

Exploring the use of Fermi Problems and the FPAT-framework with pre-service primary teachers to bring real-life contexts into classrooms

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Introduction

- Mathematical modelling is now a common curricula goal in many countries, to, among other things, show the potential of mathematics to describe complex phenomena and to promote critical thinking (Niss & Blum, 2020)
- However, modelling has not yet been established as a regular classroom activity (Borromeo Ferri, 2021) <- the openness of mathematical modelling activities puts new demands on the teacher to cope with and handling the students' diverse thinking and decisions about how to approach the problems being modelled.
- Specific training for teachers and available suitable resources are essential to enable teachers to implement mathematical modelling in the classroom (Garfunkel et al., 2021).

Introduction

- One way to make modelling and mathematics relevant to students, is for teachers to connect their everyday mathematics teaching to interesting real-life contexts and current affairs and topics in the news in ways that normally transcends what is offered in textbooks and traditional curricula materials.
- However, to first identify instances in out-of-school-contexts where mathematics comes to the fore or plays an important role, and then to make the connections to school mathematics, is non-trivial and requires solid and broad mathematical knowledge (Geiger et al., 2021).
- From a teacher education perspective, the challenge is how to prepare pre-service teachers for this task, in particular in the case of the training of primary teachers, since normally only a limited number of mathematics courses are part of their teacher training programmes.

Background: Fermi problems (FPs)

...are miniature-modeling problems in the sense that they are smaller, more well-defined and delimited real-world problems (Robinson 2008).

...at first glance seem impossible to solve, but that could be tackled by the *Fermi (estimates) method*:

decompose the original problem into simpler sub-problems to reach a solution of the original question by means of reasonable estimates or educated guesses (Carlson, 1997).

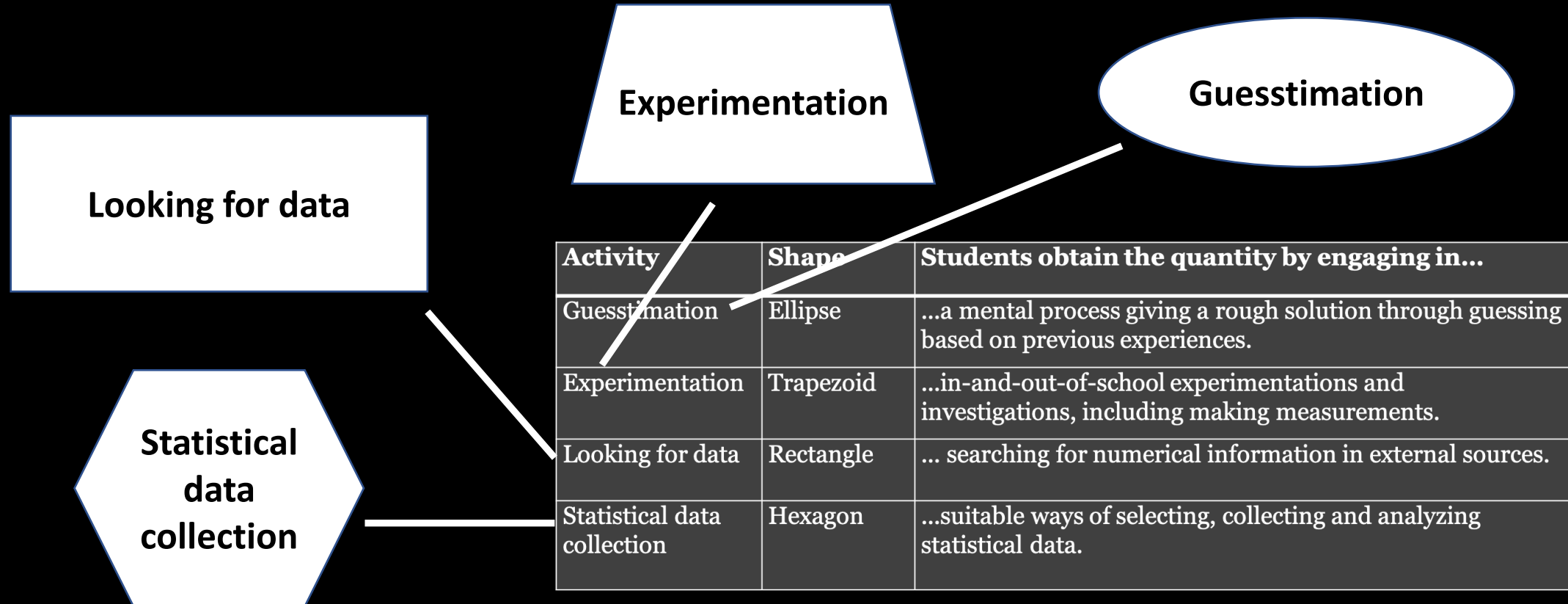
How many jelly beans can you fit in a one-litre bottle? How many pizzas are ordered in your city this year? How much gasoline is yearly consumed in our country?

Background: Mathematical activities to solve a FP

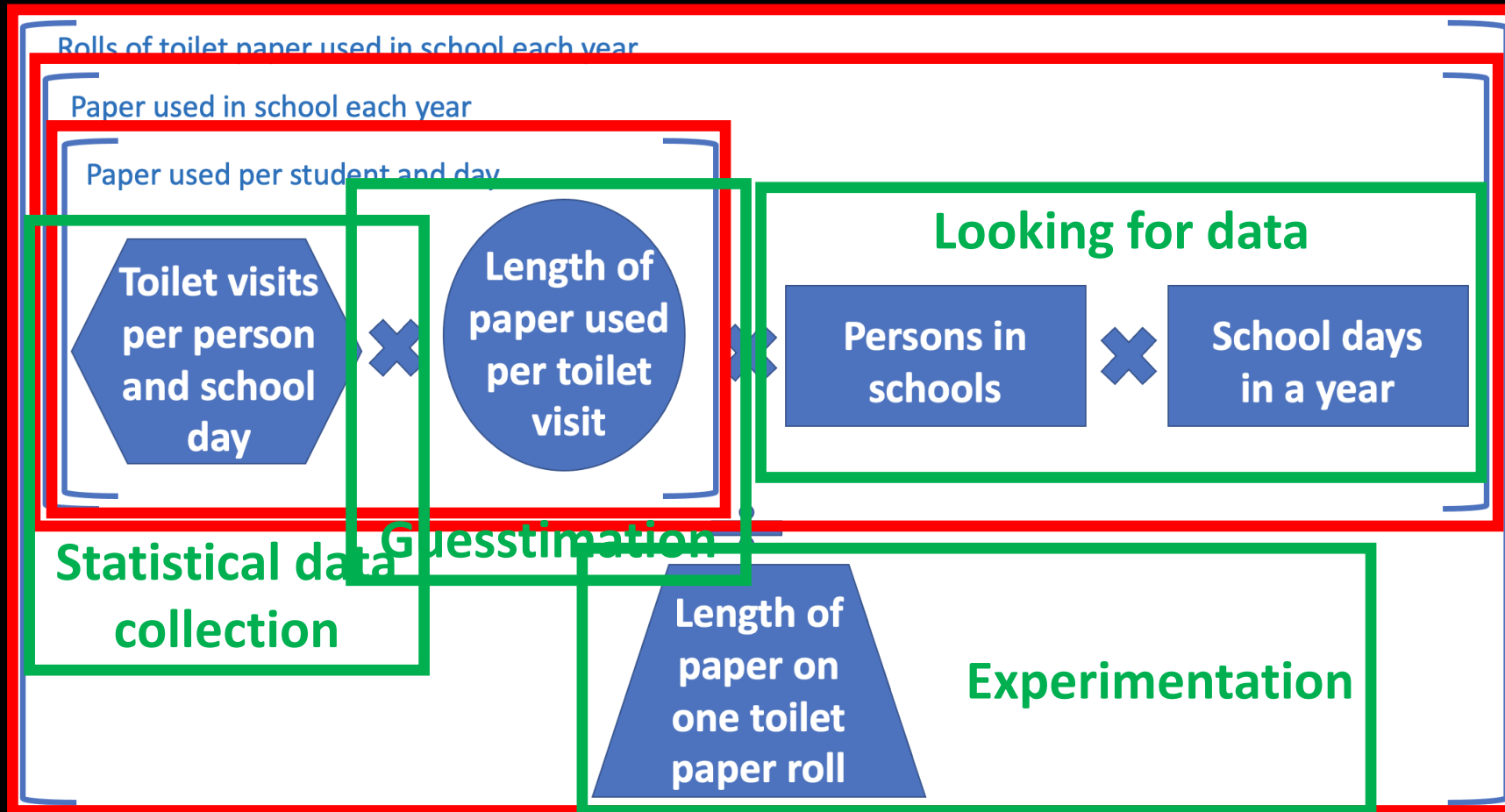
From Ärlebäck & Albarracín (2019) review on FP in STEM Education:

Activity	Shape	Students obtain the quantity by engaging in...
Guesstimation	Ellipse	...a mental process giving a rough solution through guessing based on previous experiences.
Experimentation	Trapezoid	...in-and-out-of-school experimentations and investigations, including making measurements.
Looking for data	Rectangle	... searching for numerical information in external sources.
Statistical data collection	Hexagon	...suitable ways of selecting, collecting and analyzing statistical data.

Background: FpAT – Fermi problem Activity Template



How many rolls of toilet paper are used yearly in the school?



Setting, the teaching sequence and methods

Collaborative research project involving a Catalan and a Swedish setting aiming at designing, implementing, and evaluating teaching sequences that introduce PSPTs to FPs as well as FPATs as didactical tools.

The Catalan setting:

- Teaching sequence: three 3-hour working sessions mixing brief lectures/instructions, problem solving (individually, pairs/groups), class discussions.
- 22 PSPTs in the 4th year of the primary teacher training degree; 3 courses in mathematics and mathematics education – no modelling or real-life problem solving.

Setting, the teaching sequence and methods

During these sessions the 22 PSPTs worked with a series of four FPs, including the following two problems:

- A. How many toilet paper rolls are needed at school in a year? How much space do they occupy?*
- B. How many ambulances are needed if we want to attend any emergency in less than 8 minutes in any place in Catalonia?*

Note: Problem A is set in a known and tangible situation for the PSPTs. In contrast, problem B presents a complex situation about which the PSPTs have less experience.

Setting, the teaching sequence and methods

The PSPTs worked on these tasks in five stages:

- (i) individual problem for the two
- (ii) discuss **A. How many toilet paper rolls are needed at school in a year? How much space do they occupy?** strategies;
- (iii) in pairs content and procedures needed to solve each of the problems;
- (iv) after receiving a 20 minute introduction to the FPAT framework, in pairs, created and wrote down their FPATs representing their solutions of the problems; and
- (v) reflected on their work so far and revisited stages (ii) and (iii).

Aim and research question

Aim:

To develop a characterization of the FPATs generated by the PSPTs in our first implementation of the teaching sequence to gain a basic understanding of the structure and content that came to the fore in the PSPTs' FPATs.

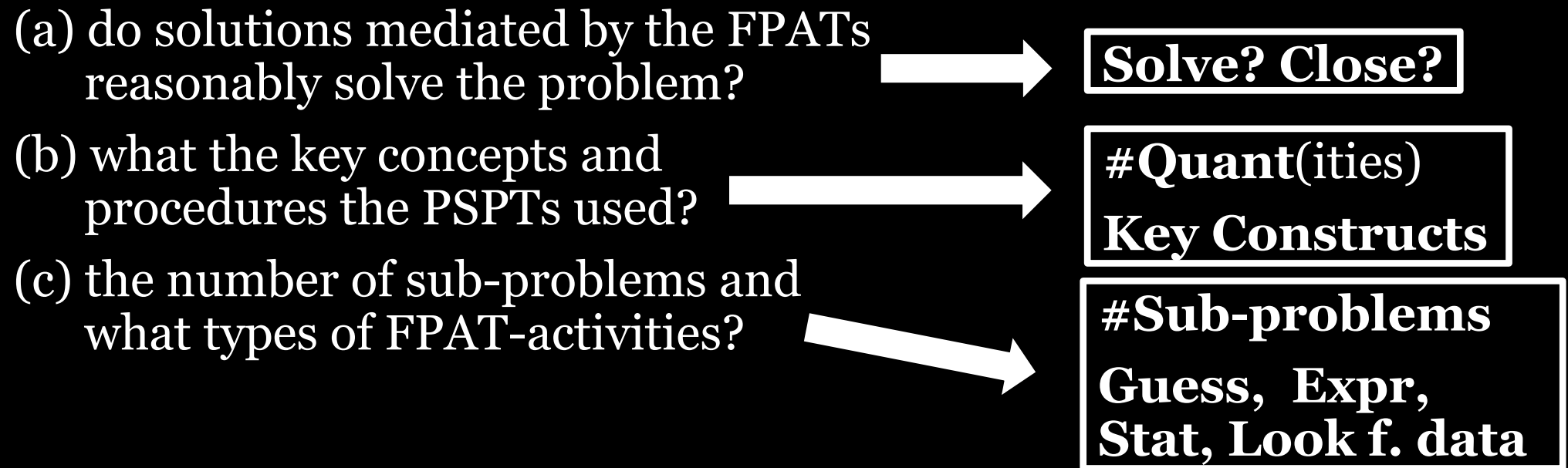
Research questions:

What characterize the FPATs produced by the PSPTs in terms of

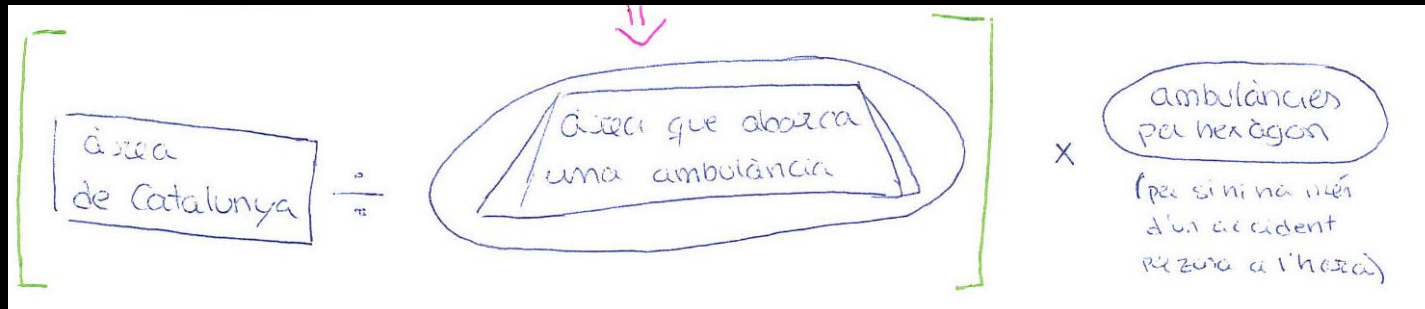
- (1) to what extent their proposed solutions reasonably solve the problems under consideration;
- (2) the key concepts and procedures that structure their solutions; and
- (3) the mathematical activities proposed in their solutions?

Analysis

Focusing on the mathematical structure of the PSPTs' solutions and if the quantities (Thompson, 1993) they chose to mathematize and include in their solutions



Example of students' work and the analysis



Results

Problem A: Toilet paper problem

Pair	Solves?/ Close?	# Sub- problems	# Quant	Guess	Expr	Stat	Look f. data	Key construct
1	Yes/No	3	3	1	2	0	0	Consumption per toilet
2	Yes/No	5*	4	3	0	0	2	Times we go to toilet
3	No/No	8*	4	2	3	0	3	Length of paper used
4	Yes/No	4*	3	1	1	0	2	Rolls used per week
5	Yes/Yes	5*	4	3	0	0	2	Length of paper used
6	Yes/No	4	3	0	0	0	4	Rolls per week
7	No/No	5*	2	1	1	2	1	Time roll lasts per class
8	No/No	4	3	1	1	1	1	Time roll lasts per class
9	Yes/No	3*	3	1	1	0	1	Rolls per toilet
10	Yes/Yes	6*	4	2	1	0	3	Length of paper used
11	Yes/No	2*	2	1	0	0	1	Rolls per week
Mean	-	4.45	3.18	1.45	0.91	0.27	1.81	-

Problem B: Ambulance problem

Pair	Solves?/ Close?	# Sub- problems	# Quant	Gues s	Expr	Stat	Look f. data	Key construct
1	No/No	4*	2	2	0	0	2	Ambulance reach (AR)
2	Yes/No	3	2	1	1	0	1	AR
3	No/No	7*	3	3	0	0	4	AR
4	No/No	3*	2	1	0	0	2	AR
5	No/No	8*	4	4	0	0	4	AR + density
6	No/No	4	3	1	0	0	3	AR
7	No/No	6*	2	4	0	0	2	AR
8	No/No	7	2	0	4	0	3	AR
9	No/No	2*	1	1	0	0	1	AR
10	No/No	2*	1	0	1	0	1	AR
11	No/No	2*	1	0	0	0	2	AR
Mean	-	4.36	2.09	1.54	0.54	0	2.27	-

Solve? Close? 8-2 / 1-0

Activities ~similar

Key construct

#Sub-prob. 4.45 / 4.36

#Quant 3.18 / 2.09

6 / “1.5”

Conclusions

- The analysis shows the complexities inherent in solving FPs set in real and everyday contexts.
- The disparity in contexts is reflected in the number of successfully proposed FPATs and the proportions of the different types of activities suggested
- The variety in key constructs seems to be an indicator of the previous knowledge the students have about the context of the problems.
- An institutional effect on the choice of activities discernible

We suggest that FPATs can be useful in teacher training both to (a) design teaching sequences with FPs; and (b) function as a didactical tool for PSPTs and teachers in connecting and bringing out-of-school contexts into the classroom as well as using the FPATs to make different ways of solving a problem explicit (Tjoe, 2019).

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Thank you for your interest!

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