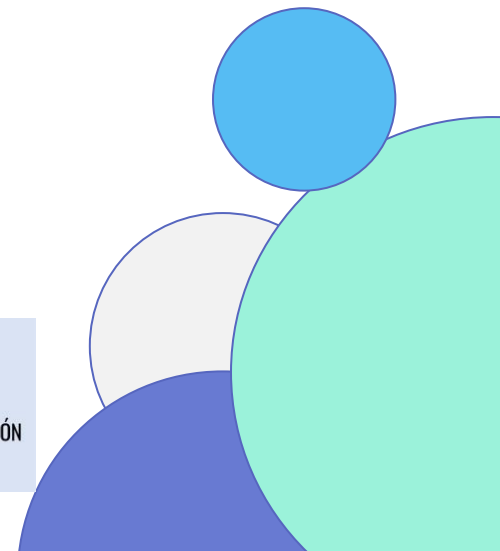


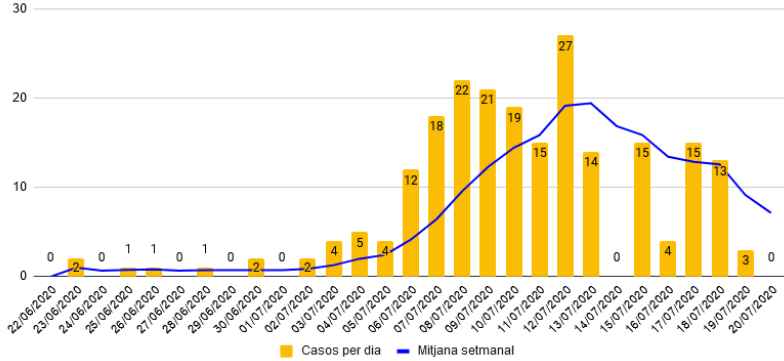
Combining analysis tools to characterize mathematical models in statistical tasks

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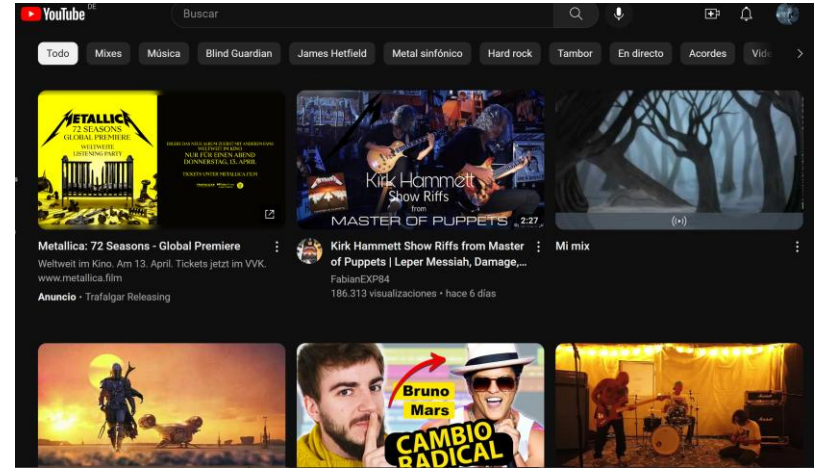
UAB Universitat Autònoma
de Barcelona



Statistics today

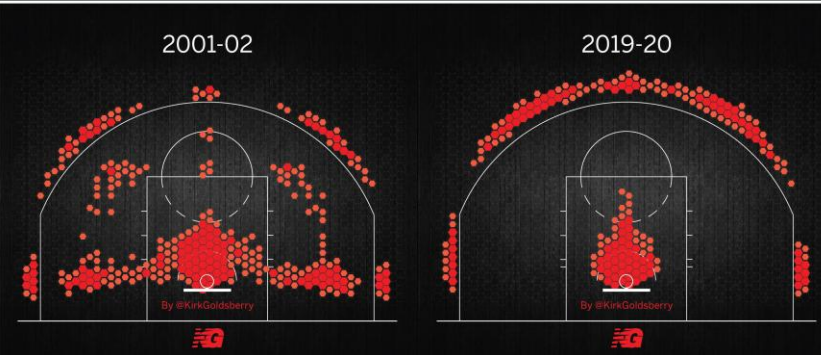


	104	38	30	340	40-27	4-36	20-60	13-76	32-00	70	38	38	30	3	4	4	72	104	-30	170
104	0	1	0	11	0-5	0-0	0-0	0-0	-	1	0	1	0	0	0	0	0	0	-0	0
38	0	1	0	4	0-0	0-0	-	0-0	-	1	0	1	0	0	0	0	0	0	0	1
30	5	0	0	4	1-5	0-1	33-30	0-0	-	0	0	0	0	0	1	0	0	5	0	1
340	5	1	1	4	0-0	0-1	0-0	5-5	100-00	0	1	1	1	0	0	0	0	5	0	3
40-27	3	3	0	10	1-5	0-0	20-00	1-5	20-00	1	5	3	0	0	0	0	0	3	-1	4
4-36	0	2	0	50	3-0	0-0	20-00	0-0	-	5	3	2	0	0	0	1	2	0	-14	0
20-60	3	3	0	53	3-0	0-3	33-30	1-5	20-00	5	1	3	0	0	1	0	0	3	14	4
13-76	3	0	1	8	5-5	1-5	32-00	0-0	-	0	0	0	0	0	0	0	0	3	-1	3
32-00	3	4	5	51	3-2	0-1	20-00	1-5	20-00	1	3	4	5	0	0	5	1	3	-4	11
70	8	5	2	52	4-3	0-3	40-00	0-0	-	1	1	5	2	1	5	0	0	8	-10	10
38	0	3	5	31	5-3	1-3	30-00	5-5	100-00	1	5	3	3	0	0	0	0	0	-15	3
38	15	4	1	13	2-2	0-3	05-20	5-5	100-00	0	4	4	1	1	0	0	1	13	-4	13
30	10	8	0	35	0-3	1-5	33-00	1-5	20-00	0	8	8	0	4	0	0	1	10	-53	53
340	52	3	5	55	10-14	1-3	00-30	5-5	100-00	0	3	3	3	1	0	0	3	52	-18	53

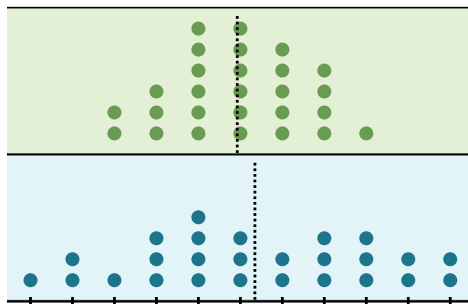
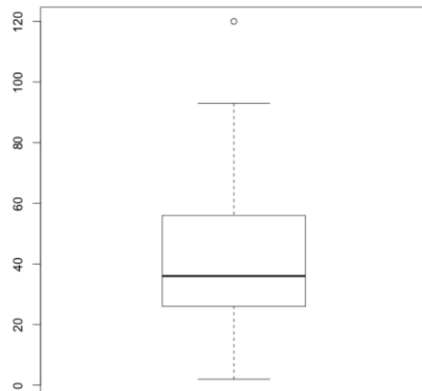
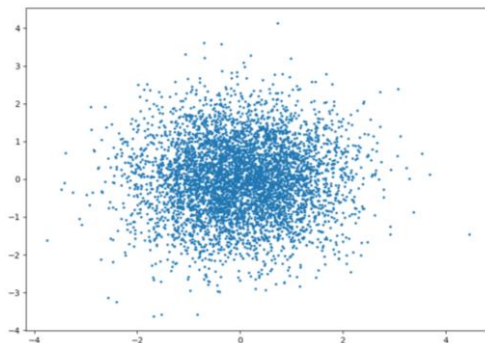


THE GAME HAS CHANGED

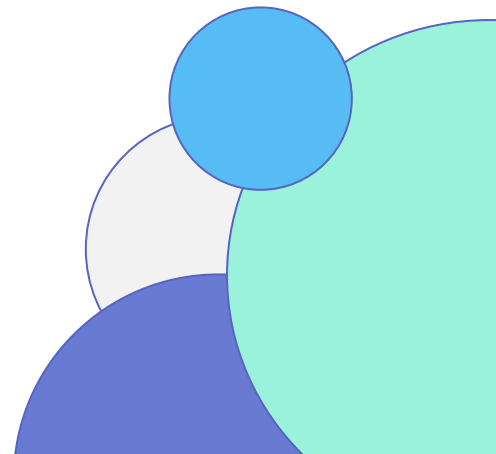
Top 200 shot locations in the NBA, 2001-02 versus 2019-20



Variation



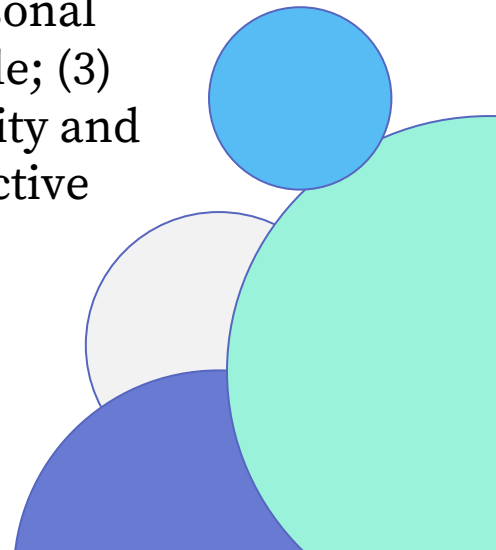
$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$



Models & Modelling Perspective

Modelling Eliciting Activities (MEAs) are designed to elicit problem-solvers' models and therefore considered as thought-revealing activities. (Lesh & Doerr, 2003).

MEAs are designed based on six principles: (1) reality (personal meaningfulness) principle; (2) model-construction principle; (3) construct documentation principle; (4) construct shareability and reusability principle; (5) self-assessment principle; (6) effective prototype principle. (Lesh et al., 1997).



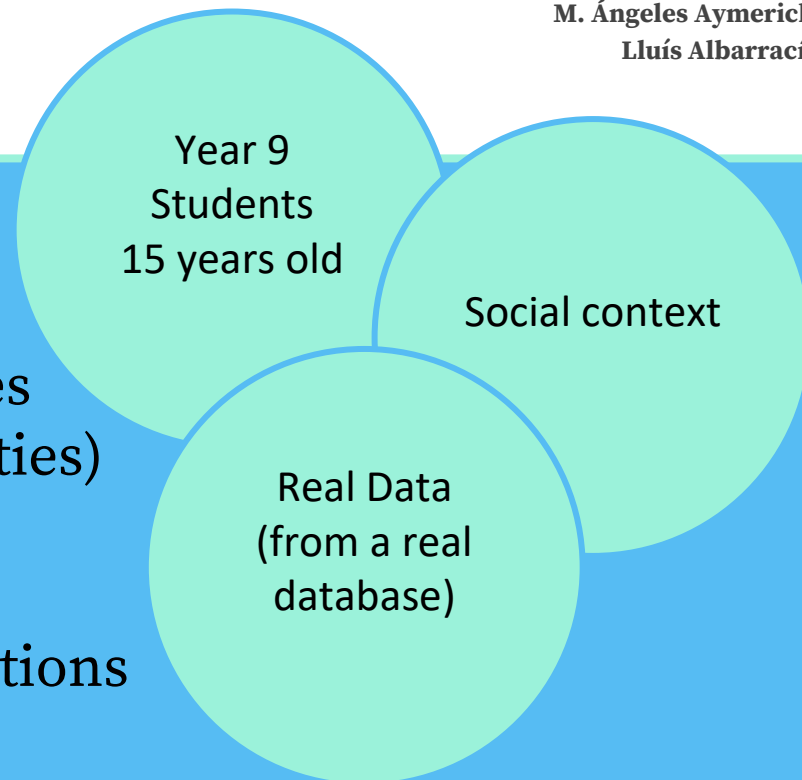
RESEARCH OBJECTIVES

1. Describe the mathematical models developed by secondary school students working in a statistical task about variation.
2. Describe the modelling activities performed by secondary school students working in a statistical task about variation.
3. Identify the key episodes in the generation of the main concepts that support mathematical models.

STATISTICAL ACTIVITY

Model Eliciting Activities Principles
Vehicle taxes in Spain (municipalities)

Students work in groups of 4
Audio recordings + written productions
12 complete sets of data
Complete analysis of just 1 group (*for now*)



TARIFAS IMPUESTO DE CIRCULACIÓN 2018											
PROVINCIA	TURISMOS (CVF)					CICLO	MOTOCICLETAS (C.C)				
	De menos de 8	De 8 a 12	De 12 a 16	Más de 16	Más de 20		Hasta 125	125 a 250	250 a 500	500 a 1.000	Más de 1.000
A CORUÑA	18,95	60,80	128,35	179,20	224,00	6,65	6,65	13,50	27,00	60,60	121,15
ALBACETE	23,98	64,75	136,69	173,84	217,28	7,87	7,87	13,47	26,97	53,92	107,83
ALICANTE	22,28	60,47	128,37	162,32	204,75	7,43	8,49	13,79	27,58	55,17	111,39
ALMERÍA	23,12	62,41	131,78	164,16	205,11	8,10	8,10	13,88	27,74	55,46	110,88
ÁVILA	20,69	57,93	124,45	163,09	200,48	8,08	8,13	14,53	29,54	60,58	119,34
BADAJOS	19,85	53,61	113,16	140,96	176,18	6,95	6,95	11,91	23,83	47,65	95,29
BARCELONA	23,47	64,06	136,69	172,05	217,28	8,39	8,39	14,38	28,78	58,16	117,53
BILBAO	25,08	67,92	(*)	268,32	343,20	10,08	10,08	17,88	37,08	74,88	155,04
BURGOS	23,19	62,62	132,20	164,66	205,80	8,13	8,13	13,91	27,84	55,66	111,32
CÁCERES	17,00	50,00	100,00	125,00	140,00	8,00	8,50	11,00	23,00	43,00	84,00
CÁDIZ	24,90	67,25	142,00	177,05	220,90	8,80	8,80	14,85	29,90	59,85	119,60
CASTELLÓN	24,99	67,48	142,44	177,43	221,76	8,75	8,75	14,99	30,00	59,97	119,95
CEUTA	12,60	34,10	71,95	89,60	112,00	4,40	4,40	7,55	15,15	30,30	60,60
CIUDAD REAL	25,24	68,16	143,88	179,22	224,00	8,84	8,84	15,14	30,30	60,58	121,16
CÓRDOBA	24,22	65,40	143,88	179,22	224,00	8,57	8,57	14,69	29,39	58,76	117,53
CUENCA	22,09	59,64	125,89	179,22	224,00	7,74	7,74	13,25	26,51	53,01	106,01
GIRONA	25,24	67,24	142,40	178,00	224,00	8,84	8,84	15,04	30,20	60,24	121,16

Within the same region, even in the same province, the differences are obvious.

Which provinces are the *fairer*, and therefore, does the fact of registering in one town or another not represent a comparative disadvantage for the owners of the vehicles?

The tax authorities are asking you for a report to assess this situation and to decide whether the state should intervene. It has to include a compilation of proposals assessing different solutions.

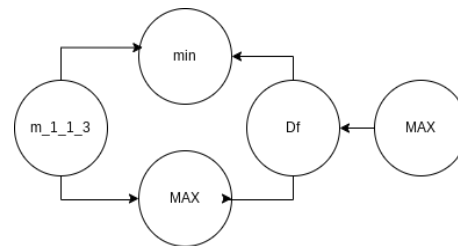
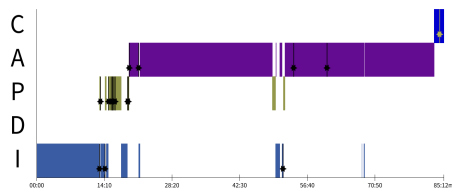
ANALYSIS TOOLS

MAD
(Modelling
Activity
Diagram)

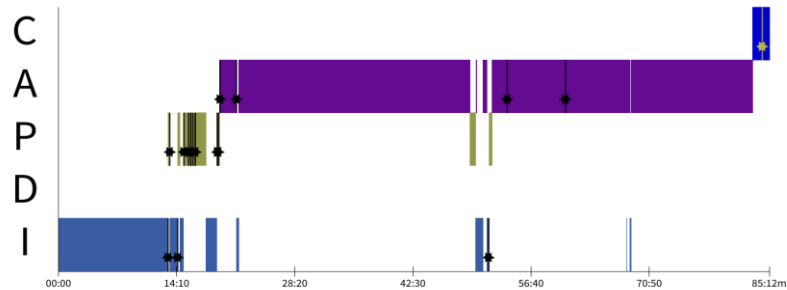
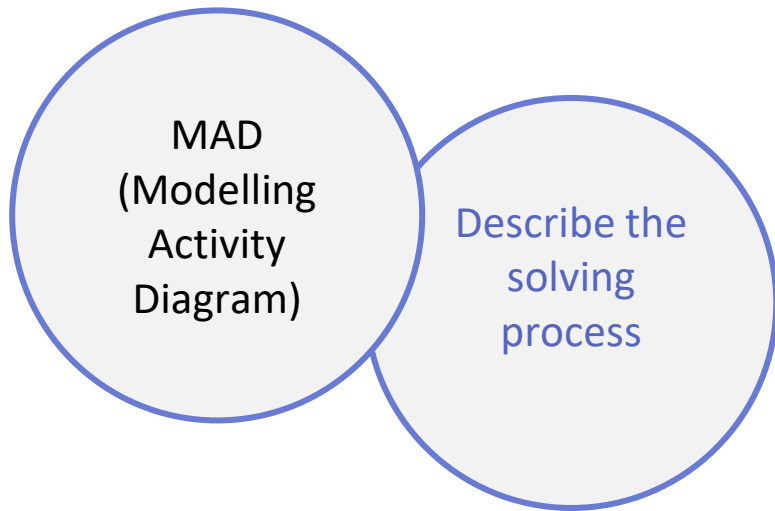
Describe the
solving
process

Characterize
mathematical
models

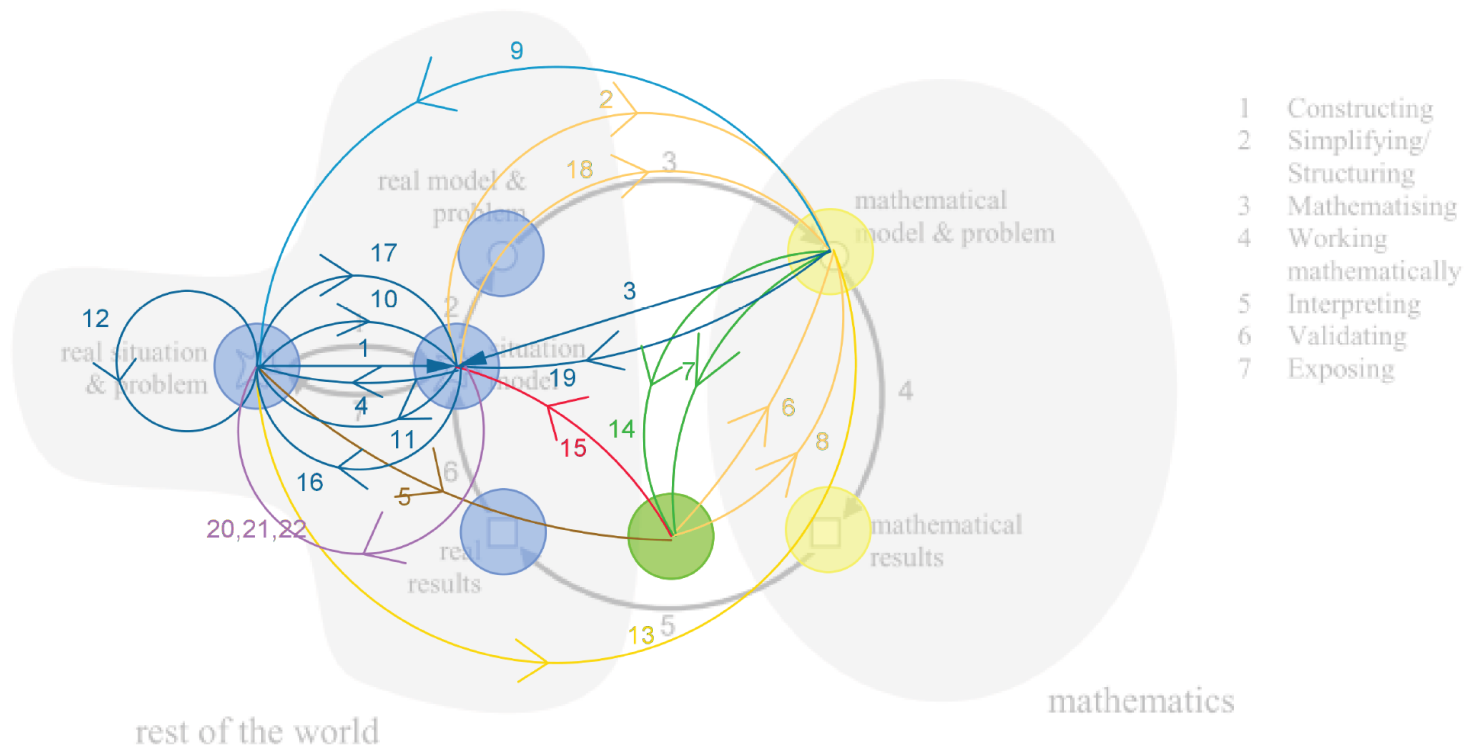
Model
Structure
Graph



ANALYSIS TOOLS

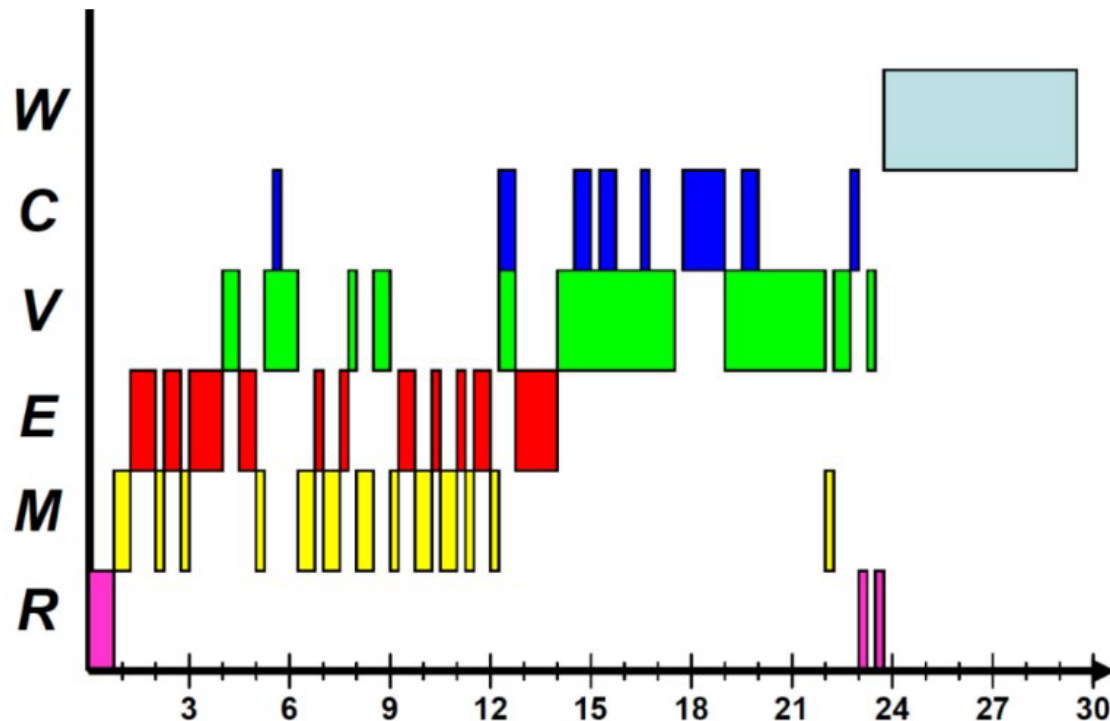


Modelling Cycle?



Aymerich, À., & Albarracín, L. (2016). Complejidad en el proceso de modelización de una tarea estadística. *Modelling in science education and learning* 9(1), 5-24.

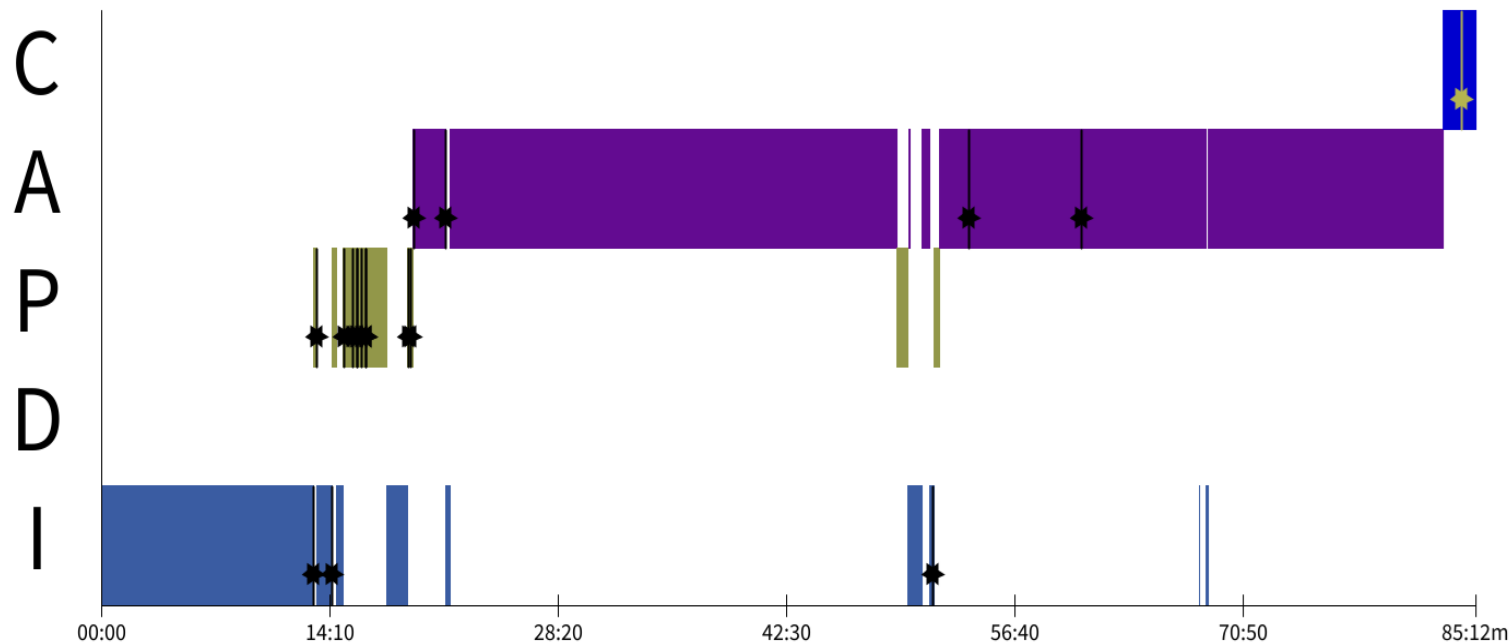
Modelling Activity Diagrams (MAD)



Ärlebäck, J. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Mathematics Enthusiast*, 6(3), 331-364.

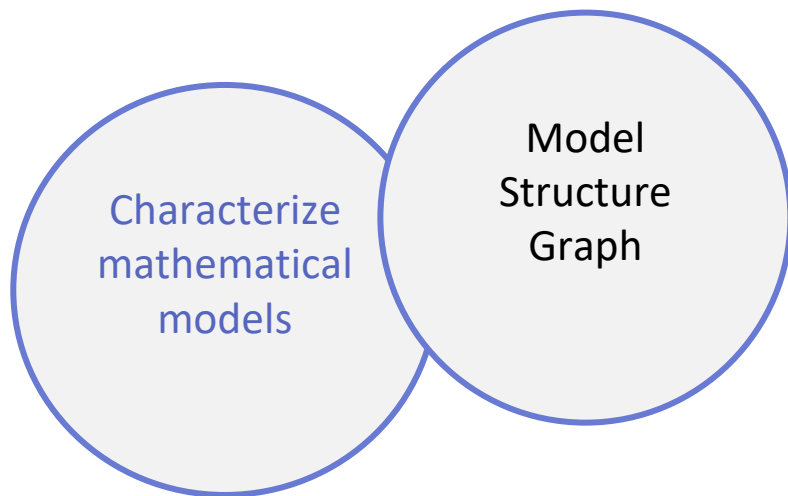
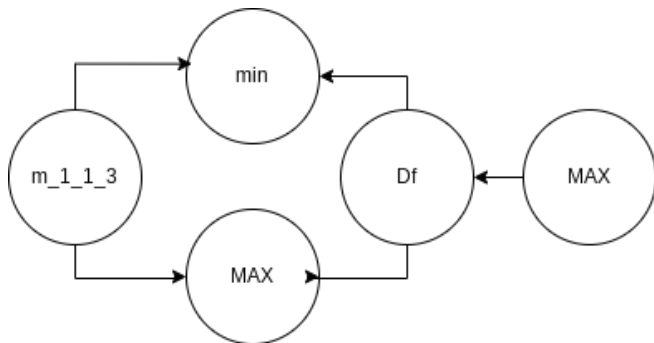
Adapted MAD categories

Interpreting (I)	They read, listen and look. They adapt the language to their vocabulary. They summarise. Compare with other similar situations. Connect with previous ideas. They look for information to better understand the statement.
Exploring data (D)	Visual exploration of data. Data curation. Exploratory calculations with data.
Planning (P)	They make a plan of action which may include mathematics or statistics contents. Modifications to the action plan.
Implementing the analysis (A)	Calculations, organization of data, generation of graphs, any activity that produces results.
Generating conclusions (C)	From the observation and interpretation of the data and the results of the analysis, generate conclusions and write them up.



Aymerich, À., & Albarracín, L. (2022). Modelización matemática en actividades estadísticas: Episodios clave para la generación de modelos. *Uniciencia*, 36(1), 264-279.

ANALYSIS TOOLS



Models are conceptual systems that generally tend to be expressed using a variety of interacting representational media, which may involve written symbols, spoken language, computer-based graphics, paper-based diagrams or graphs, or experience-based metaphors. Their purposes are to construct, describe or explain other system(s).

Models include both:

- (a) a **conceptual system** for describing or explaining the relevant mathematical objects, relations, actions, patterns, and regularities that are attributed to the problem-solving situation
- (b) **accompanying procedures** for generating useful constructions, manipulations, or predictions for achieving clearly recognized goals

Model Structure Graph v1.0

graf G1_g02

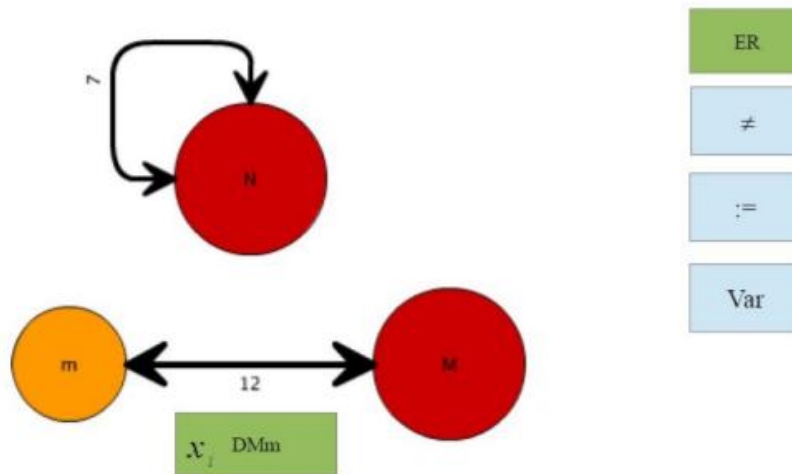
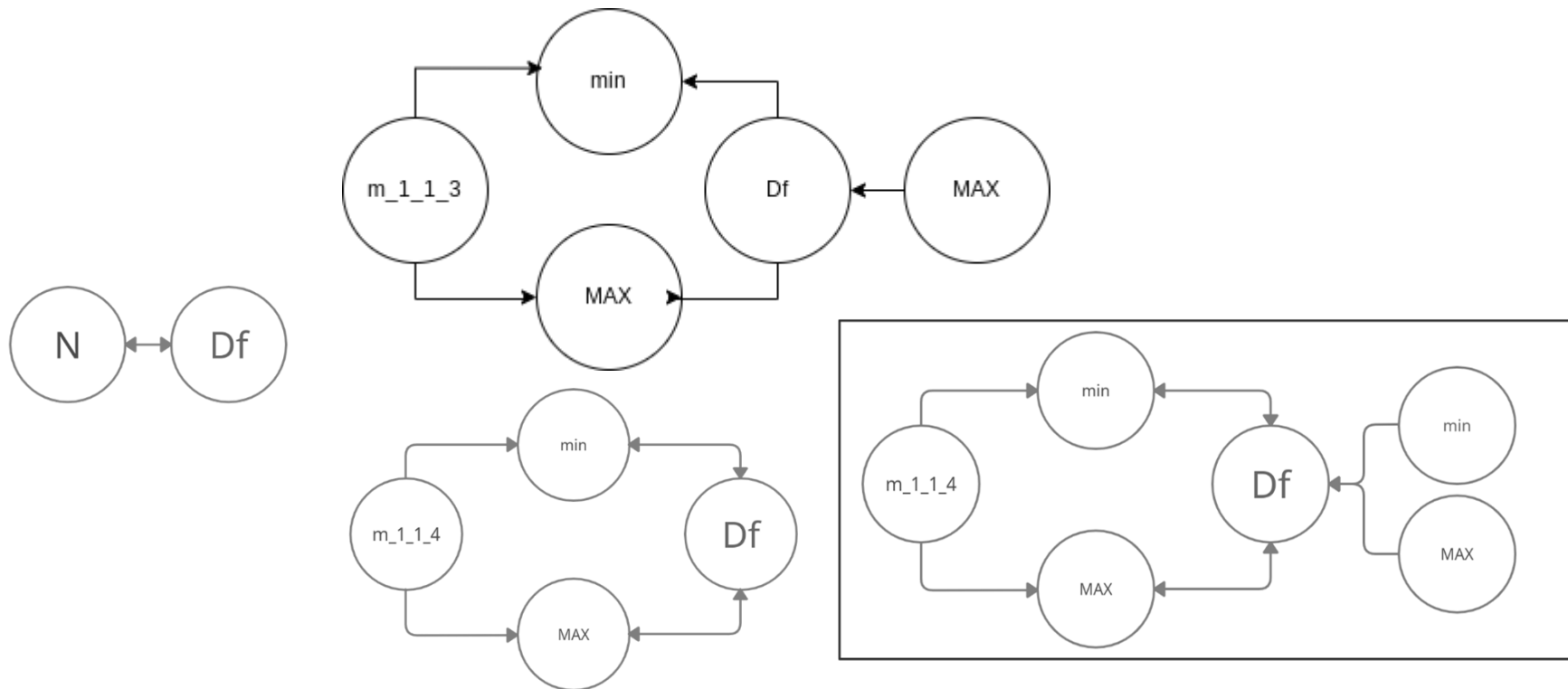


Figura 15: Graf G1_g02

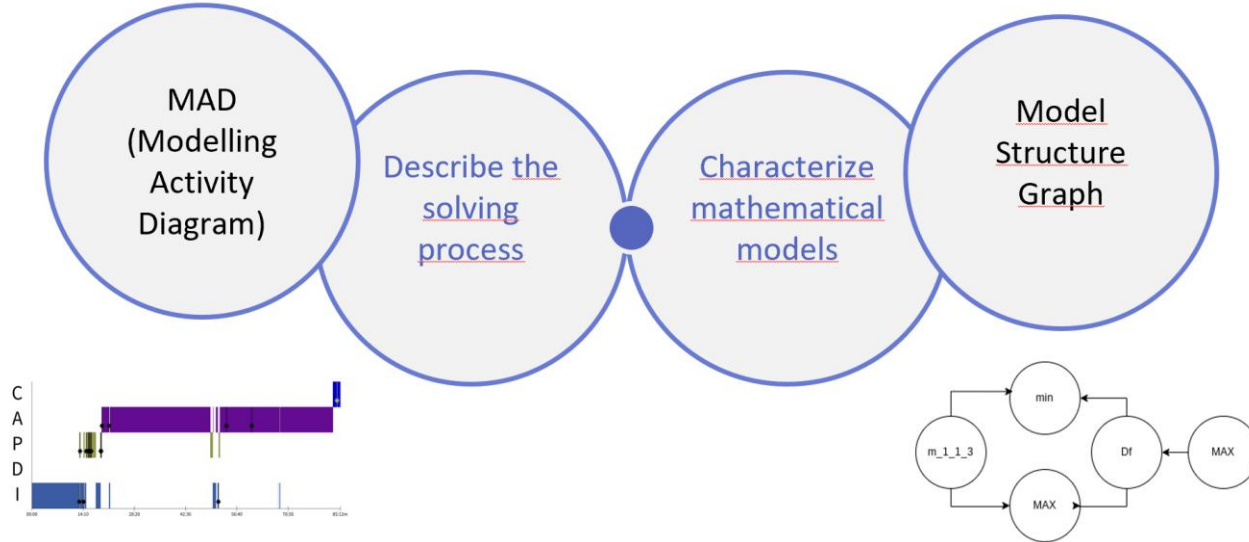
Aymerich, À., Gorgorió, N., & Albarracín, L. (2017). Modelling with statistical data: Characterisation of student models. *Mathematical Modelling and Applications: Crossing and Researching Boundaries in Mathematics Education*, 37-47.

Model Structure Graph v2.0



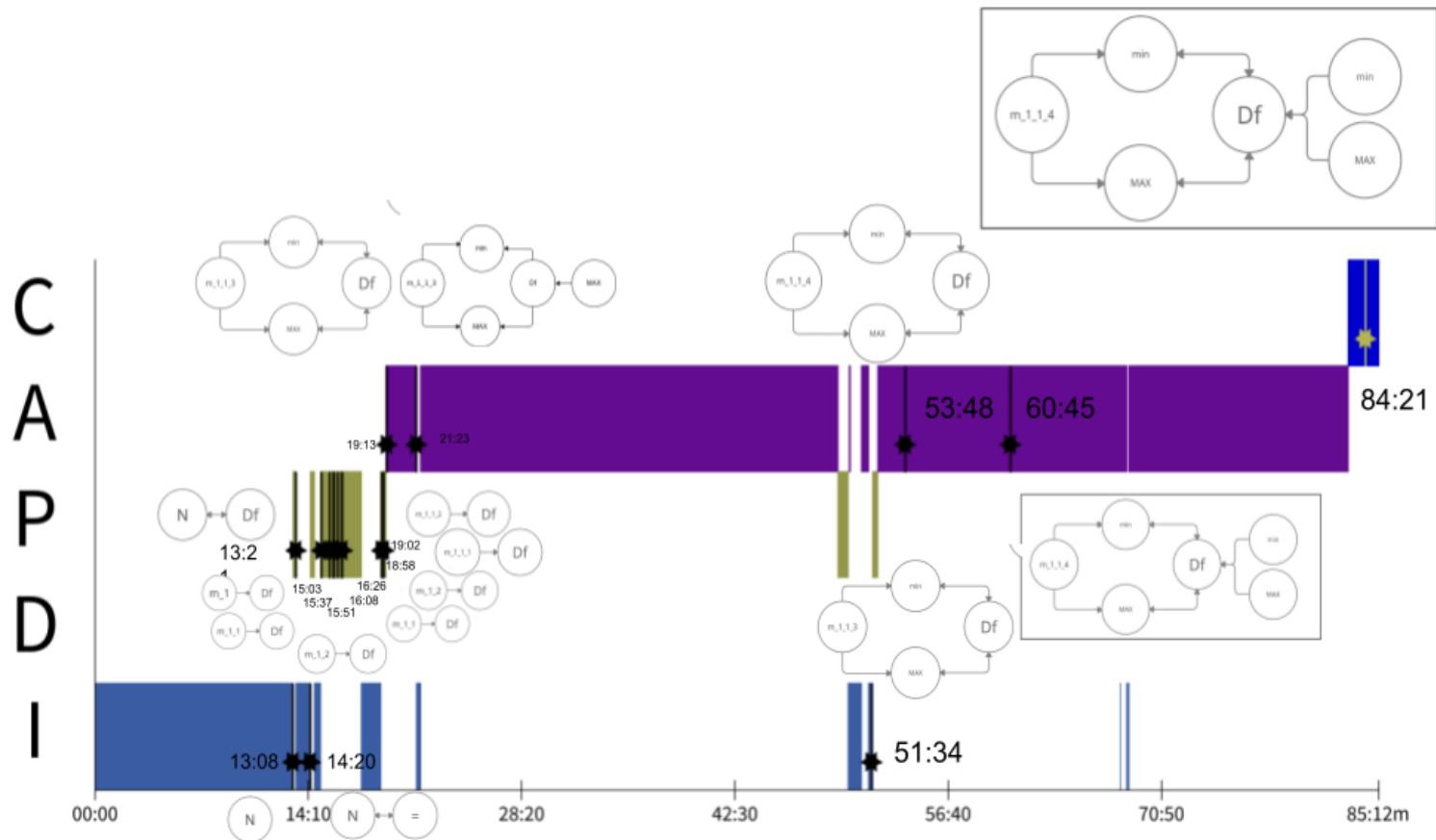
Aymerich, À., & Albarracín, L. (2021). Modelización matemática en actividades estadísticas: identificando elementos clave del diseño para promover la generación de modelos en el estudio de la variabilidad. *Quadrante*, 30(2), 179-199.

Combining the two analytical tools...



This slide is just designed to create some tension...

Combining the two analytical tools...



Preliminary results

1. These two combined analytical tools allow us monitoring model construction
 2. We identify three key elements in statistical tasks to promote mathematical models:
 - Complexity of the context/situation/phenomenon
 - Dealing with a big amount of data
 - Ambiguous wording (fair taxes)
- *What about data collection? And data cleaning?*

References

- Aymerich, À., & Albarracín, L. (2016). Complejidad en el proceso de modelización de una tarea estadística. *Modelling in science education and learning* 9(1), 5-24.
- Aymerich, À., & Albarracín, L. (2021). Modelización matemática en actividades estadísticas: identificando elementos clave del diseño para promover la generación de modelos en el estudio de la variabilidad. *Quadrante*, 30(2), 179-199.
- Aymerich, À., & Albarracín, L. (2022). Modelización matemática en actividades estadísticas: Episodios clave para la generación de modelos. *Uniciencia*, 36(1), 1-18.
- Aymerich, À., Gorgorió, N., & Albarracín, L. (2017). Modelling with statistical data: Characterisation of student models. *Mathematical Modelling and Applications: Crossing and Researching Boundaries in Mathematics Education*, 37-47.
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- Ärlebäck, J. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Mathematics Enthusiast*, 6(3), 331-364.
- Lesh, R. E., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Lawrence Erlbaum Associates Publishers.
- Lesh, R., Amit, M., & Schorr, R. Y. (1997). Using “real-life” problems to prompt students to construct statistical models for statistical reasoning. In I. Gal & J. Garfield (Eds.), *The assessment challenge in statistics education* (pp. 65-84). IOS Press.

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