QUANTUM FIELD THEORY

N-body theory vs. field theory. The rigid rod. The wave equation of the rod as a non relativistic field equation. Suggested homework: derive the wave equation and the lagrangean density for a string of constant tension.

Classical Field Theory. The stationary action principle. The Euler-Lagrange equations for continuous media. Suggested homework: derive the wave equations from the stationary action principle.

The Schrödinger equation as a wave equation in non-relativistic classical field theory. Suggested homework: show that $L = i\psi^*\partial_t\psi - \frac{1}{2m}\nabla\psi^* \cdot \nabla\psi$ leads to the familiar time-dependent Schrödinger equation.

Relativistic classical field theory. An example: the electromagnetic field. Suggested homework: Show that the wave equation for the electromagnetic 4-potential follows from the lagrangean density $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$.

The Klein-Gordon equation. Positive (negative) energy solutions. The failure of relativistic quantum mechanics (no probabilistic interpretation of wavefunctions; the Klein paradox). Suggested homework: find the lagrangean density that leads to the K-G equation. Show that

$$dw_k \equiv \frac{d^3k}{(2\pi)^3 2k_0}$$

is a Lorentz scalar.

The Schrödinger quantum field: a non-relativistic quantum field theory. Equal time field commutators. The field as a collection of quantum oscillators. Bosonic nature of excitations. Connection to non-relativistic N-body quantum mechanics. Anticommutators: a theory of fermions. Suggested homework: prove that the Hamiltonian for the Schrödinger theory reads,

$$H = \int d^3x\psi^+(\vec{x},t)\left(-\frac{\nabla^2}{2m}\right)\psi(\vec{x},t) = \int d^3k a_k^+ a_k \ .$$

The second quantized free Klein-Gordon field. Creation and annihilation operators. The Hamiltonian operator. Infinite energy of the vacuum. Normal ordering. Two-point functions and their properties. The Feynman propagator. The Feynman propagator as a Green's function for the K-G equation. Physical interpretation of propagators. Suggested homework: show by explicit calculation that the creation (annihilation) operators for the free Klein-Gordon field are time independent.

Scattering in the (non-relativistic) Schrödinger quantum field theory . "IN" and "OUT" states. The S matrix. Asymptotic condition. The LSZ reduction formula. Perturbation theory. The 2-point Green's function. The Born series for the transition amplitude. From the amplitude to the cross section. An example: Rutherford scattering.

Relativistic Quantum Field Theory for interacting fields. Introducing interactions with a few examples: the electromagnetic field with sources, Yukawa theory of nuclear interactions and the theory of two mutually coupled Klein-Gordon fields. Second quantization and the non-trivial action of operator fields on the vacuum. Suggested homework: i) derive Maxwell's equations from the lagrangean density

$$-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^{\mu}A_{\mu} \text{ and}$$

ii) find the explicit form of the Yukawa propagator in the static limit.

The asymptotic condition for interacting fields. Reduction formula for the transition amplitude of m incoming scalar particles into n outgoing scalar particles. The interacting n-point Green's function. Suggested homework: derive the LSZ formula step by step reducing out the m and n particles from the IN and OUT states, respectively.

Generating functional. A little bit of functional analysis. Definitions and rules for functional differentiation. Back to free fields: generating functional, Green's functions and Wick's theorem. Path integral representation of generating functionals. Suggested homework: proof Wick's theorem inductively.

An aside: Path Integral formulation of Quantum Mechanics for a single degree of freedom. Wave functions and propagators. The propagator amplitude as a sum over paths. Trivial extensions: the matrix element for a time ordered product of position operators. Suggested homework: derive the propagator function for a harmonic oscillator using the path integral method.

Path integrals in QM continued. Perturbing the system with an external source. A very special object: the ground state to ground state functional and how to get it.

The Path Integral formulation of Quantum Field Theory. From the Quantum Mechanics of one degree of freedom to the Quantum Mechanics of n degrees of freedom. The field theory limit: an infinite number of degrees of freedom. The quantum-mechanical sum over field configurations. The vacuum functional in the presence of an external source. The vacuum functional as a generating functional of n-point Green's functions.

A simple exercise: explicit evaluation of the path integral for the vacuum functional in the free Klein-Gordon field case.

Perturbation theory revisited. Z [J] in $\lambda\phi^4$. The 4-point Green's function to first order in perturbation theory. Connected and disconnected diagrams. Suggested homework: write down the 2-point Green's function to 2nd order perturbation theory.

Cross section for $\phi\phi \to \phi\phi$ scattering to lowest order in $\lambda\phi^4$ theory. Feynman rules in momentum space. Suggested homework: re-derive the cross section for $\phi\phi \to \phi\phi$ in the lab frame.

Short briefing on the Dirac equation and its solutions. Second quantization: field anti-commutators. The spin-statistics connection. The Feynman propagator for spinors.

The vacuum functional for free fermions ($Z_0^e[\eta,\overline{\eta}]$) . The Dirac current. Suggested homework: i) construct the Hamiltonian operator and the charge operator in terms of creation and annihilation operators. ii) show explicitly that the propagator $\langle 0|T(\psi_\alpha(x)\overline{\psi}_\beta(y))|0\rangle$ is a Green's function of the Dirac operator.

Quantum Electrodynamics (QED). Photons and gauge invariance. The photon propagator and the vacuum functional for free photons ($Z_0^\gamma[J]$). The interaction lagrangean for QED. The asymptotic condition and the reduction formula for QED. An example: the transition amplitude for Compton scattering. The vacuum functional for QED ($Z_{QED}[J,\eta,\overline{\eta}]$). $Z_{QED}[J,\eta,\overline{\eta}]$ in terms of the free Maxwell ($Z_0^\gamma[J]$) and free Dirac ($Z_0^e[\eta,\overline{\eta}]$) functionals. Application: the Green's function for Compton scattering to lowest order. Feynman rules for QED in momentum space. Suggested homework: i) write down the amplitude for Compton scattering to $O(e^2)$ and ii) try to obtain the Klein-Nishina formula for the Compton cross section.