

Fermi Problems Activity Templates (FPAT) as a Didactical Tool in Primary Teacher Training

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ICTMA20

Würzburg (Bavaria, Germany) 24th-27th September 2022



Introduction

- Mathematical modelling is now a common curricula goal in many countries, to, among other things, show the potential of mathematics to describe complex phenomena and to promote critical thinking (Niss & Blum, 2020)
- However, modelling has not yet been established as a regular classroom activity (Borromeo Ferri, 2021) → the openness of mathematical modelling activities puts new demands on the teacher to cope with and handling the students' diverse thinking and decisions about how to approach the problems being modelled.
- Specific training for teachers and available suitable resources are essential to enable teachers to implement mathematical modelling in the classroom (Garfunkel et al., 2021).

Introduction

- One way to make modelling and mathematics relevant to students is to connect mathematics teaching to interesting real-life contexts and current affairs and topics in the news in ways that normally transcends what is offered in textbooks and traditional curricula materials.
- However, to first identify instances in out-of-school-contexts where mathematics comes to the fore or plays an important role, and then to make the connections to school mathematics, is non-trivial and requires solid and broad mathematical knowledge (Geiger et al., 2021).
- From a teacher education perspective, the challenge is **how to prepare pre-service teachers for this task**, in particular in the case of the training of primary teachers, since normally only a limited number of mathematics courses are part of their teacher training programmes.

The models and modelling perspective

In the *models and modeling perspective* (MMP) a (mathematical) model is defined as a general system consisting of elements, relationships, rules and operations that can be used to make sense of, predict, describe or explain some other system (Lesh & Doerr, 2003).

From this perspective learning is understood as developing useful and generalized models that are made up of

- (1) a set of concepts used to describe or explain the mathematical objects and aspects in context relevant to the phenomenon studied; and
- (2) procedures that can be used or re-used to create useful constructions, manipulations, or predictions for achieving clearly recognized goals in a range of contexts (Lesh & Harel, 2003).

Background: Fermi problems (FPs)

FPs are realistic and open estimation problems (Ärlebäck, 2009)

...are miniature-modeling problems in the sense that they are smaller and delimited real-world problems (Robinson 2008).

...at first glance seem impossible to solve, but that could be tackled by the *Fermi (estimates) method*:

decompose the original problem into simpler sub-problems to reach a solution of the original question by means of reasonable estimates or educated guesses (Carlson, 1997).

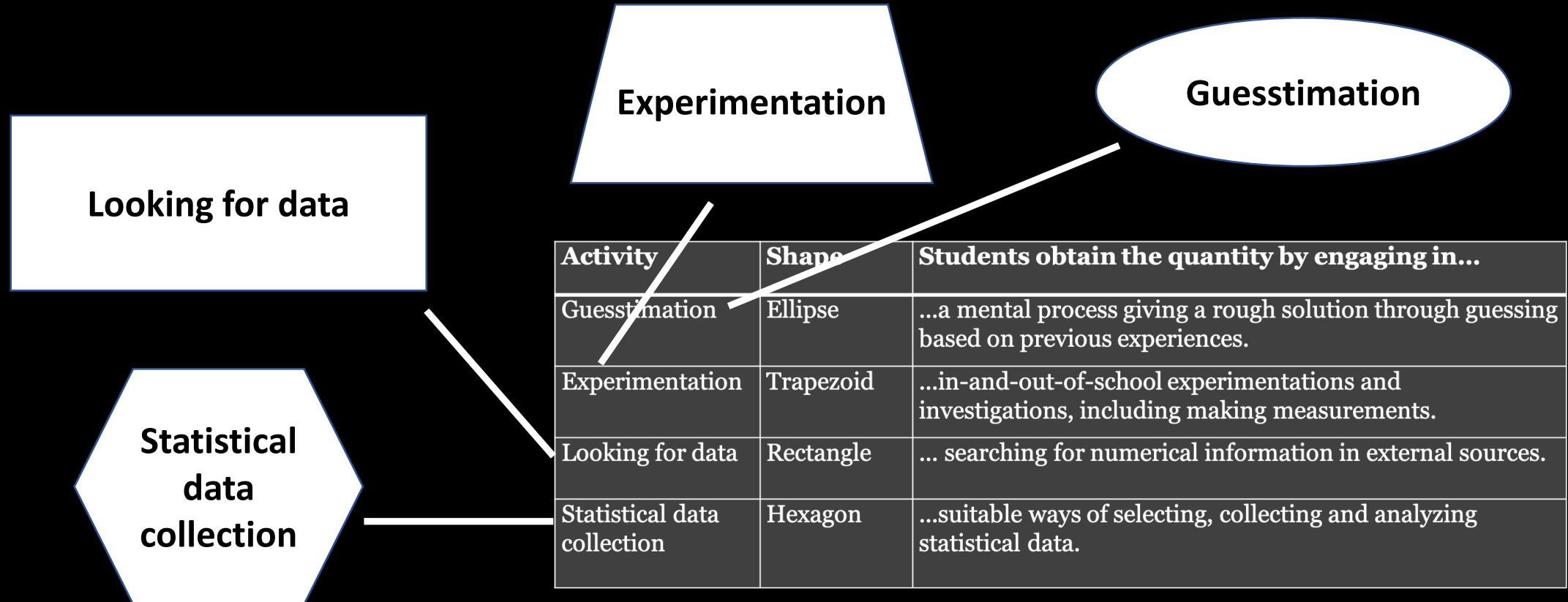
How many jelly beans can you fit in a one-litre bottle? How many pizzas are ordered in your city this year? How much gas is yearly consumed in our country?

Background: Mathematical activities to solve a FP

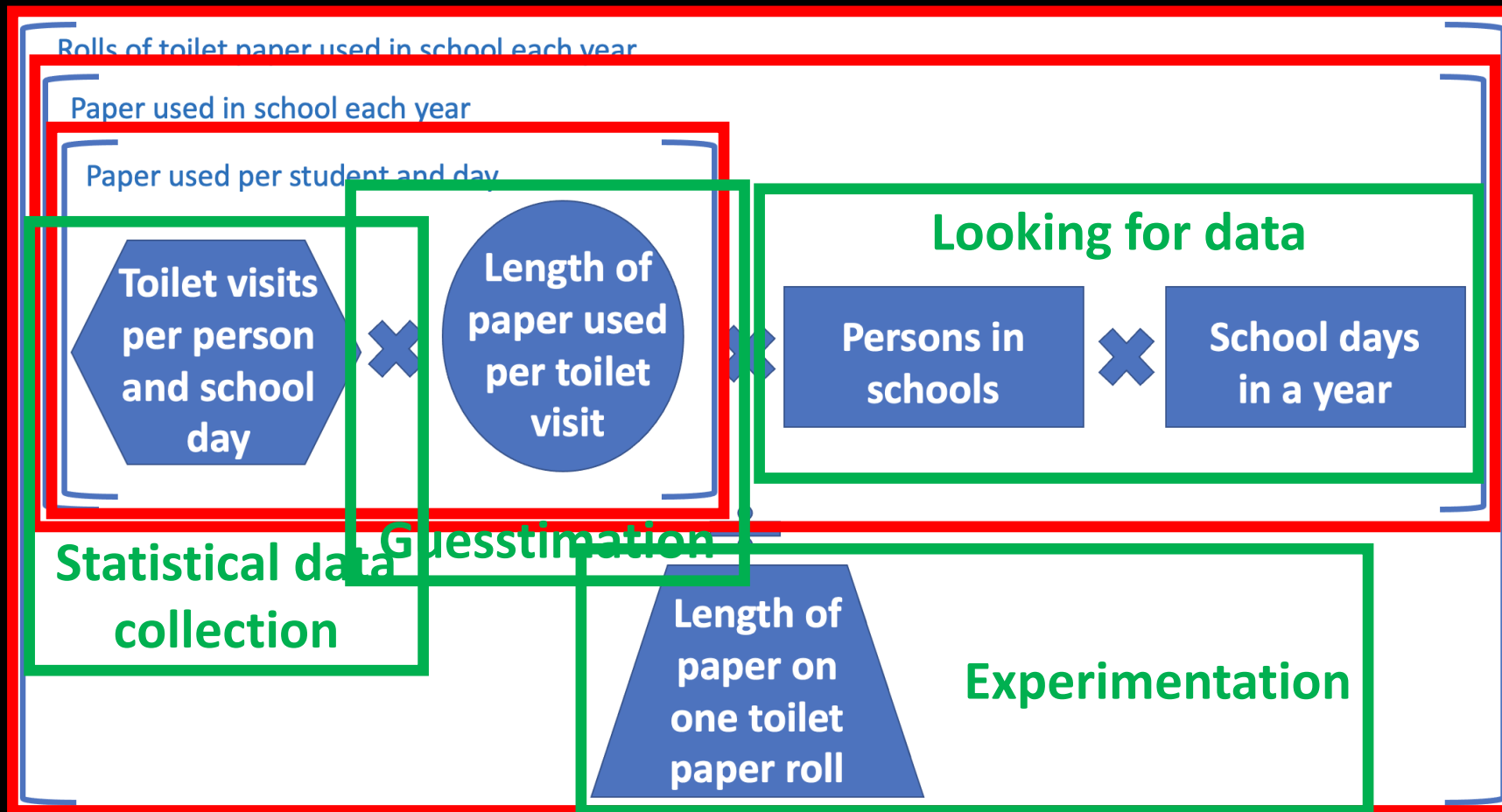
From Ärlebäck & Albarracín (2019) review on FP in STEM Education:

Activity	Shape	Students obtain the quantity by engaging in...
Guesstimation	Ellipse	...a mental process giving a rough solution through guessing based on previous experiences.
Experimentation	Trapezoid	...in-and-out-of-school experimentations and investigations, including making measurements.
Looking for data	Rectangle	... searching for numerical information in external sources.
Statistical data collection	Hexagon	...suitable ways of selecting, collecting and analyzing statistical data.

Background: FpAT – Fermi problem Activity Template



How many rolls of toilet paper are used yearly in the school?



Setting, the teaching sequence and methods

Collaborative research project involving a Catalan and a Swedish setting aiming at designing, implementing, and evaluating teaching sequences that introduce PSPTs to FPs as well as FPATs as didactical tools.

The Catalan setting:

- Teaching sequence: three 3-hour working sessions mixing brief lectures/instructions, problem solving (individually, pairs/groups), class discussions.
- 22 PSPTs in the 4th year of the primary teacher training degree; 3 courses in mathematics and mathematics education – no modelling or real-life problem solving.

Setting, the teaching sequence and methods

During these sessions the 22 PSPTs worked with a series of four FPs, including the following problem:

- A. If a city council wanted to buy all the cars in your town, how much money would it have to spend?*

Setting, the teaching sequence and methods

The PSPTs worked on these tasks in five stages:

- (i) individually wrote their proposed solution plan for the problem;
- (ii) in pairs, PSPTs discussed their proposals and agreed on joint solution strategy;
- (iii) in pairs, identified and connected the curricular mathematical content and procedures needed to solve the problem;
- (iv) after receiving a 20 minute introduction to the FPAT framework, in pairs, created and wrote down their FPAT representing the solutions of the problem; and
- (v) reflected on their work so far and revisited stages (ii) and (iii).

Aim and research question

Aim:

To develop a characterization of the FPATs generated by the PSPTs in our first implementation of the teaching sequence to gain a basic understanding of the structure and content that came to the fore in the PSPTs' FPATs.

Research questions:

What characterize the FPATs produced by the PSPTs in terms of

- (1) to what extent their proposed solutions reasonably solve the problems under consideration;
- (2) the key concepts and procedures that structure their solutions; and
- (3) the mathematical activities proposed in their solutions

Analysis

Focusing on the mathematical structure of the PSPTs' solutions and if the quantities (Thompson, 1993) they chose to mathematize and include in their solutions

(a) do solutions mediated by the FPATs reasonably solve the problem?



Solves on paper?
Feasible? Close?

(b) what the key concepts and procedures the PSPTs used?



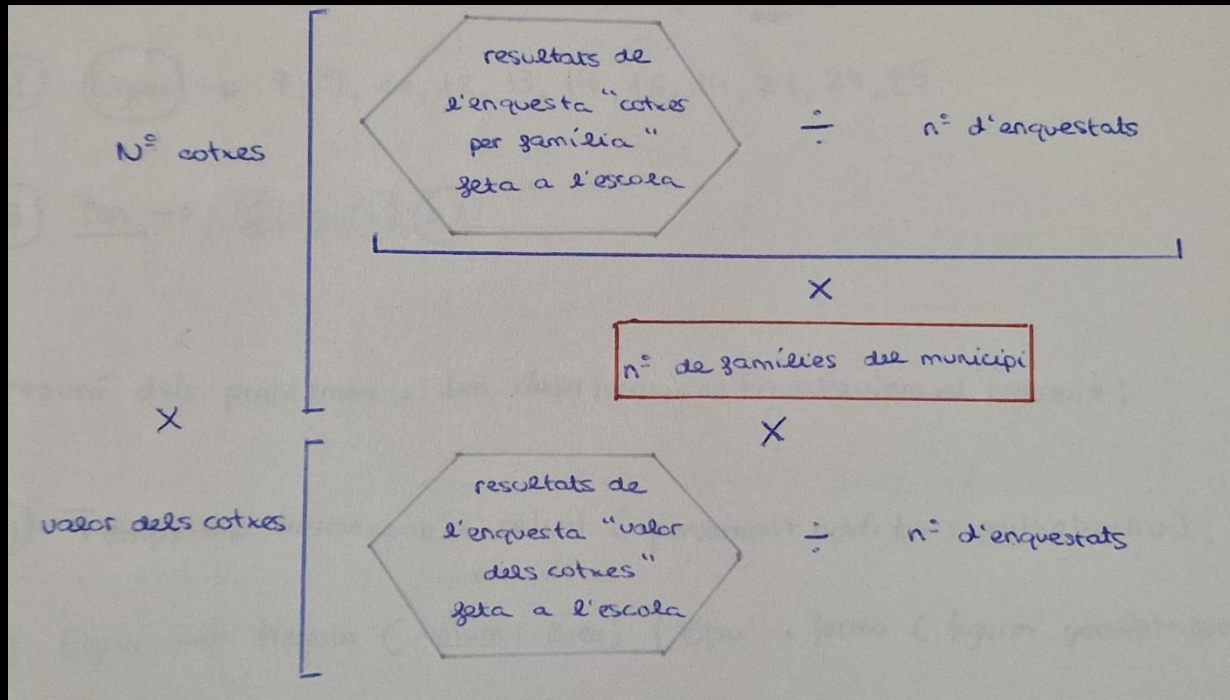
#Quant(ities)
Key Constructs

(c) the number of sub-problems and what types of FPAT-activities?



#Sub-problems
Guess, Expr,
Stat, Look f. data

Example of students' work and the analysis



The handwritten calculation shows the cost of cars based on various factors. The formula is:

$$\text{Lletres B, C, D} \times \text{Km} \times \text{ITV} \times \text{Percentatge} = \text{Diners que es gasten}$$

The calculation is broken down into several rows, each representing a different combination of factors. The factors are:

- Lletres B, C, D:** Letters B, C, and D represent different car categories.
- Km:** Kilometers, categorized as <100 Km, >100 Km, >200 Km, and >200 Km.
- ITV:** Impuesto de Tráfico de Vehículos, categorized as Apte (Apt), NO Apte (Not Apt), and NO Apte (pendent) (Not Apt (pending)).
- Percentatge:** Percentages, categorized as 35%, 5%, 10%, 15%, and 20%.

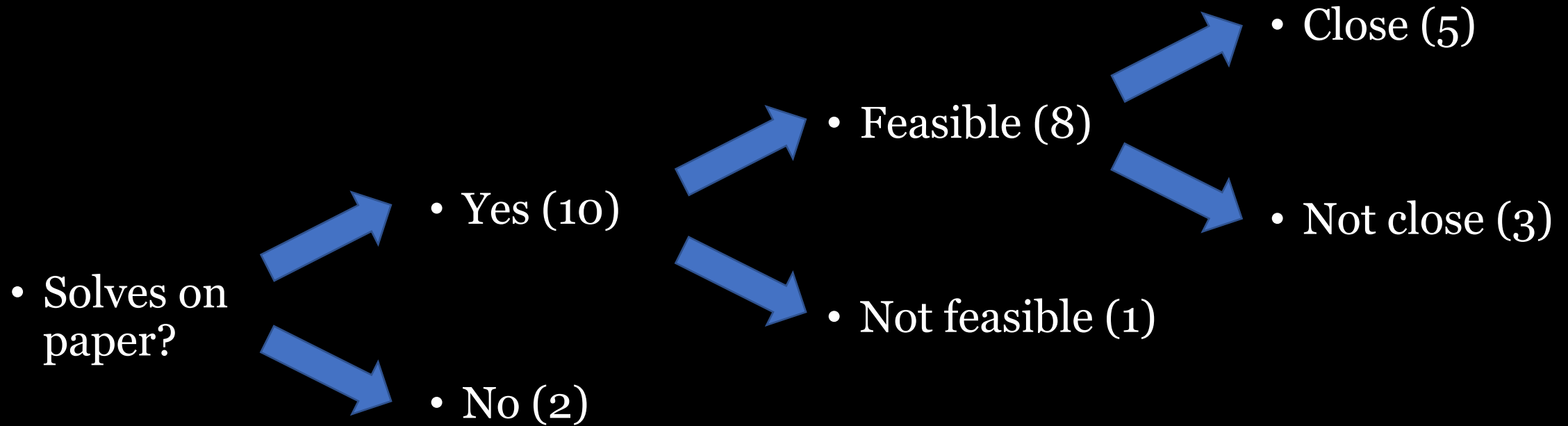
The final result is "Diners que es gasten" (Money that is spent).

Llegenda:

- Lletres B, C, D $\rightarrow$ 35%
- Lletres F, G, H, J $\rightarrow$ 25%
- Lletres K, L $\rightarrow$ 15%
- <100 Km $\rightarrow$ 5%
- >100 Km $\rightarrow$ 10%
- >200 Km $\rightarrow$ 20%
- ITV Apte $\rightarrow$ 5%
- NO Apte (pendent) $\rightarrow$ 10%
- NO Apte $\rightarrow$ 15%

* Tindran en compte els cotxes anteriors a la Lletra B

Results – Solution characterization - 11 FpATs



Pair	Solves on paper?	Feasible?	Close?	# Sub-problems	# Quant	Guess	Expr	Stat	Look f. data	Key construct
1	Yes	Yes	Yes	3	3	0	0	2	1	average price of a car
2	Yes	Yes	Yes	2	2	0	0	1	1	stratification of cars by price
3	Yes	Yes	Yes	3	2	0	1	1	1	average price of a car and devaluation
4	No	No	No	5	3	0	1	0	4	stratification of cars by price and devaluation
5	Yes	Yes	No	3	2	2	0	0	1	stratification of cars by price and devaluation
6	Yes	Yes	Yes	2	2	0	0	1	1	stratification of cars by price
7	No	No	No	2	2	1	0	0	1	average price of a car
8	Yes	Yes	No	2	2	1	0	0	1	average price of a car
9	Yes	Yes	Yes	2	2	0	0	1	1	average price of a car
10	Yes	Yes	No	2	2	1	0	0	1	stratification of cars by price
11	Yes	No	No	4	1	0	0	0	3	stratification of cars by price and use

Pair	Solves on paper?	Feasible?	Close ?	# Sub-problems	# Quant	Guess	Expr	Stat	Look f. data	Key construct
1	Yes	Yes	Yes	3	3	0	0	2	1	average price of a car
2	Yes	Yes	Yes	2	2	0	0	1	1	stratification of cars by price
3	Yes	Yes	Yes	3	2	0	1	1	1	average price of a car and devaluation
6	Yes	Yes	Yes	2	2	0	0	1	1	stratification of cars by price
9	Yes	Yes	Yes	2	2	0	0	1	1	average price of a car
5	Yes	Yes	No	3	2	2	0	0	1	stratification of cars by price and devaluation
8	Yes	Yes	No	2	2	1	0	0	1	average price of a car
10	Yes	Yes	No	2	2	1	0	0	1	stratification of cars by price
11	Yes	No	No	4	1	0	0	0	3	stratification of cars by price and use
4	No	No	No	5	3	0	1	0	4	stratification of cars by price and devaluation
7	No	No	No	2	2	1	0	0	1	average price of a car

Main Conclusions

- The analysis shows the complexities inherent in solving FPs set in real and everyday contexts.
- The variety in key constructs seems to be an indicator of the previous knowledge the students have about the context of the problems.
- Not all future teachers know how to develop a successful resolution proposal in terms of FPAT.
- Feasible proposals are those that are based on a good use of experimentation and statistical data collection activities.

References

- Albarracín, L., & Ärleback, J. B. (2025). [Exploring the role of assumptions in mathematical modeling teacher training using Fermi problems](https://doi.org/10.1007/s11858-025-01677-0). *ZDM – Mathematics Education* 57(2-3), 213–228. <https://doi.org/10.1007/s11858-025-01677-0>
- Ärleback, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Mathematics Enthusiast*, 6(3), 331–364.
- Ärleback, J. B., & Albarracín, L. (2019). The use and potential of Fermi problems in the STEM disciplines to support the development of 21st century competencies. *ZDM Mathematics education*, 51(7), 979-990.
- Ärleback, J. B., & Albarracín, L. (2024). [Fermi problems as a hub for task design in mathematics and stem education](https://doi.org/10.1093/teamat/hrado02). *Teaching Mathematics and its Applications*, 43(1), 25-37. <https://doi.org/10.1093/teamat/hrado02>
- Ärleback, J. B., & Albarracín, L. (2022). [FPAT—A Framework for Facilitating the Teaching and Learning of Using Fermi Problems Originating in Mathematics Education Research](https://doi.org/10.1080/00207179.2022.2088888). *The Physics Teacher* 60(7), 569-571.
- Borromeo Ferri, R. (2021). Mandatory Mathematical Modelling in School: What Do We Want the Teachers to Know?. In F. K. S. Leung, G. Stillman, G. Kaiser & K. L. Wong, *Mathematical Modelling Education in East and West* (pp. 103-117). Springer, Cham.
- Doerr, H., & English, L. D. (2003). A Modeling perspective on students' mathematical reasoning about data. *Journal of Research in Mathematics Education*, 34(2), 110-136.
- Garfunkel, S., Niss, M., & Brown, J. P. (2021). Opportunities for modelling: An extra-curricular challenge. In F. K. S. Leung, G. Stillman, G. Kaiser & K. L. Wong (Eds.), *Mathematical Modelling Education in East and West* (pp. 363-375). Springer, Cham.
- Geiger, V., Galbraith, P., Niss, M., & Delzoppo, C. (2022). Developing a task design and implementation framework for fostering mathematical modelling competencies: Geiger V. et al. *Educational Studies in Mathematics*, 109(2), 313-336.
- Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical thinking and learning*, 5(2-3), 157-189.
- Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. London: Routledge.
- Thompson, P. W. (1993). Quantitative reasoning, complexity, and additive structures. *Educational Studies in Mathematics*, 25, 165–208.

Thank you for your interest!

Esta publicación es parte del proyecto PID2021-126707NB-I00, financiado por MICIU/AEI/10.13039/501100011033/FEDER, UE.

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