TOWARD A THEORY OF THE STRONG INTERACTIONS

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I. Introductory Instanton Physics

Let us consider the pure Yang-Mills theory based on a Lie group G [in particular SU(2) or SU(3)] and described by the Lagrangian density

$$\mathcal{L}(x) = -\frac{1}{4g^2} F^a_{\mu\nu}(x)\, F^{a\mu\nu}(x) \tag{1}$$

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) - f_{abc}\, A^b_\mu(x)\, A^c_\nu(x)$$

Hence the Hamiltonian density is

$$\mathcal{H}(x) = \frac{1}{2g^2} \left\{ \left[E^i_a(x) \right]^2 + \left[B^i_a(x) \right]^2 \right\} \tag{2}$$

$$E^i_a(x) = -F^{0i}_a(x) \qquad , \qquad B^i_a(x) = -\frac{1}{2}\, \epsilon_{ijk}\, F^{jk}_a(x)$$

The equation of motion are easily derived from (1)

$$\mathcal{L}(x) = -\frac{1}{2g^2} \Big\{ \partial_\mu A^a_\nu(x)\, \partial^\mu A^{a\nu}(x) - \partial_\mu A^a_\nu(x)\, \partial^\nu A^{a\mu}(x) - f_{abc}\, \partial^\mu A^{a\nu} A^b_\mu(x)\, A^c_\nu(x)$$

$$+ f_{abc}\, \partial^\nu A^{a\mu}(x)\, A^b_\mu(x)\, A^c_\nu(x) + \frac{1}{2} f_{abc}\, f_{ade}\, A^b_\mu(x)\, A^c_\nu(x)\, A^{d\mu}(x)\, A^{e\nu}(x) \Big\}$$

$$\frac{\partial \mathcal{L}}{\partial A^a_\mu(x)} = -\frac{1}{g^2} f_{abc}\, \partial^\mu A^{b\nu}(x)\, A^c_\nu(x) + \frac{1}{g^2} f_{abc}\, \partial^\nu A^{b\mu}(x)\, A^c_\nu(x)$$

$$- \frac{1}{g^2} f_{bea}\, f_{bed}\, A^\nu_c(x)\, A^e_\nu(x)\, A^{d\mu}(x)$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\nu A^a_\mu(x))} = -\frac{1}{g^2} \partial^\nu A^{a\mu}(x) + \frac{1}{g^2} \partial^\mu A^{a\nu}(x) + \frac{1}{g^2} f_{abc}\, A^{b\nu}(x)\, A^{c\mu}(x)$$

$$\partial_\nu \partial^\nu A^{a\mu}(x) - \partial_\mu \partial_\nu A^{a\nu}(x) - f_{abc}\, \partial_\nu A^{b\nu}(x)\, A^{c\mu}(x) - f_{abc}\, \partial^\mu A^{b\nu}(x)\, A^c_\nu(x)$$

$$- 2 f_{abc}\, A^{b\nu}(x)\, \partial_\nu A^{c\mu}(x) - f_{bca}\, f_{bed}\, A^\nu_c(x)\, A^e_\nu(x)\, A^{d\mu}(x) = 0$$