MODELLING THE COOLING OF CONCRETE BY PIPED WATER

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ABSTRACT. Piped water is used to remove hydration heat from concrete blocks during construction. In this paper we develop an approximate model for this process. The problem reduces to solving a one-dimensional heat equation in the concrete, coupled with a first order differential equation for the water temperature. Numerical results are presented and the effect of varying model parameters shown. An analytical solution is also provided for a steady-state constant heat generation model. This helps highlight the dependence on certain parameters and can therefore provide an aid in the design of cooling systems.

INTRODUCTION

Large concrete structures are usually made sequentially in a series of blocks. After each block is poured it must be left to cool and shrink for a period depending on its size, but typically for around one week, before the next block is poured. The reason for the delay is that the mixture of cement and water, which constitute the binding agent of the concrete, results in a series of hydration reactions that generate heat. The chemical reaction can lead to temperature rises in excess of 50K and it can take a number of years before the concrete cools to the ambient temperature. Prior to construction of the Hoover dam engineers at the US Bureau of Reclamation estimated that if the dam were built in a single continuous pour the concrete would require 125 years to cool to the ambient temperature and that the resulting stresses would have caused the dam to crack and fail (US Bureau of Reclamation). This highlights the main problem of the heat generation, that of thermal stress, which can then lead to cracking, leakage and resultant structural weakening. The development of thermal stresses in hydrating concrete has been extensively discussed by Springenschmid 1994. Neville 1995 points out that high temperatures lead to porous, weak concrete. Lawrence 1998 states that temperatures greater than 70°C lead to microcracks.

Key words and phrases. Drying; Temperature; Modelling.

In order to limit the maximum thermal stresses, it is therefore necessary, during the construction process, to remove as much of the heat of hydration as possible, particularly before the next concrete block is poured. As construction time is usually an important consideration, it is essential to carry out the heat removal as quickly as possible.

There are a number of ways of minimising the temperature development in large concrete structures. One of the more effective methods, particularly for very large construction such as concrete dam walls, is to introduce an interconnected pipe network into the concrete during construction. Chilled water is then circulated through this pipe system until it is deemed sufficient energy has been removed from the concrete, see Liu 2004 for example. When designing the pipe system, engineers have to make five major decisions:
1) the type of pipe to use (metal, plastic, wall thickness, etc);
2) the diameter of the water pipe;
3) the spacing between pipes;
4) the temperature of the inlet water;
5) the flow rate of the water.

The first two decisions are based on economics, construction methodology and the need to avoid displacing so much concrete by the empty pipe that the strength of the structure is compromised. The third decision will be based on efficiency of heat removal and, effectively, how tolerant the project time-lines are of delays caused by the process of heat removal from the concrete. Heat removal not only reduces thermal stresses, it also shortens the time that the contractor has to wait for construction joints to be grouted. If this is done while the internal concrete temperature is significantly greater than ambient, the grouted joints will open upon subsequent cooling of the concrete.

Decisions 4 and 5 are the only adjustable parameters during the operation of the cooling system and therefore allow for some error in the design decisions taken regarding parameters 1 to 3. In determining these parameters, engineers rely on empirically developed design codes such as ACI 207.4R-05 (American Concrete Institute 2005). In large measure, these approaches suffer the weakness of not being able to account for differing construction conditions, differing cement types and differing thermal characteristics of concrete making materials.

In the operation of an internal water cooling system, contractors would typically monitor the inlet and outlet water temperatures to assess the quantum of heat being removed. This then allows the inlet temperature and/or flow rate to be adjusted in response to changes in the measured temperature of the concrete.

In the following work we develop a model of a simplified pipe network with the intention of providing a rational approach to the design, management and operation of an internal concrete cooling system. We assume that the network consists of a series of straight pipes, separated by a distance $2R$. Heat transfer occurs at the pipe walls from the concrete to the water. As the water travels along a pipe it becomes hotter. So, if the second pipe is downstream of the first then the heat transfer into the first pipe will be greater than into the second. Between the pipes there will be a point where the temperature gradient is zero. Provided the temperature difference is not too great this will be close to the mid-point. For simplicity we will therefore take the boundary condition that the temperature gradient is zero at the mid-point between pipes. It is important to note that this simplification will not qualitatively affect the results presented later which show the actual mechanisms for heat removal.

Governing equations

The problem configuration is shown in Figure 1. Water flows through a pipe of radius a, which is encased in a cylindrical sleeve of concrete, of radius R. The concrete temperature is denoted by T', the water temperature by θ' (primes denote dimensional variables), the flow rate is Q. The problem is governed by heat equations in the concrete and water:

$$\rho_c c_c \frac{\partial T'}{\partial t'} = \kappa_c \nabla^2 T' + q' \tag{1}$$

$$\rho_w c_w \left(\frac{\partial \theta'}{\partial t'} + \mathbf{u}' \cdot \nabla \theta' \right) = \kappa_w \nabla^2 \theta' \ , \tag{2}$$

where q' is the rate of heat production per unit volume in the concrete, and (ρ_c, c_c, κ_c), (ρ_w, c_w, κ_w) are the density, specific heat and conductivity of concrete and water respectively.

While acting to change the temperature of the concrete system, as an exothermic chemical reaction, the rate of heat production q' is itself temperature and time dependent. Ballim and Graham 2003 have shown that the way to deal with this time-temperature duality is to express the rate of heat evolution in terms of an Arrhenius maturity. This form of the heat rate can then be expressed on a time basis by monitoring and adjusting for the rate of change of maturity. However, for the purposes of this present analysis, the complexity of dealing with a maturity form of the heat rate expression was excluded. The reason for this is that we intend to demonstrate an analytical approach to understanding heat exchange in a cooling pipe system for mass concrete structures. The maturity expression of the heat rate function can be added as a second level of complexity for the actual analysis in a real concrete structure.

A typical form for q' for cement is shown in Figure 2. Initially there is a rapid increase to the maximum of around 1200 W/m^3 after around 10 hours. This is followed by an exponential decay. After around 80 hours the heat production is not measurable, but does not actually reach zero for a much longer period. From this graph it is clear that the temperature increase can be significant, particularly during the early stages of the drying process, when the heat production is very high, $q' \sim 10^3$W/m^3 . In the following analysis we will approximate q' by the following relation

$$q' = q'_m \frac{t'}{t'_m} e^{-(t'^2-(t'_m)^2)/(2(t'_m)^2)} \ , \tag{3}$$

where q'_m is the maximum value of q', which occurs at time t'_m. Since we eventually solve the problem numerically the approximation to q' can be made more accurate without increasing the solution difficulty. However, the exact choice of q' will not affect the main results.

The heat equation in the water may be simplified significantly on obvious physical grounds. The water flow is turbulent provided the Reynolds number $Re = 2Ua/\nu > 2300$. Typical values for the pipe radius and velocity are $a =$ 2.5cm, $U =$ 10cm/s, the kinematic viscosity of water $\nu = 10^{-6}$m^2/s (see Table 1) and so $Re \sim 5000$ and the flow is well into the turbulent regime. One consequence of this is that the water will be well mixed and therefore the temperature will be independent of the radial co-ordinate (except perhaps for in a narrow boundary layer near the pipe wall). If we write $\overline{\theta}'$ as the average temperature at a given z' co-ordinate then

$$\overline{\theta}' = \overline{\theta}'(z,t) = \frac{2}{a^2}\int_0^a \theta'(r',z',t') r \ dr \ .$$

Further, the average radial velocity must be zero and the mean flow is in the z'-direction, $\overline{\mathbf{u}}' = (0, w)$. Since the fluid is incompressible we can state $w = Q/\pi a^2$ is constant. Under these conditions the heat equation in the water can be integrated to give

$$\pi a^2 \rho_w c_w \left(\frac{\partial \overline{\theta}'}{\partial t'} + \frac{Q}{\pi a^2} \frac{\partial \overline{\theta}'}{\partial z'} \right) = 2\pi \kappa_w \left(a \left. \frac{\partial \theta'}{\partial r'} \right|_{r'=a} + \frac{a^2}{2} \frac{\partial^2 \overline{\theta}'}{\partial z'^2} \right) . \tag{4}$$

At the boundary between the water and the concrete a cooling condition applies

$$\kappa_w \left. \frac{\partial \theta'}{\partial r'} \right|_{r'=a} = H \left(T'|_{r=a} - \overline{\theta}' \right) \tag{5}$$

The heat transfer coefficient H is an approximate value

$$H = 2\pi \kappa_p \frac{a}{s} + H_{wp} , \tag{6}$$

where κ_p is the the thermal conductivity of the pipe, s the pipe thickness, and H_{wp} the heat transfer coefficient of water on the pipe, see Carslaw & Jaeger p. 111 1959 for example. Finally we may write the governing equation in the form

$$\left(\frac{\partial \overline{\theta}'}{\partial t'} + \frac{Q}{\pi a^2} \frac{\partial \overline{\theta}'}{\partial z'} \right) = \frac{2H}{\rho_w c_w a} (T'|_{r=a} - \overline{\theta}') . \tag{7}$$

Comparing equation (7) with the full heat equation (2) we see that the convective terms have been simplified by the removal of the radial velocity component, while w is constant and given in terms of the flux. The diffusive terms have been replaced by a term proportional to the temperature jump across the pipe wall. Pre-empting the non-dimensionalisation of the following section we neglect the diffusion term in the z direction which has a typical magnitude $\mathcal{O}(10^{-9})$ less than the terms retained in (7). This derivation is discussed in more detail in Charpin *et al* 2004.

Necessary boundary conditions for the problem are as follows. At $z = 0$ the water enters at a known temperature θ_0. At the pipe wall, $r' = a$, the concrete loses heat to the pipe,

$$\kappa_c \frac{\partial T'}{\partial r'} = H(T' - \overline{\theta}') .$$

At the edge of the domain, $r' = R$, symmetry requires

$$\frac{\partial T'}{\partial r'} = 0 .$$

Initially the concrete is assumed to be at a constant temperature T_0' and the water temperature is set to $\overline{\theta}_0'$ everywhere.

Non-dimensional analysis. We now non-dimensionalize equations (1, 7) using the scales

$$r' = Rr \quad z' = Zz \quad t' = \tau t \quad T' = T_0' + \Delta T\ T \quad \overline{\theta}' = T_0 + \Delta T\ \overline{\theta} \quad q' = q_m' q,$$

where ΔT is a typical increase in temperature within the concrete (above the initial temperature) and Z, τ are the length and time scales for significant temperature variations in the pipe; $\Delta T, \tau, Z$ are yet to be determined. The heat equation in the concrete becomes

$$\frac{\rho_c c_c \Delta T}{\tau}\frac{\partial T}{\partial t} = \kappa_c \Delta T \left(\frac{1}{R^2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{Z^2}\frac{\partial^2 T}{\partial z^2}\right) + q_m' q \ . \tag{8}$$

Anticipating the fact that radial diffusion is the dominant method for heat transferral in the concrete we rearrange this to

$$\frac{\rho_c c_c R^2}{\tau \kappa_c}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{R^2}{Z^2}\frac{\partial^2 T}{\partial z^2} + \frac{q_m' R^2}{\kappa_c \Delta T}\ q \ . \tag{9}$$

In the water we expect energy to be carried along with the fluid and so rearrange equation (7) accordingly to give

$$\frac{\pi a^2 Z}{Q\tau}\frac{\partial \overline{\theta}}{\partial t} + \frac{\partial \overline{\theta}}{\partial z} = \frac{2\pi a H Z}{\rho_w c_w Q}(T|_{r=\epsilon} - \overline{\theta}) \ , \tag{10}$$

where $\epsilon = a/R \ll 1$.

There are three unknown scales in equations (9, 10), the length-scale Z, the time-scale τ and the temperature scale ΔT. Clearly the temperature rise is driven by heat production in the concrete, so we choose

$$\Delta T = \frac{q_m' R^2}{\kappa_c} \ . \tag{11}$$

In the water the temperature rise is due to forced convection at the boundary, so we choose

$$Z = \frac{\rho_w c_w Q}{2\pi a H} \ . \tag{12}$$

The time derivatives indicate two distinct time scales. In the concrete

$$\tau = \tau_c = \frac{\rho_c c_c R^2}{\kappa_c} \ , \tag{13}$$

and in the water

$$\tau = \tau_w = \frac{\pi a^2 Z}{Q} \ . \tag{14}$$

A third time-scale appears due to the heat production $\tau = \tau_h = t'_m$.

Substituting typical values, as given in Table 1, into the expressions for the temperature scale and length-scale indicates $\Delta T \sim 54.7$K, $Z \sim 10.69$m. The temperature scale is of the order of increase observed in practice. The time-scale for significant changes in the concrete temperature is $\tau_c \sim 9.4 \times 10^4$s ~ 26 hours. The time-scale $\tau_h = t'_m \sim 3.6 \times 10^4$s $= 10$ hours. Evidently, for effective heat removal we should expect $\tau_c \sim \tau_h$. Finally, the flow time-scale $\tau_w \sim 104.9$s, $\tau_w \ll \tau_h, \tau_c$. The different time-scales indicate different possible perspectives. The movement of heat within the concrete takes of the order of hours whereas the time taken for a water particle to travel through the system is of the order of a minute. Consequently a water particle will not notice the heat movement or production within the concrete, merely that the concrete is hotter than the water and hence is a source of energy. The concrete on the other hand is only affected by the water since the supply is being continuously renewed and this occurs over a sufficiently long time-scale (much greater than τ_w) for significant hydration heat to be removed. Since our interest lies in the removal of heat from the concrete we will focus on the time-scale τ_c. The model for temperature variation in the water on the time-scale τ_w is discussed in Charpin *et al* 2004.

In the following section we solve the governing equations

$$\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{t}{t_m}e^{-(t^2-t_m^2)/(2t_m^2)} \tag{15}$$

$$\frac{\partial \overline{\theta}}{\partial z} = \left(T|_{r=\epsilon} - \overline{\theta}\right) \ . \tag{16}$$

The time t_m is non-dimensional, $t_m = 10 \times 3600/\tau_c \approx 0.38$.

The non-dimensional boundary conditions for these equations are: at the water concrete interface, $r = \epsilon$,

$$\frac{\partial T}{\partial r} = \overline{H}(T - \overline{\theta}) \ , \tag{17}$$

where $\overline{H} = HR/\kappa_c$; at $r = 1$

$$\frac{\partial T}{\partial r} = 0 \ ; \tag{18}$$

at $z = 0$ the water temperature $\overline{\theta} = \overline{\theta}_0' = (\theta_0 - T_0')/\Delta T$. The initial condition for the concrete is $T = 0$.

The coefficient $\overline{H}$ in equation (17) can be quite large, $\mathcal{O}(100)$, indicating that a better scaling would be to take $R = \kappa_c/H \ll 1$. If we choose this scale then the governing equations retain the same terms. The time and temperature scales do change and the coefficient in (17) becomes $\overline{H} = 1$. However, we choose to take the natural scale, where R is the radius of the concrete sleeve. This means that our results may exhibit high gradients in the temperature near $r = \epsilon$ but means that we do not have to carry out calculations over a large r domain or re-scale for an outer region away from $r = \epsilon$. This will be discussed later.

The non-dimensional governing equations involve a number of parameters: the scaled time at which the temperature is maximum, t_m, the ratio of the pipe radius to the pipe spacing, ϵ, the heat transfer coefficient, $\overline{H}$, and the initial temperature, $\overline{\theta}_0$. The length of the pipe z_e defines the computational domain in the z direction and can also vary. For a given type of concrete certain physical parameters may be easily changed, thus affecting the non-dimensional parameters. The most important parameter appears to be the pipe spacing. Changing this changes the time-scale τ_c and so t_m, ϵ and $\overline{H}$ all change, the temperature scale ΔT also changes and this affects $\overline{\theta}_0$. The pipe radius, a, affects ϵ and the length-scale Z. The flow rate Q also affects Z. The heat transfer coefficient H affects $\overline{H}$ and Z. Finally, the initial temperatures affect $\overline{\theta}_0$. Consequently there are many possibilities for improving the heat removal from the concrete.

In the following section we investigate a simplified model involving steady-state heat flow with a constant heat source. This allows us to obtain an analytical solution which then shows explicitly how the heat removal depends on the problem parameters. Given the large number of problem parameters this will give us a much clearer indication of the relative effect of the parameters than a numerical solution. In the numerical section we verify certain conclusions of the analytical model but show that it does not present a complete picture of the process.

Steady-state solution for constant heat generation

In general we can only solve the system of equations numerically but then the number of parameters in the governing equations makes it difficult to carry out a full parametric study. So, in order to better understand the role of the various physical parameters and to make analytical progress we now introduce an approximate form of (15), where the source term is taken as constant. Effectively this means we are working on a time-scale, τ, such that $\tau_w \ll \tau \ll \tau_h$. While this approximation is quite restrictive in the time for which it is valid, the driving mechanisms are the same as for the full problem. Hence information gained from this analysis will be relevant to the full time-dependent problem. To further simplify the problem we examine the steady-state

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + 1 \ . \tag{19}$$

This is coupled to the energy equation in the water (16).

Equation (19) integrates to

$$T = -\frac{r^2}{4} + A\log r + B \ . \tag{20}$$

Applying boundary conditions (17, 18) gives

$$T = \frac{\epsilon^2 - r^2}{4} + \frac{1}{2\epsilon\overline{H}}\left(1-\epsilon^2\right) + \frac{1}{2}\log\frac{r}{\epsilon} + \overline{\theta} \ . \tag{21}$$

So the concrete temperature depends explicitly on the water temperature. The term $\epsilon\overline{H}$ indicates a possible problem with the scaling. However, $\overline{H} \gg 1$, and in general $2\epsilon\overline{H} \gg 1$, so that this term does not dominate the equation. In fact it will only play a significant role when $r \approx \epsilon$. This apparent problem arises due to choosing the length-scale as the concrete radius, as discussed previously. It may be remedied by taking the length-scale from the boundary condition at $r = \epsilon$. This choice then requires re-scaling the equations as we move away from $r = \epsilon$ and so we stick with the simpler and more natural choice of R. Further, as will be seen from the subsequent results this choice still leads to accurate results.

Equation (21) allows us to determine $T(\epsilon, z)$ which is required in (16). The temperature in the water turns out to be

$$\overline{\theta} = \frac{1}{2\epsilon\overline{H}}\left(1-\epsilon^2\right)z + \overline{\theta}_0 \ . \tag{22}$$

Again the term involving $1/\epsilon$ does not cause a problem due to the size of $\overline{H}$.

Equations (21, 22) indicate that in the steady state the temperature in the concrete and water depends solely on the non-dimensional groupings ϵ, $\overline{H}$, $\overline{\theta}_0$. If we convert equations (21, 22) back to dimensional variables the dependence on the physical parameters becomes clear:

$$\overline{\theta}' = \overline{\theta}'_0 + \frac{\pi q'_m R^2}{\rho_w c_w Q}\left(1 - \frac{a^2}{R^2}\right) z' \ , \qquad (23)$$

$$T' = \overline{\theta}' + q'_m \left(\frac{a^2 - r'^2}{4\kappa_c} + \frac{R^2}{2aH}\left(1 - \frac{a^2}{R^2}\right) + \frac{R^2}{2\kappa_c}\ln\frac{r'}{a}\right) \ . \qquad (24)$$

The water temperature depends on the initial temperature, pipe spacing and flux, and to a much lesser extent on the pipe radius (since $a^2 \ll R^2$). The dependence on the initial temperature $\overline{\theta}'_0$ shows that a decrease in initial temperature simply acts to decrease the water temperature by the same amount (in agreement with our subsequent numerical results). The heat transfer coefficient has no effect on the water temperature (we will see later that this result is misleading). The concrete temperature does involve H, however only in the term that we highlighted as being small unless $r' \approx a$. Using the values given in Table 1, at $r' = R$ increasing H by a factor 10 results in a negligible increase in the maximum temperature. Reducing it by a factor of 10, so $H = 50$, we find a change of around 7%. However, at $r' = a$, increasing H by a factor of 10 results in a temperature increase of the order 20%. Decreasing H by a factor 10 the maximum temperature doubles. So we see that varying H has a significant effect near $r' = a$, but in general the effect becomes insignificant as we move away from this point. Of course this does not hold for all H. If $H = 0$ then the effect can be seen everywhere, since there is no mechanism for heat removal. From equation (24) we can estimate the H value above which we expect little change in the solutions away from $r' = a$. The two other terms within the bracket are both of the order R^2/κ_c. For these terms to dominate over H (when r' is not close to a) requires $H \gg \kappa_c/(2a)$. With the parameter values in Table 1 this gives $H \gg 27.4$. Our numerical calculations agree with this estimate.

We can confirm the water temperature equation through a simple energy balance. At any given z' the energy change in the water from the inlet must balance the energy generated within the concrete

$$V q'_m = Q\rho_w c_w(\overline{\theta}' - \overline{\theta}'_0)$$

where $V = \pi(R^2 - a^2)z'$ is the volume of the concrete sleeve. Rearranging this expression leads to equation (23).

Numerical solution

We solve equations (15, 16) numerically in the following manner.

(1) At the first z data point, $z = z_1 = 0$, we impose the boundary condition $\overline{\theta} = \overline{\theta}_0$ and use MATLAB routine *pdepe* to solve the system (15), (17), (18) with $\overline{\theta}$ replaced by $\overline{\theta}_0$ in (17). We also impose the initial temperature $T = 0$, everywhere.

(2) We now determine the water temperature at the next data point, $z = z_2$, by integrating (16) explicitly. The concrete temperature at the pipe wall, $T(\epsilon, z_1, t)$, is required in (16). This is taken from the solution of the previous step.

(3) We now solve (15) again but at $z = z_2$. The value of $\overline{\theta} = \overline{\theta}(z_2, t)$ in the boundary condition (17) comes from step 2.

(4) Steps 2 and 3 are repeated, with z incremented each time, until we reach the end of the pipe at $z = z_e$.

In the following solutions the parameter values are as given in Table 1 unless otherwise specified. The initial water temperature is 5°C and the initial concrete temperature is 25°C, making $\overline{\theta}_0 = -0.36$. As we vary the parameter values the scales change and this makes it difficult to compare the solutions. For this reason all the following graphs are presented with dimensional axes.

Figures 3, 4 show the temperature variation with time in the concrete at $r' = R$ and $r' = a$ and at $z' = 0, 10, 20$m. In Figure 3 a) the temperature at $r' = R$ initially rises rapidly, reaching a peak at around $t' = 26.6$ hours. This is caused by the heat generation (which reaches a maximum after 10 hours) but, since this excess heat cannot be removed immediately, the concrete temperature continues to rise well after the heat generation has peaked. As t' increases the heat generation decreases and so its effect also decreases. The maximum temperatures for both cases must obviously occur at the end of the pipe where $r' = R$. When $R = 0.5$m the maximum temperature $T \approx 55.6$°C, with $R = 0.25$m the maximum temperature is close to 41.1°C. Increasing R not only has a significant effect on the maximum temperature but the time taken to reach the equilibrium also takes much longer. If we wish the concrete temperature to be the same as the initial water temperature then with $R = 0.25$m this takes around 140 hours: with $R = 0.5$m the temperatures are still well above 20° after 170 hours.

At the pipe wall the behaviour is qualitatively different to at $r' = R$, as seen on Figure 4. Initially the temperature decreases as the water removes heat from the concrete. However, as the heat production within the concrete increases and heat diffuses from the regions away from the pipe wall, the concrete starts to heat up again. The peak temperature due to heat generation in both cases is much lower than at $r' = R$, it also occurs slightly earlier (at $t \approx 23$ hours).

Except for in the vicinity of $z = 0$, where $\overline{\theta}' = \overline{\theta}'_0$ for all time, the water temperature is similar to that of the concrete temperature at $r' = a$. This may be seen by comparing Figure 5 with Figure 4. The water temperature is everywhere slightly lower than the concrete temperature. Decreasing the pipe spacing to $R = 0.25$m, as shown on Figure 5b), slightly lowers the secondary temperature peaks, but the main effect is to significantly reduce the time taken for heat removal.

The results presented in Figures 3–5 all show that reducing R reduces the maximum temperatures and also the time taken to reach equilibrium. The steady-state solutions (23), (24) both indicate a decrease related to R^2 but obviously cannot provide the time taken to reach this state. However, the decrease in time is an obvious effect shown by the time scale τ_c, which is proportional to R^2.

Figure 6 shows the effect of increasing H to 5000. We do not show the temperature profile in the concrete at $r' = a$ since it is difficult to distinguish it from the water temperature, except for at $z' = 0$ where the concrete temperature remains slightly above the water temperature (with a maximum of 5.5°C) for about 30 hours. If we compare Fig 6a) with the corresponding graph for $H = 500$, Figure 3a), then we can see some unusual features. Firstly, the increase in $\overline{H}$ has only a slight effect on the maximum temperature, but it is in fact an increase to 56.8°C after 28.2 hours (as opposed to 55.8°C after 27.2 hours). In general both the temperature at $z = 10$ and 20m remains above that for the lower heat transfer coefficient and the temperature reduction therefore takes longer. There are two reasons for this counter-intuitive behaviour. Firstly, concrete is a relatively poor conductor so, despite the improved heat removal at the pipe wall the heat generated at $r' = R$ only diffuses slowly towards the pipe. This results in the slight variation in the peak temperature. Secondly, the improved heat transfer results in the water being heated more rapidly near the pipe entrance. If we compare

Figures 6b) and 5a) then we can see that with the greater value of H the water temperature is much higher. The energy transferred at the pipe is given by $H(T'_{r'=a} - \overline{\theta}')$. In Figure 7 we show the difference $T'_{r'=a} - \overline{\theta}'$ for $H = 5000, 500$. At $t' = 0$ there is a spike due to the initial conditions, where the difference is $T'_0 - \overline{\theta}'_0 = 20°$C. When $H = 5000$ the difference is very small apart from at the two small peaks at $(t', T'_{r'=a} - \overline{\theta}') = (16{,}3.5)$ and $(32{,}2.4)$. For large times the difference is around 0.1. When $H = 500$, in general, the difference is much greater and, in particular, for large times the difference remains around 1°C. So, the increase in H is offset by a decrease in the temperature jump, leading to the counter-intuitive result that improving heat transfer between the concrete and water can actually slow down the heat removal. Of course there is a limit to this behaviour, in that allowing $H \to 0$ results in no heat removal and so the temperature will never decrease. However, this is an extreme case, at typical values of H improvement in the heat transfer, through reducing the pipe wall thickness or using a better conducting material will have little effect. In fact, in almost all our calculations the temperature at $r = 1$ takes a similar form and so in the next two examples we will only quote the peak temperature.

The steady-state solution of § indicated that the heat transfer coefficient has no effect on the water temperature. Comparison of Figures 5a), 6b) shows that this is incorrect. The problem arises as a result of studying the steady-state: all the energy generated in the concrete has to be removed by the water, independent of H. However, if we compare the temperatures for $t' > 100$ hours then it is clear that H has little effect for large times.

The steady-state analysis also indicated that changing a has little effect on the water temperature. If we compare Figures 8b) and 5a) which have $a = 0.05, 0.025$ respectively we can see that for small times there is a significant effect. With a larger value of a the water temperature is higher. At large times the temperature change is negligible (verifying the analytical conclusion for the steady state). So, although an increase in the pipe radius provides a greater surface area between the water and pipe (or concrete) the energy transfer is less. The concrete temperature at $r' = a$, shown in Figure 8a), is also higher at small times than the corresponding temperature shown in Figure 4a). The maximum temperature at $r' = R$ is around 55.5°C.

In Figure 9 the effect of decreasing the flux is shown. At $r' = R$ the maximum temperature is approximately 56°C, equilibrium is reached some time after 170 hours, as opposed to around 65 hours, shown in Figure 3a). Comparing the temperatures in the concrete at $r' = a$ and in the water we see a similar effect when reducing Q to increasing a. In the concrete the temperature initially shows a small rise. The secondary peak is higher, reaching 21°C after 28.5 hours as opposed to 14.6°C after 23 hours. The water also shows higher temperatures and a slower decrease to equilibrium. In this case the results are intuitive and agree with the steady state solution that shows the water temperature (and hence the concrete temperature) depends on Q^{-1}. The similarity to changing a can be inferred from the length scale $Z \sim Q/a$ This also indicates that any increase in the flux is equivalent to an increase in the pipe length. This has been confirmed numerically and consequently we do not show results with a different pipe length z_e. Further, the results presented at $z' = 10$m are the results that would occur with $z_e = 10$m, so effectively we have already shown a number of results for a shorter pipe.

In the introduction we mentioned that only two parameters may be adjusted once the pipe is in place, these were the flux and the inlet temperature. We will not present results for changing the inlet temperature. Looking at the non-dimensional parameters we see that $\overline{\theta}'_0$ only appears in the temperature $\overline{\theta}_0$ and consequently the effect of changing the inlet temperature is merely to shift the temperature curves down a corresponding amount. Except for near $t' = 0$, this has been confirmed by our numerical calculations.

Finally, Lawrence 1998 states that the temperature should be kept below 70°C. In Figure 10 we show the maximum temperature plotted against R and also the time at which this is reached. From Figure 10a) it is clear that under these conditions the concrete temperature tends towards an asymptote of around 59.5°C, well below the critical 70°C mark. Presumably this is an indication that these are sensible operating conditions. The time taken to reach the maximum increases monotonically with R and consequently the time taken to reach equilibrium will also increase.

Conclusions

The primary issue for an engineer building a large concrete structure is to reduce the maximum temperature in the concrete to an acceptable level and within a reasonable time, while also maintaining structural integrity. As discussed in the introduction this leads to five choices.

1/ The type of water pipe to use – the pipe material affects the heat transfer coefficient H and the associated non-dimensional grouping $\overline{H}$. Our analytical model shows that above a certain value, $H \gg \kappa_c/(2a)$, increasing H will have little effect on the heat removal, except for in the immediate vicinity of the pipe wall. The lack of dependence on H is confirmed by our numerical results.

Provided the condition is satisfied the heat transfer properties of the pipe are largely irrelevant. Further, our numerical results show that increasing the heat transfer may, counter-intuitively, act to reduce heat removal near the end of the pipe.

2/ The diameter of the pipe – if we keep the flux constant, but increase the pipe diameter then heat removal is slightly less efficient. The water temperature increases more rapidly with a wider pipe and therefore, as with increasing H, the energy removal can be reduced.

3/ The spacing between the pipes – this is clearly the most important parameter. The order of magnitude of temperature variation in the concrete $\Delta T = q'_m R^2/\kappa_c$. For a given concrete q'_m and κ_c are fixed and so R is the only variable. Since the temperature scale depends on R^2 a moderate change in R can have a large effect on ΔT. The time-scale for the process $\tau_c = \rho_c c_c R^2/\kappa_c$ also depends on R^2, so increasing R has a significant effect on the time taken for the concrete to cool down.

4/ The temperature of the inlet water – according to the analytical model a change of x degrees in the inlet water temperature will result in a change of x degrees in the water temperature along the pipe as well as in the concrete at the pipe wall. The change away from the wall will be less than x. Our numerical calculations show that this is approximately correct. Consequently, the inlet temperature does have an effect on concrete and water temperatures, but this effect is relatively small.

5/ The flow rate of the water – this has a significant effect on the water and concrete temperature. In our calculations, halving the flux increased the maximum water temperature by a factor close to 2, the maximum concrete temperature at $r' = R$ by 10°C and at $r' = a$ by 20°C. The flux appears in a single non-dimensional grouping, namely the length-scale $Z \propto Q$. Therefore changing the flux by a factor x is equivalent to lengthening the pipe by the same factor. For this reason we did not present results for varying pipe lengths. Further, the results presented at $z' = 10$m are the results that would occur with $z_e = 10$m, so effectively we presented a number of results for a shorter pipe.

Comparison of our steady state analytical model and numerical results confirms many of the findings of the analytical model. In particular, in the water the important parameters are the heat generation q_m', the pipe spacing R and the flux Q. The pipe diameter plays a relatively small role. The concrete temperature depends primarily on the water temperature, q_m' and R. The thermal conductivity κ_c and H have a lesser effect. The numerical solution shows that the pipe diameter affects the temperature profile for small times.

This simplified model of heat transfer in a concrete slab has at least two obvious deficiencies. Firstly, we neglect edge effects such as convective cooling at the edges of the slab, $z' = 0, z_e$. However, the daily temperature variation should only be felt approximately 20cm into the concrete (this is determined by setting $\tau_c = 24 \times 3600$s in $\tau_c = \rho_c c_c R^2/\kappa_c$ to find $R \approx 24$cm). Our analysis is therefore justified provided we limit it to a region more than 20cm from the block ends. Secondly, we have imposed a symmetry condition at $r' = R$, where R is half of the spacing between pipes. In reality the second pipe would not be at the same temperature and so, although the temperature gradient must be zero somewhere between pipes, it is unlikely to be at R. Provided the second pipe is not too much hotter than the first, the error from this will be small. Further, it will not have a qualitative effect on our results (the same is true of the first problem). Perhaps more to the point, it will not affect our conclusions as to what are the important parameters governing the heat removal, which, of course, is the aim of this exercise.

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Quantity	Value	SI unit	Quantity	Value	SI unit
ρ_c	2350	kg/m^3	c_c	880	J/kg °C
ρ_w	1000	kg/m^3	c_w	4200	J/kg °C
κ_c	1.37	W/m °C	R	0.25	m
q_m'	1200	W/m^3	H	500	W/m^2 °C
Q	2×10^{-4}	m^3/s	a	0.025	m
κ_w	0.59	kg/m^3	z_e'	20	m

Table 1. Parameter values

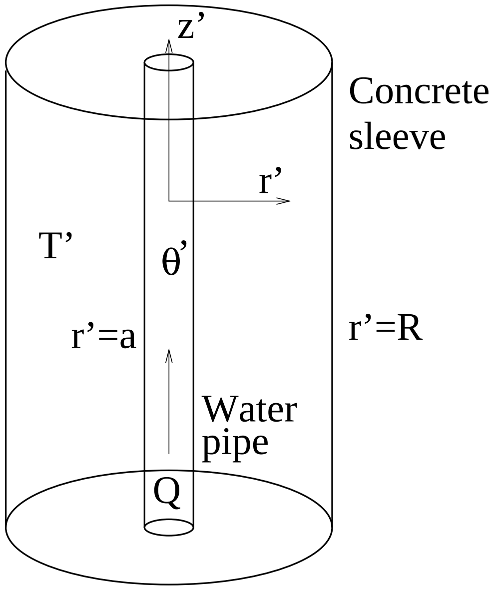

Figure 1. Problem configuration

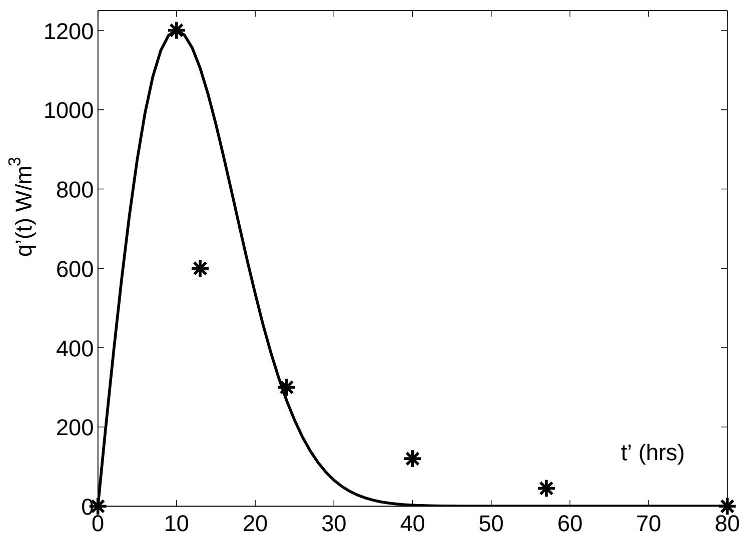

Figure 2. Typical adiabatic heat rate data and approximation given by equation (3)

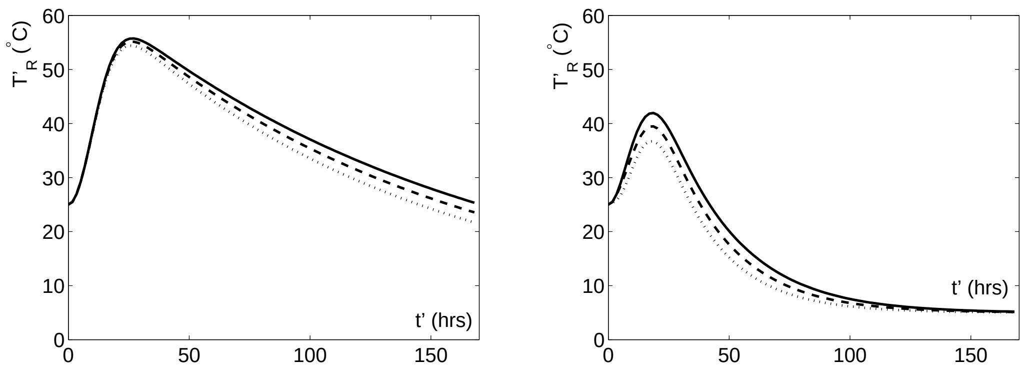

FIGURE 3. Typical temperature profiles in the concrete at $r' = R$, $z' = 0$ (dotted line), 10 (dashed line), 20 (solid line) m and a) $R = 0.5$m, b) $R = 0.25$m

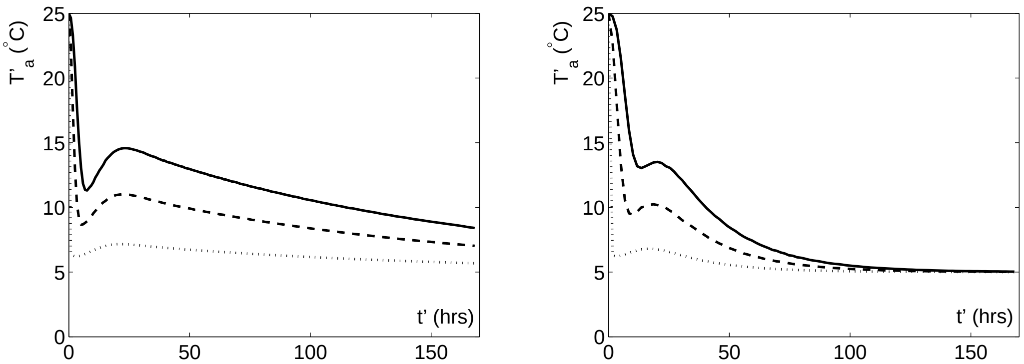

FIGURE 4. Typical temperature profiles in the concrete at $r' = a$, $z' = 0, 10, 20$m and a) $R = 0.5$m, b) $R = 0.25$m

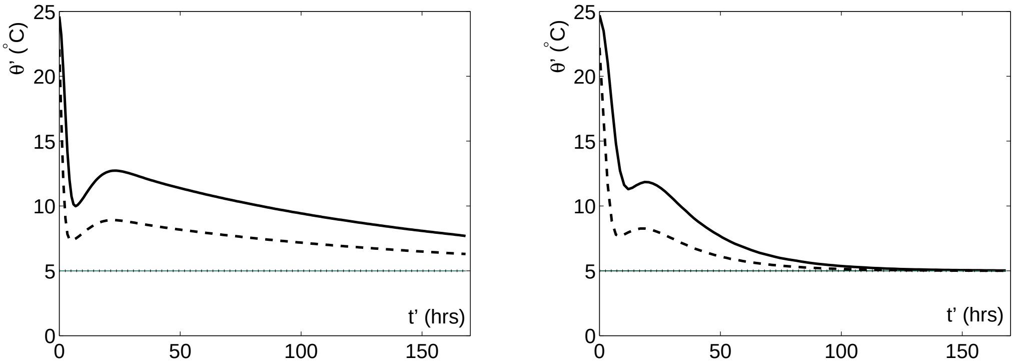

FIGURE 5. Typical temperature profiles in the water at $z' = 0, 10, 20$m and a) $R = 0.5$m, b) $R = 0.25$m

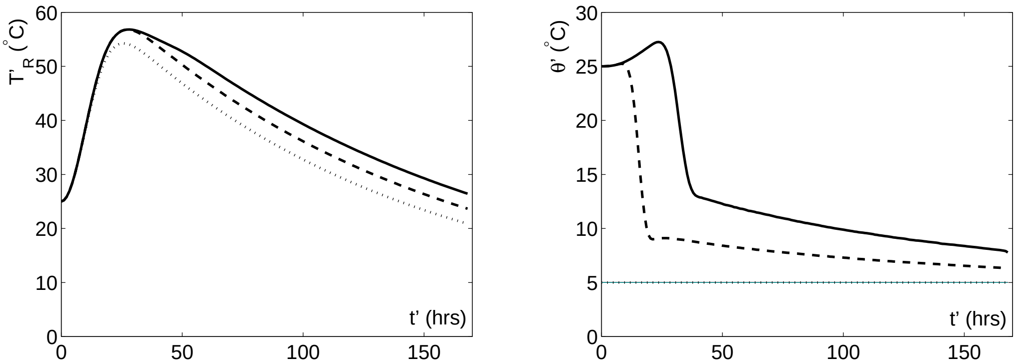

FIGURE 6. Typical temperature profiles in the concrete at $z' = 0, 10, 20$m with $H = 5000$ and a) $r' = R$ b) $r' = a$

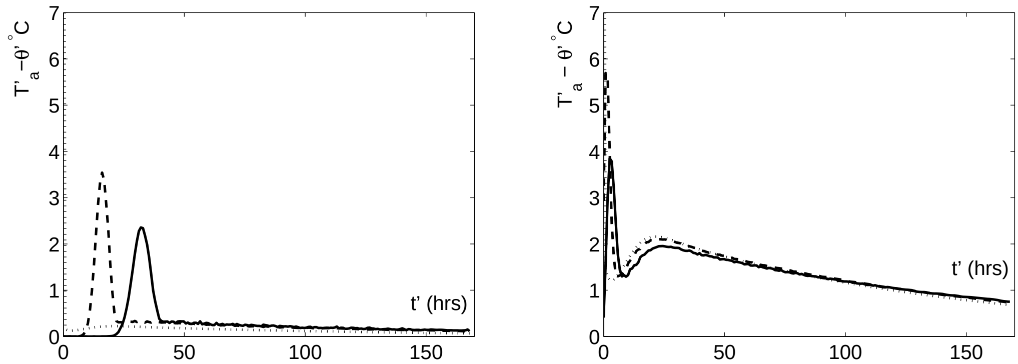

FIGURE 7. $T'_{r'=a} - \overline{\theta}'$ at $z' = 0, 10, 20$m with $H = 5000, 500$

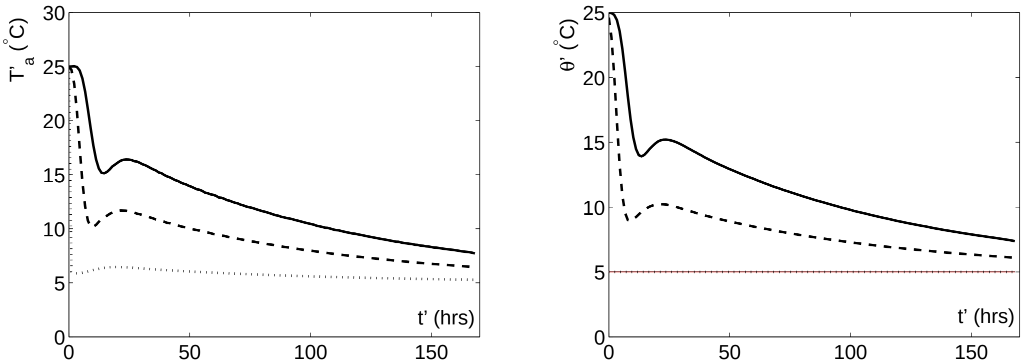

FIGURE 8. Typical temperature profiles at $z' = 0, 10, 20$m with $a = 0.05$m in a) concrete at $r' = a$ b) water

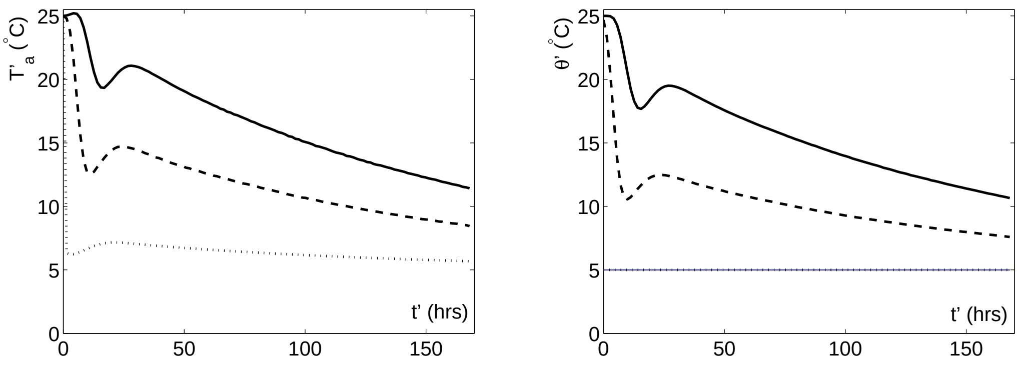

FIGURE 9. Typical temperature profiles at $z' = 0, 10, 20$m with $Q = 10^{-4}$ in a) concrete at $r' = a$ b) water

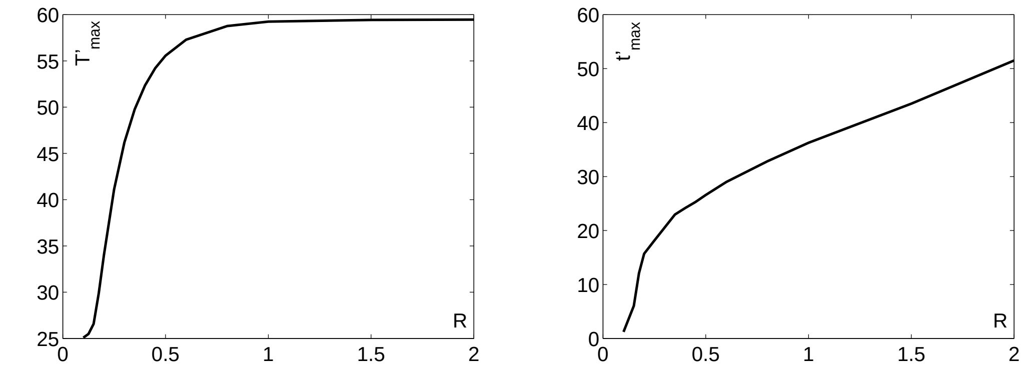

FIGURE 10. a) Maximum temperature against R, b) Time at which maximum temperature is reached

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