i$ (i.e.: $\sum_i n_i = n$). Let $M_i$ denote the achievement matrix corresponding to subgroup $i$. The axiom of *Subgroup Decomposability* (SD) states that

$$f(M;z) = \sum_i \frac{n_i}{n} f(M_i;z). \tag{5}$$

In words: Overall poverty is equal to the population weighted average of the subgroup poverty levels.

## 2.1 The dual cutoff identification method

The identification function suggested by Alkire and Foster (2011), denoted $\rho_{AF,\mathbf{a},k}$, can be written as the composite $\rho_{AF,\mathbf{a},k} = \rho^b_{AF,\mathbf{a},k} \circ \rho^w$, where $\rho^b_{AF,\mathbf{a},k}$ is in turn defined as the composite $\rho^b_{AF,\mathbf{a},k} = \iota_k \circ c_{\mathbf{a}}$, with

$$c_{\mathbf{a}} : X_d \to [0,1] \tag{6}$$

and

$$\iota_k : [0,1] \to \{0,1\}. \tag{7}$$

For any $\mathbf{x} \in X_d$, the function $c_{\mathbf{a}}$ is defined as $c_{\mathbf{a}}(\mathbf{x}) = \sum_{j=1}^{j=d} a_j x_j$, that is: $c_{\mathbf{a}}$ simply counts the weighted proportion of deprivations experienced by someone with deprivation profile $\mathbf{x}$.

Following the terminology of Alkire and Foster (2011), $c_{\mathbf{a}}(\mathbf{x})$ is referred to as *deprivation score*. Lastly, for any $s \in [0,1]$ and for any $k \in (0,1]$, $\iota_k$ is defined as

$$\iota_k(s) = \left\{ \begin{array}{c} 1 \text{ if } s \geq k \\ 0 \text{ if } s < k \end{array} \right\}. \tag{8}$$

The $\iota_k \circ c_{\mathbf{a}}$ function takes a value of 1 whenever the weighted proportion of deprivations attains a certain threshold $k$ (which is exogenously given) and a value of 0 otherwise. Summing up, the dual cutoff identification method $\rho_{AF,\mathbf{a},k}$ is defined as a composite of three functions

$$(I_1 \times \cdots \times I_d) \times (I_1 \times \cdots \times I_d) \overset{\rho^w}{\longrightarrow} X_d \overset{c_{\mathbf{a}}}{\longrightarrow} [0,1] \overset{\iota_k}{\longrightarrow} \{0,1\} \tag{9}$$

that identifies individual $i$ as being 'poor' whenever the deprivation score associated with the deprivation profile $\rho^w(\mathbf{y}_i, z)$ is not lower than $k$ (the poverty threshold across dimensions) and as 'non-poor' otherwise. Parameter $k$ indicates the proportion of weighted deprivations a person needs to experience in order to be considered multidimensionally poor. Therefore, the sets of 'AF-poor' and 'non-AF-poor' profiles can be written as

$$P_{d,AF(\mathbf{a},k)} \ : \ = \left\{ \mathbf{x} \in X_d | \sum_{j=1}^{j=d} a_j x_j \geq k \right\} \tag{10}$$

$$R_{d,AF(\mathbf{a},k)} \ : \ = \left\{ \mathbf{x} \in X_d | \sum_{j=1}^{j=d} a_j x_j < k \right\}. \tag{11}$$

The higher the value of $k$, the more difficult it is that an individual ends up being classified as poor. When $k \leq \min_j a_j$, $\rho_{AF,\mathbf{a},k}$ corresponds to the union identification approach, and when $k = 1$, $\rho_{AF,\mathbf{a},k}$ is equivalent to the intersection approach. The Hasse diagrams shown in Figures 1a and 1b illustrate examples of sets of poor profiles $P_{d,AF(\mathbf{a},k)}$ for certain combinations of $d, \mathbf{a}$ and $k$. In Figure 1a, we have chosen $d = 4, a_1 = a_2 = a_3 = a_4 = 1/4$ and $k = 1/2$ and in Figure 1b, $d = 4, a_1 = 1/2, a_2 = 1/4, a_3 = 1/8, a_4 = 1/8$ and $k = 3/4$. If one chooses equal weights, whenever a deprivation profile belongs to $P_{d,AF(\mathbf{a},k)}$, all other deprivation profiles in the same row are included in $P_{d,AF(\mathbf{a},k)}$ as well (see Fig. 1a). Alternatively, when the weights are allowed to be different it is possible that not all members of the same row are included in $P_{d,AF(\mathbf{a},k)}$ (see Fig. 1b).

Once the identification step is over, there are several methods of aggregating information to construct a multidimensional poverty index. For the purposes of this paper it will be enough to consider the multidimensional headcount ratio $H$ and the so-called adjusted headcount ratio $M_0$ suggested by Alkire and Foster (2011), which is currently being used in the construction of UNDP's MPI. Using the notation introduced in this paper, $H$ and $M_0$ can be defined as follows. For any set of poor profiles $P_d \subset X_d$, let

$$H(P_d) := \frac{1}{n} \sum_{i \in Q(P_d)} 1 = \frac{1}{n} \sum_{\mathbf{x} \in P_d} n_{\mathbf{x}} = \frac{q}{n}. \tag{12}$$

The index $H$ is simply the share of individuals that are multidimensionally poor according to $P_d$. On the other hand, the adjusted headcount ratio is defined as

$$M_0(P_d) := \frac{1}{n} \sum_{i \in Q(P_d)} c_{\mathbf{a}}(\rho^w(\mathbf{y}_i; z)) = \frac{1}{n} \sum_{\mathbf{x} \in P_d} n_{\mathbf{x}} c_{\mathbf{a}}(\mathbf{x}). \tag{13}$$

$M_0$ is simply a population average of the deprivation scores of those individuals that are multidimensionally poor according to $P_d$.

## 3. Identifying the poor: Beyond the counting approach

Roughly, the AF method is an algorithm-like procedure stipulating that if the number of deprivations experienced by an individual exceeds a certain threshold, that individual should be considered poor – irrespective of the specific combination of deprivations contributing to the count. The main aim of this section is to go beyond this counting approach suggesting more general and less stringent identification procedures that are better equipped to capture the subtelties and intrincacies involved in such a delicate matter.

Among all potential partitions of $X_d$ into the disjoint sets $P_d$ and $R_d$ (i.e., when identifying what deprivation profiles should fall into the 'poor' or 'non-poor' categories), not all possibilities are meaningful. Whenever a certain $\mathbf{x}$ belongs to $P_d$, one would expect that those deprivation profiles $\mathbf{y}$ containing at least the same set of deprivations as those in $\mathbf{x}$ should also be included in $P_d$. That is: if an individual $i$ is labeled as poor, another individual $j$ experiencing deprivations at least in the same dimensions as those where $i$ experiences deprivations, and possibly in others, should also be labeled as poor. Formally, it seems reasonable to impose that the set of poor profiles $P_d$ should respect the partial order $\preceq$ generated by vector dominance, that is:

**Definition 1**: A set of poor profiles $P_d$ satisfies the *Consistency Condition* (CC) if and only if for any $\mathbf{x} \in P_d$ and any $\mathbf{y} \in \mathbf{x}^{\uparrow}$, then $\mathbf{y} \in P_d$.

In terms of the corresponding between-dimension identification functions[9] (i.e.: in terms of $\rho^b$), the Consistency Condition stipulates that for any $\mathbf{x}, \mathbf{y} \in X_d$ with $\mathbf{x} \preceq \mathbf{y}$, one must have $\rho^b(\mathbf{x}) \leq \rho^b(\mathbf{y})$. Because of its logical solidity, we contend that the class of between dimension identification functions satisfying CC should be the universe of reference from which identification functions should be drawn. We denote by $\mathcal{P}_d$ the set of all sets of poor profiles $P_d$ satisfying CC. Given their relevance for this paper, we now characterize the elements of $\mathcal{P}_d$.

**Proposition 1**. One has that $P_d \in \mathcal{P}_d \Leftrightarrow (U(P_d))^{\uparrow} = P_d$.

[9] Given the one-to-one correspondence between 'poor profiles' and 'between-dimension identification functions', we will interchangeably use the expressions '$P_d$ satisfies CC' and '$\rho^b$ satisfies CC'.

**Proof**: See the appendix.

According to Proposition 1, the set of poor profiles satisfying CC are the sets that coincide with the up-set of their undominating elements. This implies that the sets of poor profiles $P_d$ satisfying CC are univocally characterized and represented by the corresponding subsets of undominating elements $U(P_d)$. When choosing a sensible set of poor profiles $P_d$, the subsets $U(P_d)$ are particularly important because their elements determine the least deprived conditions that individuals should experience in order to be considered as poor. Indeed, the sets $U(P_d)$ can be thought as a generalization of the concept of a poverty line to the multidimensional context (i.e.: they determine the boundary separating the poor from the non-poor). For this reason, the sets $U(P_d)$ obtained from the different $P_d \in \mathcal{P}_d$ are referred to as the sets of *boundary profiles*, and are denoted as $\mathcal{Z}$. As a consequence of Proposition 1, we say that $\mathcal{P}_d$ is the same as $\{Z^{\uparrow}\}_{Z\in\mathcal{Z}}$, that is: Any poor profile $P_d \in \mathcal{P}_d$ corresponds to the up-set of some $Z$ belonging to $\mathcal{Z}$ and vice-versa. Since $\mathcal{Z}$ contains the undominating elements of the sets of poor profiles satisfying CC, it can be written as

$$\mathcal{Z} := \{Z \subset X_d | \forall \mathbf{x} \in Z, \nexists \mathbf{y} \in Z \backslash \{\mathbf{x}\} \ s.t. \ \mathbf{y} \preceq \mathbf{x}\}. \tag{14}$$

That is: $\mathcal{Z}$ contains all subsets of $X_d$ such that any two of its members never vector-dominate one another (in particular, it contains all singletons of $X_d$).

What can be said about the dual cutoff method in this broader identification context? The sets of poor profiles $P_{d,AF(\mathbf{a},k)}$ generated by the dual cutoff method satisfy the Consistency Condition for any $\mathbf{a} \in \Delta_d$ and any $k \in (0,1]$ (i.e., $P_{d,AF(\mathbf{a},k)} \in \mathcal{P}_d$ because, whenever $\mathbf{x} \preceq \mathbf{y}$, one clearly has that $\rho^b_{AF,\mathbf{a},k}(\mathbf{x}) \leq \rho^b_{AF,\mathbf{a},k}(\mathbf{y})$). However, the following result proves that $\mathcal{P}_d$ contains other elements that cannot be generated via the AF method.

**Theorem 1**: For any $d \geq 2$ let $\mathcal{AF}_d$ be the set of all sets of poor profiles generated by the AF method, that is: $\mathcal{AF}_d := \left\{P_{d,AF(\mathbf{a},k)}\right\}_{\mathbf{a}\in\Delta_d,k\in(0,1]}$. Then, if $d \in \{2,3\}$, $\mathcal{AF}_d = \mathcal{P}_d$. For any $d \geq 4, \mathcal{AF}_d \subset \mathcal{P}_d$.

**Proof**: See the appendix.

Theorem 1 stipulates that the set of sets of poor profiles generated by the dual cutoff identification method is strictly included within the set of sets of poor profiles satisfying CC whenever the number of dimensions taken into account is greater than 3. This implies that the counting approach underlying the AF method leaves aside certain sets of poor profiles $P_d$ belonging to $\mathcal{P}_d$ that might represent a sensible way of deciding who is poor and who is not. In the following section we describe an important class of sets of poor profiles belonging to $\mathcal{P}_d$ that the AF method fails to identify.

### 3.1 Hierarchically structured composite indices

Consider a hypothetical example where the multidimensional poverty levels of individuals are assessed with the following variables: $V_1$ = 'Income', $V_2$ = 'Years of Schooling', $V_3$ = 'Self-assessed Health' and $V_4$ = 'Health insurance' (that is: $d = 4$). Assume that, for each variable, there is a threshold below which individuals should be considered deprived (in the case of $V_4$, an individual is deprived if s/he has no health insurance). In this framework, one might say that $V_1$ and $V_2$ capture alternative aspects of a broader domain one might call "Capacity to make a living" (denoted as $D_1$) while $V_3$ and $V_4$ capture different aspects within the domain of "Health " (denoted as $D_2$). When deciding who is poor and who is not, one might reasonably argue that if someone is only deprived in $V_1$, then she should not be identified as 'poor' because her high level of education might somehow compensate and potentially offer some alternatives for the current lack of income in the capacity to make a living. If someone is only deprived in $V_2$, then he might not be identified as 'poor' because his lack of education can be compensated by his high level of income. Here, one might say that in order to be identified as ‘poor’, an individual should experience deprivations at least in $V_1$ *and* $V_2$ simultaneously – something which would severly hinder that individual’s capacity to make a decent living. Analogously, one could argue that an individual is poor whenever she experiences deprivations in at least $V_3$ *and* $V_4$ simultaneously (an alarming circumstance for that individual’s health), but not poor if she only experiences deprivation in one of the two variables separately (good self-assessed health might somehow compensate for the lack of health insurance and vice-versa). Lastly, one could also argue that when an individual is only deprived in one variable within $D_1$ *and* in one variable within $D_2$, then that individual should *not* be identified as ‘poor’ because the variable within each domain where that individual attains a good achievement somehow compensates for the deprivation experienced in the other variable. For instance: an individual deprived in $V_2$ and $V_3$ only might not be classified as poor because her high income and health insurance might compensate in a way for her low levels of education and low self-assessed health respectively. Formally, all the previous arguments are summarized stating that the set of poor profiles $P_4^* = \{1100, 0011, 1110, 1101, 1011, 0111, 1111\}$ can be a reasonable choice when deciding who is poor and who is not for the case $d = 4$ (to illustrate, the Hasse diagram corresponding to $P_4^*$ is shown in Figure 2; observe that $U(P_4^*) = \{1100, 0011\}$). Interestingly, it is straightforward to check that while $P_4^*$ satisfies the Consistency Condition (i.e.: $P_4^* \in \mathcal{P}_4$), $P_4^*$ does not belong to $\mathcal{AF}_4$ for any $\mathbf{a} \in \Delta_4$ and any $k \in (0, 1]$ (see the proof of Theorem 1). In other words: No matter what weighting scheme $\mathbf{a}$ or what deprivation score threshold $k$ we choose, the AF identification method never generates a set of poor profiles like $P_4^*$.

[[[Figure 2]]]

Whenever the number of variables taken into account is greater than four, analogous examples of sets of poor profiles that cannot be generated via the AF method can be constructed. Assume some subset $S \subseteq D$ of the set of variables taken into account is partitioned into at least two exhaustive and mutually exclusive groups $S_1, \ldots, S_G$ and that each group $S_g$ contains $m_g$ variables with $m_g \geq 2 \forall g$. One can define a set of poor profiles satisfying CC (denoted as $P_d^*$) that is obtained whenever we apply the following rule for a between-dimensions identification function $\rho^b$: If an individual experiences deprivation in all variables included within some of the groups, $S_1, \ldots, S_G$, then she will be identified as poor. However, if that individual is deprived in some, but not all, variables within the different groups $S_1, \ldots, S_G$, then she will not be considered poor.[10] It turns out that $P_d^*$ can never be generated via the dual cutoff method. Formally, we have the following proposition.

**Proposition 2**. Let $d \geq 4$. Consider a set of poor profiles $P_d^* \in \mathcal{P}_d$ such that $U(P_d^*) \supseteq \{\mathbf{1}_{S_1}, \ldots, \mathbf{1}_{S_G}\}$, with $(S_1, \ldots, S_G)$ being a partition of some subset $S \subseteq D$ with $|S_i| \geq 2 \forall i \in \{1, \ldots, G\}$ and $G \geq 2$ (i.e.:$(S_1, \ldots, S_G) \in \Pi_{S,G}$). There exists no weighting scheme $\mathbf{a} \in \Delta_d$ and deprivation score threshold $k \in (0, 1]$ that are able to generate such set of poor profiles by means of the AF method.

**Proof**: See the appendix.

This kind of identification rule might be meaningful when it can be argued that the lack of deprivation in some of the variables of a given group might somehow compensate for the deprivations experienced in the other ones within the same group. Importantly for the

[10] We are not contending that such general identification rules are meaningful in *all* possible contexts. Indeed, it would be quite easy to construct examples where it is not (e.g., if there are $G = 2$ groups of variables with five variables each, the rule would not identify as poor an individual deprived in four variables within each group – totalling 8 out of 10 possible deprivations). We are simply arguing that it is possible to construct perfectly sensible and meaningful identification functions satisfying the Consistency Condition that are not covered by the dual cutoff method.

purposes of this paper, it turns out that the examples presented here are not artificial constructs that are unlikely to appear in practice, but quite the opposite: we contend that such classes of identification rules are very likely to arise whenever the multidimensional indices we are using to assess poverty levels are *hierarchically structured.*[11] Figure 3 illustrates a simple example where the different variables composing the multidimensional index are hierarchically structured around several domains.[12] The use of this kind of index is becoming increasingly popular since they provide simple comparisons of countries that can be used to illustrate complex and sometimes elusive issues in wide-ranging fields, such as environment, economy, society or technological development. In particular, UNDP's Multidimensional Poverty Index is a well-known example of a hierarchically structured composite index that has ten variables structured around three domains ('Health', 'Education' and 'Standard of Living' – with two variables in the first two domains and six in the third one, see section 4.2 for details).

[[[Figure 3]]]

Whenever a composite index of poverty is hierarchically structured, it generates a natural partition of variables into a set of domains $D_1, \ldots, D_G$. The partition of variables between domains introduces a layer of complexity that opens up the possibility of complicated interaction patterns that should be taken into account when determining the poverty status of individuals. It is not difficult to imagine situations in which one might want compensation between deprived and non-deprived variables to occur within certain domains but not in

[11] Roughly speaking, a hierarchically structured composite index can be defined as a composite index whose different indicators are organized in a tree-like manner such that each element is only linked to one higher order element in the hierarchy (e.g.: each indicator can only belong to one domain).

[12] Even if the ideas presented below apply as well for more sophisticated tree-like structures with domains, sub-domains and the like, the simple structure of indicators partitioned into different domains is sufficient to illustrate our point.

others. If one is willing to allow for the possibility of such compensation phenomena within or between certain domains, there is reason to make room for the more sophisticated identification methods proposed in this paper. In contrast to the AF method, we contend that the set of identification functions satisfying CC is rich enough to capture the intertwined relationships between groups of variables that might be observed in diverse empirical applications. We conclude that because of the simplicity of the counting approach, the dual cutoff identification methodology precludes the possibility of generating certain identification rules that researchers or policy-makers might reasonably want to incorporate in poverty eradication programs. To further illustrate this point, in section 4 we present two empirical examples that show the extent to which the dual cutoff method and some of the identification rules advocated in this paper disagree when determining the set of individuals that should be considered poor.

## 3.2 Profile decomposability

An attractive characteristic of the Multidimensional Poverty Index suggested by Alkire and Foster (2011) is its purported ability to assess the contribution of each dimension to the values of the index. Once the identification step is over, the additive separability of the index allows decomposing its values according to the percent contribution of its basic constituents, a property referred to as 'dimensional decomposability'. Clearly, this property is motivated by the desire of facilitating the design of the most effective poverty eradication strategy.

Despite the apparent simplicity and intuitive appeal behind dimensional decomposability, we contend that this property is reflective of the trading-and-piling-like identification procedure in which deprivations across dimensions are freely interchangeable as long as they add up to the corresponding deprivation score. Because of the way in which it is defined, dimensional decomposability disregards the complex patterns in which dimensions are interwoven to generate the partition of deprivation profiles ($X_d$) into poor and non-poor profiles ($P_d$ and $R_d$). In other words, it does not take into account the possibility that deprivations in some dimensions might have to be experienced jointly with deprivations in other dimensions if someone is to be identified as being multidimensionally poor. After performing a dimensional decomposability exercise, policy makers are incentivized to focus on reducing deprivations in the dimension that contributes the most to multidimensional poverty levels – e.g., $V_i$. However, the reduction of deprivations in $V_i$ might require entirely different policies if those deprivations are jointly experienced with deprivations in, e.g., $V_j$, or with deprivations in $V_l$. Therefore, we suggest complementing dimension decomposability by another decomposability property that is consistent with the identification method suggested in this paper.

The set of deprivation profiles naturally generates a partition of the population under study, $N$, into $|X_d| = 2^d$ groups (each individual $i$ is assigned via $\rho^w$ to the corresponding element in $X_d$ on the basis of her achievement vector $\mathbf{y}_i$). For any $\mathbf{x} \in X_d$ let $M_{\mathbf{x}}$ denote the achievement matrix corresponding to the set of individuals experiencing deprivations as in $\mathbf{x}$ (i.e., $N_{\mathbf{x}}$). The multidimensional poverty level corresponding to the members of $N_{\mathbf{x}}$ is written as $f(M_{\mathbf{x}}; z)$. According to the axiom of SD,

$$f(M; z) = \sum_{\mathbf{x} \in X_d} \frac{n_{\mathbf{x}}}{n} f(M_{\mathbf{x}}; z) \tag{15}$$

The percent contribution of the members of $N_{\mathbf{x}}$ to overall poverty levels is thus calculated as

$$C_{\mathbf{x}} = 100 \left( \frac{n_{\mathbf{x}}}{n} f(M_{\mathbf{x}}; z) \right) / f(M; z). \tag{16}$$

Clearly, $\sum_{\mathbf{x} \in X_d} C_{\mathbf{x}} = 100$. The exercise of breaking down overall poverty into the set of contributions $\{C_{\mathbf{x}}\}_{\mathbf{x}\in X_d}$ is referred to as profile decomposability. Profile decomposability conveys a clearer message than its dimension-wise counterpart with respect to understanding the articulation of multidimensional poverty. Since the different population subgroups in $\{N_{\mathbf{x}}\}_{\mathbf{x}\in X_d}$ might require profile-specific anti-poverty strategies (i.e., anti-poverty strategies specifically crafted for them), profile decomposability can be particularly informative for the design of efficient poverty erradication programs.

# 4. Empirical illustrations

In this section we present two empirical examples to show the mismatch between some of the identification methods suggested in this paper and the dual cutoff method suggested by Alkire and Foster (2011). The first example uses data from the United States and the second one focuses on 48 countries from the developing world.

## 4.1 United States

In order to illustrate the usefulness of their multidimensional poverty measures, Alkire and Foster (2011) presented an empirical exercise using the 2004 National Health Interview Survey from the US. In that exercise, the authors used the following four variables to assess multidimensional poverty levels among adults aged 19 and above: $V_1$ = 'Income measured in poverty line increments and grouped into 15 categories', $V_2$ = 'Years of Schooling', $V_3$ = 'Self-assessed Health' and $V_4$ = 'Health insurance'. The dimension-specific deprivation thresholds were defined as follows. A person is deprived in $V_1$ if she lives in a household falling below the standard income poverty line, in $V_2$ if he lacks a high school diploma, in $V_3$ if she reports 'fair' or 'poor' health and in $V_4$ if he lacks health insurance. The population is partitioned into four groups (Hispanic/Latino, (Non-Hispanic) White, (Non-Hispanic) African American / Black and Others) and the sample size is $n$ =45884.

To identify poor individuals, Alkire and Foster basically use the dual cutoff method assuming equal weights across dimensions (i.e.: $a_1 = a_2 = a_3 = a_4 = 1/4$) and a deprivation threshold $k = 1/2$.[13] This way, whenever an individual is deprived in at least two dimensions (any), she will be considered poor. With the notation introduced in this paper, this generates the set of poor profiles $P_4^1 = \{1100, 1010, 1001, 0110, 0101, 0011, 1110, 1101, 1011, 0111, 1111\}$. However, if one is willing to allow for the role of compensation within domains (see section 3.1), there are good reasons to argue that in order to be considered poor an individual has to experience deprivation at least in $V_1$ and $V_2$ or in $V_3$ and $V_4$ simultaneously, therefore generating the set of poor profiles $P_4^* = \{1100, 0011, 1110, 1101, 1011, 0111, 1111\}$. Since $P_4^*$ can never be generated via the AF identification method, it is informative to compare the poverty levels derived from it with the poverty levels reported by Alkire and Foster (2011) when using $P_4^1$.

We start by reporting the shares of individuals that are coherently identified as poor or non-poor according to $P_4^1$ and $P_4^*$ together with the shares of individuals that are misclassified according to the two criteria across the four racial groups (see Table 1). Since $P_4^* \subset P_4^1$, the set of individuals that are coherently identified as poor by the two methods corresponds to the set of individuals with deprivation profiles $\mathbf{x}$ belonging to $P_4^*$ (their percentages are reported in column A). The individuals that are coherently identified as non-poor by the two methods must have a deprivation profile belonging to $\{0000, 1000, 0100, 0010, 0001\}$ (their percentages are reported in column B). The individuals that are identified as poor by $P_4^1$ but as non-poor by $P_4^*$ are the ones with deprivation profiles in $\{1010, 1001, 0110, 0101\}$ (the respective percentages are reported in column C). As shown in the last row of Table 1, the shares of individuals that are coherently identified as poor and non-poor are 7.1%

[13] At the end of the exercise, they show alternative results when choosing $k \in \{1/4, 1/2, 3/4, 1\}$.

and 83.9%, respectively. The share of individuals that are misclassified according to the two identification methods is 9%, and is particularly high among Hispanics (20%). In Table 1 we also show the values of two multidimensional poverty indices resulting from alternative identification methods. One of them is the multidimensional headcount ratio $H$ (see equation (12)) and the other one is the adjusted headcount ratio $M_0$ proposed by Alkire and Foster (2011) (see equation (13)). In Columns D and E we show the values of $H$ when using $P_4^1$ and $P_4^*$ as identification methods respectively, while the analogous results corresponding to $M_0$ are shown in Columns F and G. The values of the headcount index vary substantially between $P_4^1$ and $P_4^*$: They more than halve the original levels (since $P_4^* \subset P_4^1$, the values of $H$ are necessarily smaller). A similar pattern is observed when computing the values of $M_0$: When moving from $P_4^1$ to $P_4^*$ the values of the adjusted heacount ratio more than halve. We observe no changes between multidimensional poverty *rankings* when moving from one identification method to the other. However, the set of people that could potentially be the target of anti-poverty programs varies substantially across methods.

[[[Table 1]]]

We turn now to the issue of decomposability. According to dimension decomposability, the values of $M_0$ can be broken down by the contribution of the four variables taken into account.[14] More specifically, we write $M_0 = \sum_j H_j/d$, where $H_j$ is the share of the respective population that is both poor (according to the AF identification method using $P_4^1$) and deprived in variable $j$. However, among the people that are both 'AF-poor' and deprived in variable $j$, there is a subgroup of individuals that are not poor according to the identification method $P_4^*$. To compute the relative size of this subgroup for the case

[14] See Table 2 in Alkire and Foster 2011 for the specific results.

where $j = 1$ (i.e.: in the case of $V_1$ = 'Income') we simply need to compute the following quantity: $(N_{1010} + N_{1001})/(N_{1100} + N_{1010} + N_{1001} + N_{1110} + N_{1101} + N_{1011} + N_{1111})$. The respective denominator contains all individuals that are 'AF-poor' *and* deprived in terms of income while the numerator counts how many of them are considered to be non-poor according to $P_4^*$. That would be the share of people contributing to $H_1$ that are mistargeted according to $P_4^*$. In Table 2 we show the percentage of mis-targeted individuals for the four variables taken into account across the different racial groups. The presence of mis-targeted individuals is quite substantial, on many occasions with values above 50%. This suggests that the alternative methods discussed in this section identify groups of individuals differing to a great extent.

[[[Table 2]]]

We conclude this empirical illustration with the results of the profile decomposability exercise suggested in section 3.2. In Table 3 we show the multidimensional poverty levels (as measured with $M_0$; they are reported in the third column) corresponding to each group $N_{\mathbf{x}}$ for the different $\mathbf{x} \in P_4^*$ and the corresponding contribution to overall poverty ($C_{\mathbf{x}}$, shown in the fourth column). The shares of the different groups $N_{\mathbf{x}}$ are reported in the second column. The deprivation profile experienced by the largest share of individuals is 1110 (that is: those having health insurance but deprived in all other variables) and the one experienced by the smallest share of individuals is 1011 (i.e., those having a high school diploma but deprived in all other variables). As expected, the groups experiencing more deprivations tend to be poorer, so their contribution to overall poverty levels is higher. This is the reason why even if the set of individuals experiencing deprivation in income and education only are four times more numerous than those individuals deprived in all dimensions ($N_{1100}/N = 1.68\%$ and

$N_{1111}/N = 0.43\%$), the contribution of the former to overall poverty levels is barely twice that of the latter ($C_{1100} = 19.44\%$ vs. $C_{1111} = 8.33\%$).

[[[Table 3]]]

## 4.2 Developing World

Since 2010, the UNDP presents the values of the Multidimensional Poverty Index (MPI) on a yearly basis to rank more than a hundred countries in terms of multidimensional poverty levels (see Alkire and Santos 2010). The UNDP's MPI mainly draws from three sources of data: the Demographic and Health Surveys (DHS), the Multiple Indicators Cluster Survey and the World Health Survey. In order to avoid the comparability problems arising from the use of alternative sources of data, in this paper we focus our attention on 48 out of the 50 DHS used in the construction of the 2014 MPI [15] (totaling $n$=761,909 households, which are the basic units of analysis).

The MPI is a hierarchically structured index of multidimensional poverty, with ten variables partitioned in three domains: 'Health' (H), 'Education' (E) and 'Standard of Living' (S). In Table 4 we show the variables included in each domain. The 'Health' and 'Education' domains are composed of two variables each: One referring to adults and the other to children in the corresponding household. The six variables in the 'Standard of Living' domain

[15] The DHS for Nicaragua 2012 and Tajikistan 2012 were not accessible to the authors of this paper. The remaining 48 countries included in the dataset and the year/s in which the DHS was taken are: Albania 2008/2009; Armenia 2010; Azerbaijan 2006; Bangladesh 2011; Benin 2006; Bolivia 2008; Burkina Faso 2010; Burundi 2010; Cambodia 2010; Cameroon 2011; Colombia 2010; Congo 2011/2012; Cote d'Ivoire 2011/2012; Dominican Republic 2007; Egypt 2008; Ethiopia 2011; Gabon 2012; Guinea 2005; Guyana 2009; Haiti 2012; Honduras 2011/2012; India 2005/2006; Indonesia 2012; Jordan 2009; Kenya 2008/2009; Lesotho 2009; Liberia 2007; Madagascar 2008/2009; Malawi 2010; Maldives 2009; Mali 2006; Moldova 2005; Mozambique 2011; Namibia 2006/2007; Nepal 2011; Niger 2012; Pakistan 2012/2013; Peru 2012; Philippines 2008; Rwanda 2010; Sao Tome and Principe 2008/2009; Senegal 2010/2011; Tanzania 2010; Timor-Leste 2009/2010; Uganda 2011; Ukraine 2007; Zambia 2007 and Zimbabwe 2010/2011.

include several household characteristics. In Table 4, we also show the conditions that must be met in order to consider a household deprived in the corresponding variable. Lastly, the table also shows the weight that the AF method assigns to each variable.

[[[Table 4]]]

To decide whether a household should be identified as poor or not, the MPI uses the AF method with the weights shown in Table 4. The three domains are equally weighted at 1/3, with a deprivation threshold of $k = 1/3$. This allows trading and piling deprivations across dimensions to decide who is poor and who is not. For instance, a household experiencing deprivation in one of the Health variables and in one of the Education variables (each with a weight of 1/6) is identified as poor. Analogously, a household experiencing deprivation in one of the Health or Education variables and in any three of the Standard of Living variables is identified as poor. However, if one considers that the lack of deprivation in some variables within some domain could somehow compensate for the deprivations experienced in the other variables of that domain, then the AF identification method is not the most appropriate (for instance: one might argue that the deprivation experienced by parents might somehow be compensated by the lack of deprivation of the children). Instead, one might prefer to define a houshold as being poor whenever the corresponding deprivation profile belongs to the set of poor profiles $P^*_{10} = \{1100000000, 0011000000, 0000111111\}^{\uparrow}$. Observe that in order to avoid trading and piling deprivations across domains, a '$P^*_{10}-$poor household' must be deprived in all variables of at least one domain. Since one might argue that requiring a household to be deprived in all six 'Standard of Living' indicators to be considered as poor is too stringent a condition, one could also relax this assumption and define another set of poor profiles, $Q^*_{10}$, as follows. A '$Q^*_{10}-$poor household' must be deprived in two variables in at least one of the domains of 'Health' and 'Education' (i.e.: $U(Q^*_{10}) \supset \{1100000000, 0011000000\}$) *or* it must be deprived in at least four of the six variables comprising the 'Standard of Living' domain. Because of the way in which it is defined, $Q^*_{10}$ can be seen as a mixed case between $P^*_{10}$ and the counting approach. While both $P^*_{10}$ and $Q^*_{10}$ satisfy the Consistency Condition, neither of them can be generated via the AF identification method (see Proposition 2). Since both represent reasonable criteria to identify poor households, in this section we compare how they perform vis-à-vis the dual cutoff method.

We first perform a validation check to assess the quality and soundness of the 48-country dataset created for this section of the paper. More specifically, we compare the official UNDP's 2014 MPI value, restricted to the 48 countries whose MPI values where estimated using DHS, with the MPI values obtained using the Alkire and Foster (2011) $M_0$ index applied to this dataset. Unsurprisingly, both sets of measures give highly consistent results. As shown in Figure 4 both measures tend to rank countries in a strongly linear fashion: The correlation coefficient is as high as 0.94. The differential treatment of missing values and some slight differences in the definition of the Nutrition variable[16] explain the differences observed between both measures. These results confirm that the dataset we use is of reasonable quality.

[[[Figure 4]]]

In the first three columns of Table 5 we present the country values of Alkire and Foster's poverty index $M_0$ shown in equation (13) under three different poor identification functions: (i) The 'classical' dual cutoff method that weights the three domains of the MPI equally

[16] In the official MPI, a household is deprived in the nutrition variable if any adult or child for whom there is nutritional information is malnourished. In the MPI measure constructed in this paper, the nutritional information has only been collected for the adult household members.

at 1/3 and uses a deprivation threshold of $k = 1/3$; (ii) $P^*_{10}$ and (iii) $Q^*_{10}$. We denote them as $M_0(P_{10,AF((1/3,1/3,1/3),1/3)})$, $M_0(P^*_{10})$ and $M_0(Q^*_{10})$, respectively. As can be seen, the values of the different $M_0$ go in the same direction for the three cases: Countries with low or high poverty levels coincide substantially. The correlation coefficient between the 48 values of $M_0(P_{10,AF((1/3,1/3,1/3),1/3)})$ and $M_0(P^*_{10})$ and the correlation coefficient between the 48 values of $M_0(P_{10,AF((1/3,1/3,1/3),1/3)})$ and $M_0(Q^*_{10})$ are very high, at 0.95 and 0.98, respectively. Since these correlation coefficients implicitly depend on the weights used for each domain $\mathbf{w} = (w_1, w_2, w_3) \in \Delta_3$ and the deprivation threshold $k \in (0, 1]$, they are denoted as $r_{AF,P^*_{10}}((w_1, w_2, w_3), k)$ and $r_{AF,Q^*_{10}}((w_1, w_2, w_3), k)$, respectively. Therefore, we can write $r_{AF,P^*_{10}}((1/3, 1/3, 1/3), 1/3) = 0.95$, and $r_{AF,Q^*_{10}}((1/3, 1/3, 1/3), 1/3) = 0.98$.

Even if the three measures tend to rank countries in a highly consistent way, it turns out that the corresponding poor identification functions operate in a distinct manner. In Table 5 we show for each country the percentage of households where the AF-method and $P^*_{10}$ disagree (i.e., we quantify the share of households that are misclassified as 'poor' or 'non-poor' according to the $P_{10,AF((1/3,1/3,1/3),1/3)}$-method and $P^*_{10}$). Since these percentages implicitly depend on the weights that are used for each domain $\mathbf{w} = (w_1, w_2, w_3) \in \Delta_3$ and the deprivation threshold $k \in (0, 1]$, we denote them as $m_{AF,P^*_{10},c}((w_1, w_2, w_3), k)$, where $c$ indexes the 48 countries taken into account. It turns out that the degree of disagreement between both identification methods is substantial: Averaging across countries (i.e., computing $\overline{m}_{AF,P^*_{10}}((1/3, 1/3, 1/3), 1/3) := \sum_{c=1}^{c=48} m_{AF,P^*_{10},c}((1/3, 1/3, 1/3), 1/3)/48$), we find that 24% of households are classified inconsistently between the two criteria. In some countries the percentage of disagreement is greater than 50%. Repeating the same exercise comparing the AF-method with $Q^*_{10}$, we obtain a cross-country average of $\overline{m}_{AF,Q^*_{10}}((1/3, 1/3, 1/3), 1/3) = 13\%$ of misclassified households. The size of these percentages implies that the potential beneficiaries of poverty alleviation programs can differ dramatically when choosing one identification method or the other.

[[[Table 5]]]

In Table 5 we have compared the performance of $P_{10}^{*}$ and $Q_{10}^{*}$ with the 'official' AF-method that weights the three domains of the MPI equally at 1/3 and uses $k = 1/3$. In this context, one might wonder whether the results shown in Table 5 are highly dependent on the specific choice of these parameters or if they are robust to other specifications. Since the dual cutoff method does not *a priori* impose any restrictions on the choice of weights $\mathbf{w}$ or the deprivation threshold $k$, we complete our comparative analysis allowing these parameters to take all possible values within their respective domains. In other words, we compare the performance of $P_{10}^{*}$ and $Q_{10}^{*}$ with the dual cutoff method considering *all* possible weighting schemes for the three domains of the MPI[17] , and under *any* deprivation threshold $k \in (0, 1]$ (that is, considering all possible elements of $\mathcal{AF}_d := \{P_{d,AF(\mathbf{a},k)}\}_{\mathbf{a}\in\Delta_d,k\in(0,1]}$). In the left-hand side of Figure 5 we have plotted the values of the correlation coefficient $r_{AF,P_{10}^{*}}((w_1, w_2, w_3), k)$ when the weights assigned to the domains E, H and S are allowed to take any value within the unitary simplex $\Delta_3$ and $k$ takes the values of $0.2, 0.4, 0.6, 0.8$ and $1$. In the right-hand side of the same Figure we have the analogous results for $r_{AF,Q_{10}^{*}}((w_1, w_2, w_3), k)$. As seen on both sides of the panel (i.e., both for $P_{10}^{*}$ and $Q_{10}^{*}$), the correlation coefficients with the values of $M_0(P_{10,AF((w_1,w_2,w_3),k)})$ are quite high when $k = 0.2$. These tend to increase even further for $k = 0.4$ and $k = 0.6$ and then decrease for higher values of $k$. Though high, these correlation coefficients never reach the value of 1 (a consequence of the fact that neither

[17] To simplify matters, we allow all possible weights across domains only (i.e.: not across all 10 indicators). Once a weight is assigned to each domain, we assume that all indicators within that domain are weighted equally.

$P_{10}^*$ nor $Q_{10}^*$ belong to $\mathcal{AF}_d$). There is much complexity in the patterns shown in Figure 5, but in all cases the correlation coefficients tend to be relatively high. The average of $r_{AF,P_{10}^*}((w_1,w_2,w_3),k)$ and $r_{AF,Q_{10}^*}((w_1,w_2,w_3),k)$ across the entire domain $\Delta_3 \times (0,1]$ equal 0.91 and 0.89 respectively. From this analysis, we conclude that the identification methods $P_{10}^*$ and $Q_{10}^*$ tend to rank countries in the same direction as the dual cutoff method does.

The fact that $M_0(P_{10,AF((w_1,w_2,w_3),k)})$, $M_0(P_{10}^*)$ and $M_0(Q_{10}^*)$ tend to rank countries similarly does not necessarily imply that the three methods agree when deciding whether a given household should be considered 'poor' or 'non-poor'. In Figure 6a we show the centiles of $\overline{m}_{AF,P_{10}^*}((w_1,w_2,w_3),k)$ (the 48-country average of misclassified households according to the $P_{10,AF((w_1,w_2,w_3),k)}$-method and $P_{10}^*$) when the weights assigned to the domains E, H and S are allowed to take any value within the unitary simplex $\Delta_3$ for all possible values of $k \in (0,1]$. In Figure 6b, we show the analogous results for $\overline{m}_{AF,Q_{10}^*}((w_1,w_2,w_3),k)$. As $k$ increases we observe a U-shaped distribution of the different centiles of $\overline{m}$, both for $P_{10}^*$ and $Q_{10}^*$. When $k$ is slightly above 0, the values of $\overline{m}$ tend to be high, with medians around 70% and 50% for $P_{10}^*$ and $Q_{10}^*$, respectively (i.e., high levels of disagreement). Then, the centiles of $\overline{m}$ decrease until they reach a minimum (with medians around 15% and 25% for $P_{10}^*$ and $Q_{10}^*$, respectively). The centiles of $\overline{m}$ increase again for higher values of $k$. Since neither $P_{10}^*$ nor $Q_{10}^*$ belong to $\mathcal{AF}_d$, it turns out that all values of $\overline{m}$ are strictly positive. Indeed, the average value of $\overline{m}_{AF,P_{10}^*}((w_1,w_2,w_3),k)$ and $\overline{m}_{AF,Q_{10}^*}((w_1,w_2,w_3),k)$ across the entire domain $\Delta_3 \times (0,1]$ equal 27% and 32%, respectively. From these analyses we can conclude that the level of disagreement between the identification functions considered here are generally quite substantial, a result with strong implications for the identification of the potential beneficiaries of poverty eradication programs.

[[[Figure 5: Panel triangles with correlation coefficients for $P^*_{10}$ and $Q^*_{10}$]]]

[[[Figure 6a: Centile distributions with the values of $\overline{m}$ for $P^*_{10}$]]]

[[[Figure 6b: Centile distributions with the values of $\overline{m}$ for $Q^*_{10}$]]]

## 5. Discussion and concluding remarks

The success of any poverty eradication program crucially depends on its ability to identify who is poor and who is not. In this paper, we have shown that *the* state-of-the-art methodology that is pervasively used to identify the poor in multidimensional contexts, the dual cutoff or AF method (Alkire and Foster 2011), is an all too often simplistic method that precludes many of the subtle and complex considerations that should be incorporated in such consequential decisions. One of the main findings of this work is that the simplicity of the counting approach that underlies the dual cutoff method – an algorithm-like approach that simply counts the number of deprivations experienced by individuals to decide about their poverty status – precludes the possibility of generating 'poor-identification rules' that are sensitive to potential interactions between the sets of dimensions taken into account. Depending on the nature of the variables considered, it could be the case that one might want the lack of deprivation in some dimension $X$ to compensate for the deprivation experienced in some dimension $Y$ *but* not in $Z$. We contend that such patterns of dimension-specific interactions naturally arise when multidimensional indices are hierarchically structured in exhaustive and mutually exclusive domains, as is increasingly the case in all areas of the social sciences.

As acknowledged by Alkire and Foster (2011: 482), the dual cutoff method is fundamentally related to the axiomatic literature on freedom, and more specifically to the work of Pattanaik and Xu (1990). Pattanaik and Xu axiomatically characterize a counting approach to measure freedom that ranks opportunity sets according to the number of options they contain. Given the dismal view expressed by the authors after the triviality of the quantitative nature of their results, it is somewhat surprising that this approach is used to justify the ethical foundations of the dual cutoff method.

To overcome the blind aggregation underlying the dual cutoff method, we suggest a much broader and less stringent identification approach that contains the latter as a particular case. More specifically, we only impose that whenever an individual $i$ is labeled as poor, another individual $j$ experiencing deprivations at least in the same dimensions as those where $i$ experiences deprivations (and possibly in others) should also be labeled as poor. This is the so-called Consistency Condition (CC). Because of its logical validity, we contend that the identification methods satisfying CC should be the universe of reference from which poor identification functions should be drawn. We show that the dual cutoff method is *not* able to generate certain identification rules satisfying the Consistency Condition that researchers or policy-makers might reasonably want to incorporate in poverty eradication programs. The conditions imposed under CC are flexible enough to allow capturing the intertwined relationships between groups of variables between and within domains observed in diverse empirical applications. In addition, since the CC approach makes room for the possibility of compensation within some domains and non-compensation in others, it represents an improvement with respect to the current state of the literature, which assumes the same degree of complementarity or substitutability across deprivations (see Alkire and Foster 2011, p. 485-486).

Another attractive characteristic of the dual cutoff method is its purported ability to explain the contribution of each dimension to the overall values of the poverty index (a property known as 'dimensional decomposability'). However, this property implicitly ignores the interaction patterns existing between dimensions (that is, the fact that deprivations in some dimensions must be experienced jointly with deprivations in other dimensions if someone is to be identified as being multidimensionally poor). Decision-makers guided by 'dimensional decomposability' have incentives to allocate resources to reduce deprivations in the dimension contributing the most to overall poverty levels (say, $X$), irrespective of the huge difference it may make to experience deprivations in $X$ jointly with deprivations in $Y$ rather than experiencing deprivations in $X$ and $Z$. We suggest complementing 'dimensional decomposability' with 'profile decomposability', another property that is naturally derived from the CC identification methods suggested in this paper and which conveys a clearer message to understand the articulation of multidimensional poverty. More specifically, 'profile decomposability' explicitly accounts for patterns of joint deprivation, so it is particularly useful for the design of 'profile-specific anti-poverty strategies', i.e., anti-poverty strategies specifically crafted for a group experiencing a certain pattern of multiple deprivations.

In the empirical section of the paper we investigate the performance of alternative poor-identification rules in two separate studies. The first uses data from the US in 2004, and the second uses data from 48 Demographic and Health Surveys collected around 2010. The findings for both cases are robust. It turns out that the dual cutoff method and the alternative CC methods that can not be generated via the AF methodology tend to consistently rank the populations compared in terms of poverty levels. In other words: the populations experiencing high or low poverty levels using both identification methods coincide substantially. Even if the relative position of the populations that are being compared does not change substantially, what *does* substantially change is the corresponding *level* of poverty observed under alternative identification methods. The percentage of individuals or households that are inconsistenly identified as 'poor' according to both criteria is considerably high (for the

48 developing countries example, it is around 30%). We reiterate that these differences can have enormous implications for the identification of the potential beneficiaries of poverty eradication programs.

The main advantage of the CC approach is its flexibility in capturing the complications and intrincacies involved in the identification of the poor. Unlike the AF method, the CC poor-identification rules suggested here do not follow an algorithm-like procedure that can be quasi-automatically implemented in a wide variety of settings. Instead, it forces analysts and policy makers to think about the meaning of being multiply deprived in different contexts. With the target date of the Millennium Development Goals (MDGs) rapidly approaching, many scholars and policy-makers are currently engaged in an intense debate about what kind of headline poverty indicator should be the most appropriate to guide poverty eradication strategies in the post-2015 global development agenda. Like its predecessor, the first of the so-called Sustainable Development Goals (the SDGs) aims to 'End Poverty in all its forms everywhere'. This is a good moment to take stock and reflect before uncritically extending use of the dual cutoff method. Other procedures, such as the ones suggested here, exist to identify recipients under one of the greatest international endeavours of our time to eradicate poverty.

## 6. Appendix

### Proofs

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