BASIC MATHEMATICAL KNOWLEDGE OF STUDENTS ENROLLING FOR PRIMARY EDUCATION UNIVERSITY DEGREES

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Our main object of research is the mathematical knowledge of students who enrol at University to begin their degree in Primary School Teaching. By Basic Mathematical Knowledge (BMK) we understand the initial mathematical background we desire students to have at the start of their education to become Primary School teachers. In this paper we present a first approach to the notion of BMK and provide empirical data that justify the relevance of this concept as an element to evaluate and reflect upon the knowledge of candidates for the Degree in Primary Education.

INTRODUCTION

To this date, University entrance exams in Spain are identical for all degrees and for all students, regardless of the type of study they want to pursuit. However, soon there will apply a new legislation and universities will have to design their own entrance tests. Therefore, it is essential to find a more precise way of establishing what these tests should aim to measure for the entrance to each university degree.

Several international studies have proved the importance of mathematical training of Primary teachers. However, there is little research about students' mathematical background at the beginning or their education to become teachers. Our attention is focused on the mathematical knowledge of the students enrolling for Primary Teaching degrees in Catalonia. We aim to open a reflection with the final aim of constructing a tool to evaluate the mathematical knowledge of students who begin their studies to become teachers. By Basic Mathematical Knowledge (BMK) we understand the initial mathematical background we would like students to possess when starting their education as Primary school teachers. Determining what their BMK should be to start a Primary Teaching degree is essential, since this is what the students' mathematical knowledge for teaching should be built upon.

In this communication we would like to introduce a first definition and characterization of the concept of BMK together with some results obtained in a first test taken by students that begin their degree in Primary Education at the Universitat Autònoma de Barcelona.

BASIC MATHEMATICAL KNOWLEDGE

It cannot be expected of students who start their degree to have received a previous education providing them with a deep understanding of the mathematics concepts in their syllabus, or the ability to transmit their knowledge to others. For this reason, the BMK would only be a part of Shulman's (1986, 1987) Content Knowledge or a part of Fennema and Franke's (1992) Knowledge of Mathematics. In Ball, Thames and Phelps's (2008) model about around Mathematical Knowledge for Teaching model, the BMK would be part of the common knowledge and, at the same time, the starting point to develop the horizon content knowledge.

However, all the above mentioned models study elements identified in experienced teachers' practices while in the case of the present study we are interested in the knowledge of future educators before starting their teacher training. Since the education the students have received before reaching University level should have allowed them to deal with more advanced knowledge than what they are going to teach in Primary School, the BMK should comprise the knowledge of mathematical practice and structures dealt with throughout all the years of schooling (Castro et al. 2014).

Linsell and Anakin (2013) suggest that the models developed to describe the professional knowledge of teachers have limitations when it comes to the analysis of the knowledge of beginning undergraduate students. In addition, these models are based on an analysis in terms of lack of knowledge, instead of considering an evaluation of the students' background knowledge as a starting point to be developed further during undergraduate training. Our notion of BMK differs little from the Foundation Content Knowledge of Linsell and Anakin (2012), since they refer to the knowledge of future educators at the beginning of their training. Linsell and Anakin (2012) propose the concept of Foundation Content Knowledge to refer to the knowledge of mathematical content that future educators possess when starting their training programme. This type of knowledge includes as inseparable conditions, both conceptual knowledge and procedural knowledge.

METHODOLOGY

After developing a preliminary theoretical approach to BMK, as a preliminary study, we set out to elaborate a diagnostic test to be administered to student teachers. For this purpose we revised different pre-existing tests aimed at the evaluation of mathematical knowledge of teachers in different moments of their training or professional development. A wide range of exercises have been chosen to represent the 5 blocks of the mathematics curriculum of the Spanish educational system: Numbers and Arithmetic, Space and Shape, Relations and Change, Measurement, and Statistics and Randomness. The exercises are designed with an open-question format to avoid suggesting possible answers, as may be the case when using a multiple-choice question format. We selected 18 activities that constitute a balanced test with respect to content blocks and levels of mathematical knowledge.

The test was administered to 291 students of the first year of the Primary Education degree at the Universitat Autònoma de Barcelona (UAB). Among the University-entrance exam grades for these studies in the Universities of Catalonia, the UAB has the highest. Therefore, we can state that not only have our students successfully passed their educational stages previous to University admission, but have also obtained higher University-entrance qualifications.

SOME RESULTS

As follows, we present the analysis of some of the data from the answers of the 291 students to two of the questions included in the aforementioned test. The main interest of our study is focussed on determining the type of mathematical background desired for students recently admitted to the Degree in Primary Education and diagnosing errors made in their learning process. For this reason, we wish to make a qualitative analysis of the type of mathematical content knowledge these tests reveal.

Decimal numbers, hours and minutes

The second question of the test, amongst other changes of measurement units, asks students the equivalence of 1.4 hours in minutes. Table 1 shows the results obtained and their absolute and relative frequencies.

Answer	Frequency	Percentage	Answer	Frequency	Percentage
NR	3	1.0%	84 minutes	79	27.1%
60 minutes	1	0.3%	90 minutes	3	1.0%
64 minutes	7	2.4%	100 minutes	154	52.9%
75 minutes	9	3.1%	120 minutes	3	1.0%
80 minutes	3	1.0%	Other	29	10.0%

Table 1: Answers to question 2

We note that the correct answer is not the most frequent, but the most recurrent answer states that 1.4 hours equals 100 minutes. We may attribute this error to the lack of knowledge of the sexagesimal system. While revising, the answers we conclude that students add the 60 minutes of a complete hour to the 40minutes they erroneously attribute to 0.4 hours. This result is greater than the 90 minutes that correspond to an hour and a half, which indicates that these students don't relate their answer to common mathematical knowledge.

This lack of a practical everyday meaning is more evident in the case of students who answer 60, 90 or 120 minutes, which are values used outside the context of the classroom. Other results on the list, such as 60.4, may be related to conceptual errors of a similar nature. Those students that answer 75 have neither constructed a meaning for 0.4 hours and present a more complex misinterpretation. Since when telling the time, both in Spanish and in Catalan (the home languages of the students), 15 minutes past the hour is verbalized as "the hour and a quarter" (e.g. 3:15 is expressed as three and a quarter) the students misinterpret "0.4" to be a quarter of an hour and therefore 15 minutes

Contextualised problems with verbal formulation

Question 5 of the test is: *"When going on a school outing it is required for children to be accompanied by adults. Each adult can be responsible, at the most, for a group of 16 children. In an outing with 54 children, how many adults are needed to accompany them?"* Table 2 summarizes the answers of the students to this question and shows their relative and absolute frequencies.

Answer	Frequency	Percentage	Answer	Frequency	Percentage
NA	17	5.8%	4	154	52.9%
3	41	14.1%	4 with errors	20	6.9%
3.375	24	8.2%	5	6	2.1%
3.4	12	4.1%	Other	10	3.4%
3.5	7	2.4%			

Table 2: Answers to question 5

We can see that 52.9% of the students answer this question correctly, calculating the ratio in excess, thus giving an answer that considers the context of the formulation. Some of the students give an answer of 4, based on erroneous calculations or invalid arguments. We also find students (14.8%) that consider the direct result of the division (3.375, 3.4 and 3.5) as the only answer to the question, thus overlooking the meaning of the situation. There is another group of students (14.1%) who answer that 3 adults should accompany the group of children, not considering that the context situation would require finding the ratio of the division in excess.

CONCLUSIONS

Our students at the start of their Degree in Primary Education at the UAB have successfully passed their studies previous to University but, according to the results obtained in our empirical study, with an incomplete BMK. Specifically, we have documented a lack of competence even in the field of common mathematical knowledge, in aspects related to units of measurement or the contextualisation of mathematical knowledge.

Many of the students who enter University may have possibly forgotten the elementary mathematics they once studied. Their results may probably be better if asked to perform a derivation, or a more recent topic for them, instead of questions that require a deep understanding of a more basic mathematic content. The results obtained show that many of our students have not developed an adequate and solid construction of mathematical knowledge during their previous education, and are therefore not able to reconstruct when forgotten those processes they learned by heart and without searching for their meaning within a practical context.

The proof of the lack of elementary mathematical knowledge of students at the start of their teacher training justifies the notion of BMK and leads us to the need of its characterization, establishing its form and content to be able to evaluate it.

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