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LEPTON FLAVOUR VIOLATING DECAY OF THE Z°
IN THE SCALAR TRIPLET MODEL

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A B S T R A C T

The lepton flavour violating $Z \to \mu e$, $Z \to \mu\tau$ and $Z \to e\tau$ decays are evaluated in the framework of the Majoron triplet model of Gelmini and Roncadelli. The process is dominated by the diagrams with charged scalars in the loops, so the GIM cancellation is avoided. We obtain branching ratios for $Z \to \mu\tau$ and $Z \to e\tau$ which might be detectable at SLC and/or LEP.

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It has long been recognized that flavour changing weak neutral currents, which are very strongly suppressed in the quark and the lepton sectors, constitute a sensitive tool to search for the direction in which the Standard Model will have to be extended. We will study here lepton flavour non conservation and the prospects for its detection at the Z° peak.

A model which extends the standard theory by adding new neutral fermions was considered in Ref. 1), studying its implications for lepton flavour non conservation. In this work we shall discuss this problem in the context of extended Higgs sector. At the level of the electroweak SU(2) U(1) gauge theory, this enlargement is forced if one wishes to have neutrino masses without right-handed neutrinos. The lepton flavour changing neutral currents due to non vanishing neutrino masses and mixings have too small amplitudes to be detected, but the effects of the new scalars could be important. A model to get neutrino Majorana masses was proposed by Gelmini and Roncadelli[2)], in which a scalar triplet develops a small vacuum expectation value v. The value v has strong bounds coming from Majoron emission by stellar objects [v < 100 keV[3)], v < 1 MeV[4)] and v < 9 keV[5)]].

In the scalar triplet model seven physical scalars remain after the Higgs mechanism. One of them is the massless Goldstone boson, the Majoron, associated with the global lepton number U(1) breakdown. Neutrinoless $\beta\beta$-decay of ^{76}Ge with Majoron emission was claimed recently by Avignone et al.[6)], but their results are in disagreement with more sensitive experiments[7)] which show no evidence for this Majoron channel. There are two more neutral scalars, a heavy one which corresponds to the neutral Higgs boson of the standard theory and a light scalar with mass proportional to the triplet vacuum expectation value. The remaining fields are a single charged scalar ω and a doubly charged one ϕ with masses related by $m_\phi \simeq \sqrt{2} m_\omega$. These new charged Higgs fields have Yukawa couplings with the leptons, leading to additional contributions to charged current processes.

All the neutral currents are diagonal at tree level due to the fact that each vacuum expectation value gives masses separately to only one kind of leptons, so the diagonalization of the fermion mass matrices implies the diagonalization of the neutral Yukawa couplings[8)]. Therefore processes like $\mu \to e\gamma$ must be produced at the one-loop level and are a priori suppressed by the GIM mechanism. However, the presence of charged scalars allows to avoid the GIM cancellations in diagrams with these scalars running in the loop. In fact, it has been shown that the contributions to $\mu \to e\gamma$ of the charged Higgs sector are larger than the ordinary gauge ones by at least twenty four orders of magnitude[9)]. This opens the door to processes like $Z^\circ \to l_i \bar{l}_j$, (i = j). If the SLC and /or LEP colliders produce several millions of Z's per year it would be possible to study some rare decays with lepton flavour non conservation. The detection of these leptonic channels seems much cleaner[10)] than that for the corresponding quark flavour non-conservation[11)]. We study the lepton flavour violating decays of the Z° induced by the charged scalars of the triplet Majoron model. The dominance of these contributions for the $Z \to l_i \bar{l}_j (i \neq j)$ decay is due to the same reason than for the $\mu \to e\gamma$ decay. We consider the case of three generations because the results[12)] of the CERN $p\bar{p}$ collider for Γ_z/Γ_w place stringent limits[3),13)] if Majorons exist.

The diagrams contributing to the process are depicted in Fig. 1, which contains the single charged scalar in the loop, and Fig. 2, which contains the doubly charged scalar. As mentioned before, additional diagrams with gauge bosons are suppressed due to the unitarity of the mixing matrix and they vanish with the neutrino mass (with respect to the W-mass). We give the Yukawa couplings needed for the calculation, keeping only the parts proportional to (m_ν/v) which dominate the process, so[8)]

$$L_1 = \frac{1}{v} \bar{\nu} D_\nu U L \ell \, \omega^{(+)} - \frac{1}{v\sqrt{2}} \bar{\ell} U^T A^* D_\nu U L \ell \, \phi^{(++)} + h.c. \qquad (1)$$

where L is the left projection operator $\frac{1}{2}(1-\gamma_5)$, D_ν is the diagonal mass of the physical neutrinos, U is the same leptonic mixing matrix which appears in the charged gauge sector and it can be parametrized à la Kobayashi-Maskawa, and A is a diagonal matrix of phases characteristic of Majorana neutrinos $\nu = -A\nu^c$.

The gauge couplings of the Z to the leptons are

$$L_2 = \frac{g}{4\cos\theta_W} \left\{ \bar{\ell} \gamma^\mu (-1 + 4\sin^2\theta_W + \gamma_5) \ell - \bar{\nu} \gamma^\mu \gamma_5 \nu \right\} Z_\mu \qquad (2)$$

where one must notice that ν are Majorana neutrino fields and g is the SU(2) gauge coupling constant. Finally, we need the gauge couplings of the Z to charged scalars

$$L_3 = \frac{g}{\cos\theta_W} i Z_\mu \left\{ (1 - 2\sin^2\theta_W) \phi^{(--)} \overleftrightarrow{\partial} \phi^{(++)} - \sin^2\theta_W \, \omega^{(-)} \overleftrightarrow{\partial} \omega^{(+)} \right\} \qquad (3)$$

The total effective amplitude for the process has the form

$$T = \frac{g}{\cos\theta_W} \bar{u}(p_1) \Gamma^\mu v(p_2) \varepsilon_\mu(q) \qquad (4)$$

where $\varepsilon_\mu(q)$ is the polarization vector of the Z°. In the limit of vanishing lepton masses, the vertex Γ^μ can be written as

$$\Gamma^\mu = -\frac{\langle m_\nu^2\rangle_{ij}}{64\pi^2 v^2}\ \gamma^\mu L\ F(M_Z, m_\omega, m_\phi) \tag{5}$$

where F is a form factor depending on the boson masses and the subindices (i,j) refer to the generation pair of outgoing charged leptons. The Yukawa couplings appear in the form

$$\langle m_\nu^2\rangle_{ij} / v^2 \equiv \sum_r U_{ri}^* U_{rj} (m_{\nu_r}/v)^2 \tag{6}$$

Each of the diagrams in Fig. 1 is divergent. However, when we add their contributions, the divergences cancel and the result is finite, giving origin to a form factor

$$\begin{aligned} F_\omega = & \left[\tfrac{3}{2} + f_a(s_\omega) + g_a(s_\omega)/2\right] \\ & -2\sin^2\theta_W\left[-\tfrac{1}{2} + f_b(s_\omega) + g_b(s_\omega)/2\right] \end{aligned} \tag{7}$$

where the functions $f_a(s_\omega)$, $g_a(s_\omega)$, $f_b(s_\omega)$ and $g_b(s_\omega)$ depend on the charged scalar mass through the variable $s_\omega = (M_Z/m_\omega)^2$ and can be expressed in terms of the Spence functions and logarithms

$$\begin{aligned} f_a(s) &= \log s - 2 - i\pi \\ g_a(s) &= -\frac{4}{s}\left\{\log s - 1 + \frac{1}{s}\left[\mathrm{Li}_2\left(\frac{s}{1+s}\right) - \log(1+s)\log\frac{s}{1+s}\right]\right\} \\ &\quad + i\,\frac{4\pi}{s}\left\{1 - \log\frac{1+s}{s}\right\} \\ f_b(s) &= \begin{cases} -2 + \sqrt{\frac{4}{s}-1}\left[\pi - 2\arctan\sqrt{\frac{4}{s}-1}\right] & ; \quad s \leq 4 \\ -2 + \sqrt{1-\frac{4}{s}}\ \log\dfrac{1+\sqrt{1-\frac{4}{s}}}{1-\sqrt{1-\frac{4}{s}}} - i\pi\sqrt{1-\frac{4}{s}} & ; \quad s \geq 4 \end{cases} \\ g_b(s) &= -\frac{4}{s}\left\{f_b(s) + 1 - \frac{2}{s}\left[\mathrm{Li}_2\left(\frac{1}{y_1}\right) + \mathrm{Li}_2\left(\frac{1}{y_2}\right)\right]\right\} \end{aligned} \tag{8}$$

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where the arguments of the Spence functions in $g_b(s)$ are

$$y_{1,2} = \begin{cases} \frac{1}{2}\left(1 \pm \sqrt{1-\frac{4}{s}}\right) \pm i\epsilon & ; \quad \epsilon \to 0^+ \;, \quad s \geq 4 \\ \frac{1}{2}\left(1 \pm i\sqrt{\frac{4}{s}-1}\right) & ; \quad s \leq 4 \end{cases} \tag{9}$$

The sum of the diagrams of Fig. 2 also gives a finite result, which can be written in terms of the functions of Eq. (8), but changing the variable s_ω by $s_\phi = (M_Z/m_\phi)^2$

$$F_\phi = 2(2\sin^2\theta_W - 1)\left\{\frac{5}{2} + f_a(s_\phi) + g_a(s_\phi)/2 - 2f_b(s_\phi) - g_b(s_\phi)\right\} \tag{10}$$

Adding the two contributions (7) and (10), and using the approximate relation between the masses of the two charged scalars $m_\phi^2 = 2m_\omega^2$, the form factor depends on one unknown parameter, say the mass ratio of Z to ϕ, $s \equiv s_\phi$,

$$F(s) = F_\omega(s_\omega = 2s) + F_\phi(s_\phi = s) \tag{11}$$

Using the amplitude (4) and (5), the lepton flavour violating width of the Z° is

$$\Gamma(Z \to l_i \bar{l}_j) = \left(\frac{g}{\cos\theta_W}\right)^2 \frac{M_Z}{24\pi} \left| \frac{\langle m_\nu^2 \rangle_{ij}}{64\pi^2 v^2} F(s) \right|^2 \tag{12}$$

The rate ratio between the lepton flavour changing decay and the diagonal decay of the Z° to electrons can be expressed in terms of the form factor F(s) and the Yukawa couplings

$$R^Z_{ij} \equiv \frac{\Gamma(Z \to l_i \bar{l}_j) + \Gamma(Z \to l_j \bar{l}_i)}{\Gamma(Z \to e\bar{e})} \simeq 4 \cdot 10^{-5} \left| \frac{\langle m_\nu^2 \rangle_{ij}}{v^2} F(s) \right|^2 \tag{13}$$

On the other hand, the Yukawa couplings are not free, they are restricted by the bounds coming from the $l_i \to l_j \gamma$ decays. The branching ratio for that flavour violating radiative process is[9])

$$R^{\gamma}_{ij} \equiv \frac{\Gamma(l_i \to l_j \gamma)}{\Gamma(l_i \to l_j \nu \bar{\nu})} = \frac{25\,\alpha}{24\pi \lambda_4^2} \left| \frac{\langle m^2_\nu \rangle_{ij}}{v^2} \right|^2 \tag{14}$$

where λ_4 is the coupling constant of the Higgs potential that gives masses to the charged scalars

$$m_\phi{}^2 = \frac{\lambda_4}{G_F\, 2\sqrt{2}} \tag{15}$$

We can rewrite Eq. (13) eliminating the Yukawa couplings in favour of the low-energy rate ratio R^{γ}_{ij}, with the result

$$R^{z}_{ij} = 1.1 \cdot 10^{-3} \left| \frac{F(s)}{s} \right|^2 R^{\gamma}_{ij} \tag{16}$$

Using the experimental bounds on R^{γ}_{ij} and calculating the functions F(s)/s we can obtain limits on R^{z}_{ij}. The experimental branching ratios[14)] for $l_i \to l_j + \gamma$ are translated into the following limits for R^{γ}_{ij} when normalized to the three body $l_i \to l_j \nu\bar{\nu}$ decays

$$R^{\gamma}_{\mu e} \leq 1.7 \cdot 10^{-10}$$

$$R^{\gamma}_{\tau\mu} \leq 3 \cdot 10^{-3}$$

$$R^{\gamma}_{\tau e} \leq 4 \cdot 10^{-3} \tag{17}$$

leading to

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$$\mathcal{R}^{Z}_{e\mu} \leq 1.9 \times 10^{-13} \; |F(s)/s|^2$$

$$\mathcal{R}^{Z}_{\mu\tau} \leq 3.3 \times 10^{-6} \; |F(s)/s|^2$$

$$\mathcal{R}^{Z}_{e\tau} \leq 4.4 \times 10^{-6} \; |F(s)/s|^2 \qquad (18)$$

We have calculated the function $|F(s)/s|^2$ in the allowed range of the charged scalar masses 30 GeV $< m_\phi <$ 240 GeV[3),15)]. For $m_\phi \gtrsim$ 150 GeV, this factor is larger than one. Using Eq. (18), we see that the process $Z \to e\mu$ is beyond the experimental possibilities. For the other two processes, the prospects to be detected depend on the mass of the doubly charged scalar. If $\lambda_4 = 1$, value which corresponds to m_ϕ = 176 GeV, we obtain $|F(s)/s|^2 = 1.8$, which leads to

$$\mathcal{R}^{Z}_{\mu\tau} \leq 5.9 \times 10^{-6} \quad ; \quad \mathcal{R}^{Z}_{e\tau} \leq 7.9 \times 10^{-6} \qquad (19)$$

These limits could only be accessible at SLC and/or LEP if several millions of Z's per year are collected. The results for the form factor $F(s)/s$ are given in Fig. 3, where we plot both its real and imaginary parts in the interesting phenomenological region for m_ϕ.

In view of the above discussion and the connection established by Eq. (16) it is important to improve the bounds for the radiative τ-decays. Finally we mention that in the $Z°$ decay to different leptons, and in this model, there is no CP-violation when we neglect the lepton masses in the loop. To have a CP-violating observable we need the interference between amplitudes with different absorptive parts. Due to this, the form factors must be different for the different leptons in the loop. When we neglect the masses of the internal leptons the form factor can be factorized and there are no effects. So, CP-violating effects are suppressed at least by additional $(m_l/m_\phi)^2$ factors.

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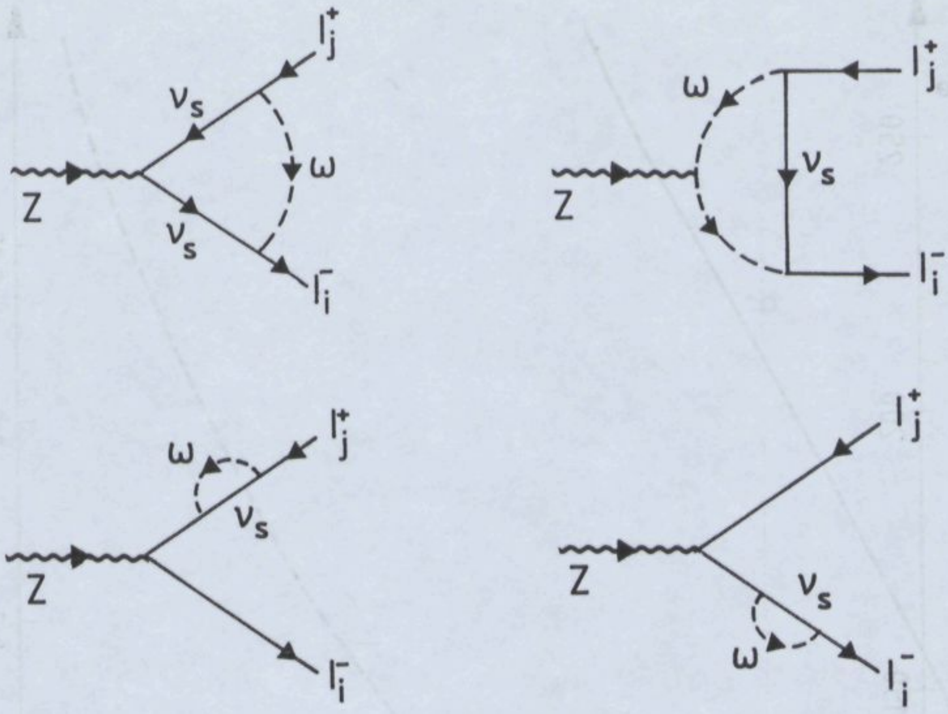
FIGURE CAPTIONS

Fig. 1: Diagrams contributing to the process $Z^\circ \to l_i^- l_j^+$ for the single charged scalar ω running in the loop.

Fig. 2: Same as Fig. 1, but for the doubly charged scalar ϕ running in the loop.

Fig. 3: The form factor $F(s)/s$, with $s \equiv (M_z/m_\phi)^2$. The two curves indicate its real part R and its imaginary part I.



l_j^+
ν_s
ω
Z
ν_s
l_i^-
ω
l_j^+
ν_s
Z
l_i^-
ω
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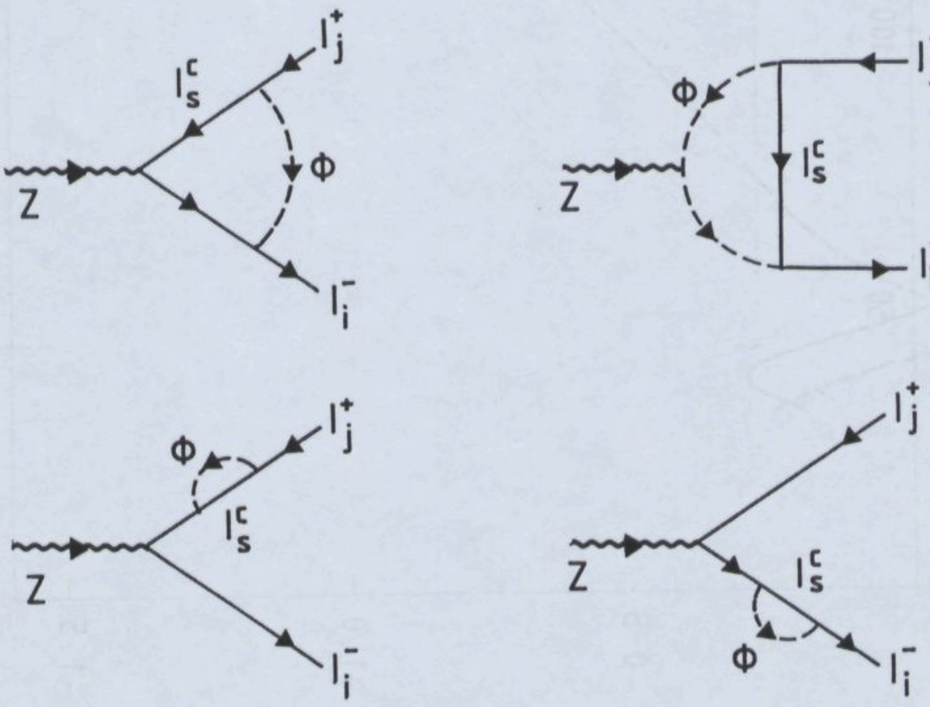
Z
l_i^-
l_j^+
Z
ν_s
ω
l_i^-

Fig. 1



l_j^+
l_s^c
Φ
Z
l_i^-
Φ
l_j^+
l_s^c
Z
l_i^-
l_j^+
Φ
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Z
l_s^c
Φ
l_i^-

Fig. 2

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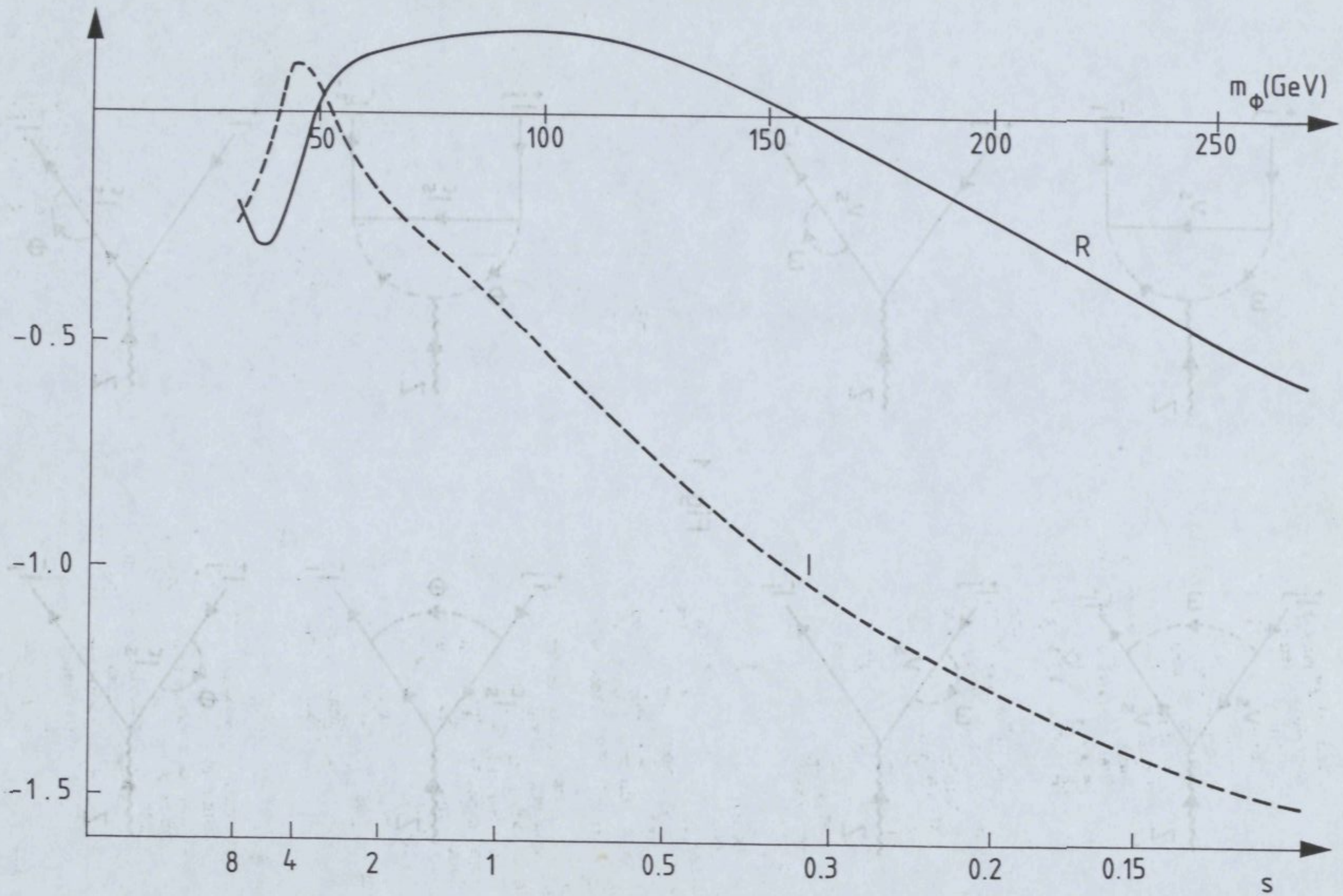

Fig. 3